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Quantum Field Theory and On-Shell Constructibility

We are in the midst of a revolution in quantum field theory (QFT) that is teaching us better ways to calculate observables, and changing how we think about the conceptual underpinnings of our theories.

At its core, this innovation is due to a change in formalism inspired by a need to more efficiently calculate cross-sections for collider experiments. For many years, such calculations all proceeded in more-or-less the same way: after figuring out the process to calculate, one imagined all the possible virtual processes that could convert the incoming particles to the outgoing ones, then calculated an amplitude for each possibility using Feynman rules, and squared the sum of all of these amplitudes. This process is tedious and difficult, having intermediate steps with often dozens or hundreds of different pieces which individually do not respect all the symmetries of the problem, before dramatic cancellations in the final step produce simple, compact answers. This alone was a good sign that there should be an easier way to calculate which obeyed all of the constraints manifestly throughout the process.

The key insight is to do away with virtual particles, and switch to variables which themselves encode that the incoming and outgoing particles are 'on-shell' - they obey $p_\mu p^\mu = m^2$, with m the particle mass. This is known as the spinor-helicity formalism and involves a factorization of Lorentz vectors into right-handed and left-handed spinors for on-shell, massless particles. This is even more useful than it has a right to be because in applications such as Large Hadron Collider scattering at 13 TeV center-of-mass energy, even protons are massless to very good approximation.

While this simple change of variables does make calculations much easier, it has also lead to a paradigm shift in our understanding of QFT. A large reason that the standard calculational framework was so difficult was due to gauge redundancy - the fact that massless vector fields don't fit into full representations of the Lorentz group quite correctly. Thus forcing gauge theories into the familiar language in which we knew how to formulate manifestly local, unitary, well-defined theories - that of a Lagrangian built out of various Lorentz representations combined into Lorentz scalars - leaves you with an infinite degeneracy of ways to write your theory. This has often been thought of as a *symmetry*, and much work has been expended on the idea of this redundancy was important and fundamental.

However, all of that gauge redundancy *vanishes* for particles which are on-shell. So if we can formulate QFT manifestly in terms of on-shell particles, then we can entirely bypass the Lagrangian formulation of QFT, and eschew all of the difficulties - both theoretical and calculational - that come along with it. The existence of an understanding of QFT in which gauge redundancy never enters shows that we need not think of it as a deep, fundamental symmetry, and provides insight into how gauge theories fit into our larger picture of QFT as a whole.

One of the crucial pieces of the effort to formulate QFT without Lagrangians is the program of constructibility - showing that you can construct arbitrary n-particle amplitudes using on-shell methods. This is obviously fundamental to our understanding, as it tells us how to calculate observables. A major development here was the discovery of on-shell recursion [1]. It turns out that the requirements of locality and unitarity demand that higher-particle amplitudes factorize into products of fewer-particle *fully on-shell* amplitudes near their singular points, and these singularities must occur where some sum of on-shell momenta has zero Lorentzian norm. This allows the rewriting of n-particle amplitudes in terms of sums of on-shell fewer-particle amplitudes, broadly akin to that seen in Feynman diagrams but for the purely on-shell program.

The constructibility story has now been completed for theories of massless particles where all amplitudes can be related to three-particle amplitudes. This includes cases where the Lagrangian description would yield a four-particle Feynman vertex (e.g. Yang-Mills), but which is secretly related to three-particle vertices (e.g. by gauge invariance), and so the theory admits a cubic description. It is understood how three-particle amplitudes are fixed purely from the helicities of the particles involved, and recently in [2] all such interactions which lead to consistent, local, unitary field theories were classified using the recursive generation of higher-particle amplitudes. Thus we now know that this class of theories does admit a fully on-shell description. However, surprisingly little effort has been made in extending this work.

This summer we will push on the program of on-shell constructibility toward the eventual goal of showing the constructibility of all local, unitary quantum field theories.

One avenue is to extend constructibility to theories with fundamental four-particle interactions. For example, scalar ϕ^4 theory, in which the coefficient of the four-particle interaction may be set independently of any three-particle amplitudes. Such interactions also appear often in effective field theories, and in QFTs in fewer dimensions. This analysis is more difficult than the three-particle case, but there is a growing tool-set of on-shell techniques for us to apply. Furthermore, this spring there is a KITP program on on-shell methods, so we have a great opportunity to learn even more from the many experts who are now here. A four-particle constructibility argument (or counterargument) would be of great value to the community.

Another branch of inquiry is to apply these arguments to three-particle amplitudes for massive particles. Multiple proposals for how to generally deal with massive particles using spinor-helicity have appeared. One construction uses massless spinor-helicity in six dimensions and dimensional reduction to cleverly handle the massive four-dimensional case, while another idea is to apply a spinor-helicity formalism to polarization vectors instead of momenta, and so eliminate the ambiguities related to polarization reference directions which appear in the current framework. This work has not yet been applied to constructibility, and so would also represent significant scientific progress.

The constructibility program is essential to showing that it is possible to reformulate QFT in a manifestly on-shell manner. Our work this summer will progress this program and contribute to the revolution in our conception of QFT itself. Not any undergraduate could work on this, but Aidan has had much preparation in field theory and in on-shell methods, having gone through the graduate series in both quantum field theory and gauge theory. I have been working with him for the past year on a different project which is now in its finishing stages, and know how hard-working and inventive he is. I have no doubt that together, and with help from Nathaniel and inspiration from the KITP program, we will be able to make significant progress this summer.

References

[1] R. Britto, F. Cachazo, B. Feng, and E. Witten, "Direct proof of the tree-level scattering amplitude recursion relation in yang-mills theory," *Phys. Rev. Lett.*, vol. 94, p. 181602, May 2005.

[2] D. A. McGady and L. Rodina, "Higher-spin massless s matrices in four dimensions," *Phys. Rev. D*, vol. 90, p. 084048, Oct 2014.