(Yet Another) Single-Observer Measure for Eternal Inflation

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The Question:

In a complicated landscape containing many dS ($\Lambda > 0$), AdS ($\Lambda < 0$) and flat ($\Lambda = 0$) vacua, what is the *measure* associated to each vacuum?

- Do you count the *Number of* or the *Volume occupied by* a given vacuum?
- Is the measure time-slice (i.e. diffeomorphism) invariant?
- Is the measure independent of initial conditions?

Strategy: motivated by Holography and Complementarity, construct a measure only from quantities visible to a *single time-like observer*.

The Setup:

Dynamics on the landscape is complicated. Different vacua will expand (or contract) at different rates, and can transition among one another.

If the typical decay time of an inflating vacuum is long compared to its Hubble time, this leads to eternal inflation:

- As the inflating vacuum expands, small bubbles of other vacua are continually nucleated, "populating" the landscape.
- At late times this leads to an infinite number of bubbles, distributed over all length scales.

$\Longrightarrow$ an *Inflationary Fractal*.

The Measure:

Consider a time-like observer who ends up in a bubble of flat space, called the reference bubble.

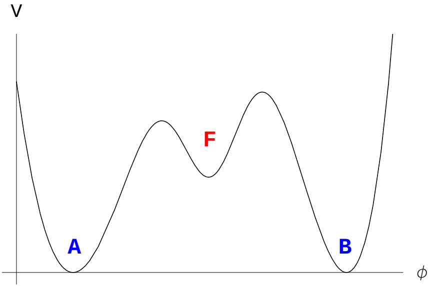
This bubble will collide with an infinite number of other bubbles of all different types.

- So the landscape is observable. Our observer can look in his past light cone and count the number of vacua colliding with his own bubble.
- He will see the same fractal structure of bubbles distributed over all scales – an imprint of the inflationary fractal on the night sky.

In a certain approximation this measure reduces to a co-moving measure of Vilenkin et al. But, as we will see, many of the details differ.

The Inflationary Fractal

Consider an inflating false vacuum F that can tunnel into two flat vacua A or B.

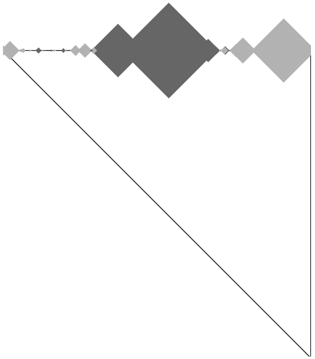


The dimensionless decay rate is

$$\epsilon_{F \to A} \sim H_F^{-4} e^{-(S_E(Instanton) - S_E(dS))}$$

Causal Structure

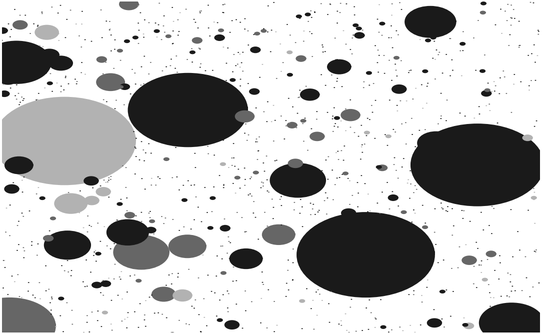
Both types of bubbles are continually nucleated, forming a fractal structure on $\mathcal{I}^+$ (Guth & Weinberg):



Almost all geodesic observers end up in a flat A or B bubble.

$$Dim(\text{inflating set}) \sim 3 - \left(\frac{4\pi}{3}\right)\epsilon + \mathcal{O}(\epsilon^2)$$

A typical slice of co-moving volume:



Each flat bubble collides with an infinite number of other flat bubbles, forming a cluster containing an infinite number of connected flat bubbles.

For small ϵ, the bubbles do not percolate. There are an infinite number of disconnected clusters, each of which contains an infinite number of bubbles.

Global Measure I

There are an infinite number of bubbles. To define a measure, we need to regulate this infinity.

If we count bubbles nucleated before a particular time t_{cut} (i.e. smaller than a particular proper volume V_{cut}) the answer depends on the choice of time slice.

Instead, impose a cutoff on *co-moving* volume rather than *proper* volume (Garriga, Schwartz-Perlov, Vilenkin & Winitzki).

This gives answers which are reparameterization invariant. This is because the co-moving volume occupied by a bubble goes to a *constant* on $\mathcal{I}^+$ rather than diverging.

Global Measure II

The co-moving volumes $\mathbf{f} = (f_a, f_i)$ $(a = 1, ..., N_{dS},\ i = 1, ..., N_{flat})$ obey linear rate equations

$$\dot{\mathbf{f}} = M \cdot \mathbf{f}, \qquad \mathbf{f}(t) \sim \mathbf{f}^0 + \mathbf{s}e^{-qt} + ...$$

where $\mathbf{f^0}$ is any zero eigenvector ($f_a^0 = 0$) and $\mathbf{s}$ is the first non-trivial eigenvector.

Since $\mathbf{f^0}$ is not unique, a co-moving volume weighted measure depends on initial conditions.

However, the *Number* of vacua larger than given comoving volume

$$N_i(t) \sim \sum_a H_a^3 \epsilon_{a \to i} e^{(3-q)t} s_b + ...$$

is determined uniquely by the transition matrix M.

Single-Observer Measure

Consider only the information accessible to the timelike observer in our reference bubble.

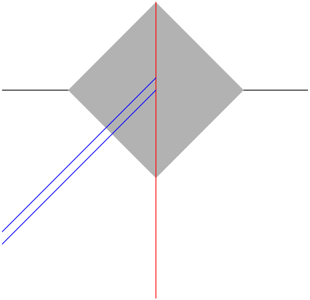
The reference observer can assemble statistics on all the bubbles in his cluster. These statistics are (sometimes) representative of the global fractal statistics.

The worldline time of the census taker provides a natural cutoff on the number of bubbles.

This is a cutoff on *co-moving* size, rather than proper size. So this measure is naturally related to the co-moving global measure.

Derivation I

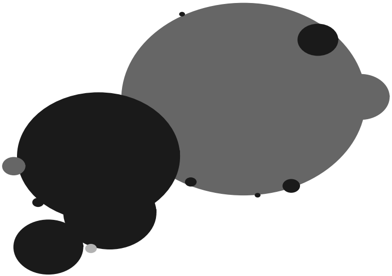
In the thin wall approximation we can calculate this measure explicitly, by developing rate equations on the boundary S^2 of the reference bubble:



These are identical to the comoving rate equations.

Derivation II

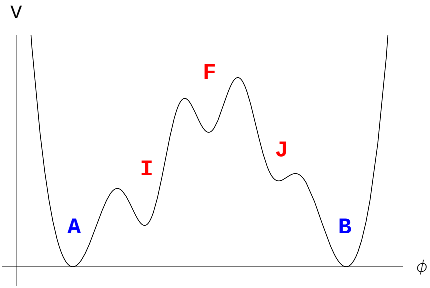
Actually, we need rate equations on the edge of a cluster, not just on the sphere:



This gives deviations of $\mathcal{O}(\epsilon^2)$.

An Example

For potentials with multiple flat and dS vacua, such as



the measure favors the progeny of long lived, small Λ vacua:

$$p_A >> p_B.$$

Thick Domain Walls

When the domain walls are not thin, the reference observer's statistics aren't necessarily representative of the global statistics.

For example, if a domain wall between two vacua goes through a region of moduli space where the potential is very flat

$$\frac{V'}{V}, \quad \frac{V''}{V^2} << 1$$

then the domain wall will inflate.

An observer in one of these vacua will never see a vacuum of the other type in his past light cone $\implies$ final state dependence.

AdS Vacua

So far, we've ignored AdS ($\Lambda < 0$) vacua, which crunch. These singularities may collide with our reference bubbles and destroy the census taker.

When $T = |\Lambda|$ AdS-crunch singularities are hidden behind horizons (Freivogel, Horowitz and Shenker). They are no worse that the spacelike singularities of a black hole.

So our census taker, in addition to seeing a distribution of flat bubbles colliding with his own, will see a pattern of black spots on the night sky – horizons hiding AdS singularities.

When $T > |\Lambda|$ the domain walls accelerate away from the census taker, and are unobservable.

dS Reference Bubble

What if the reference bubble A is de Sitter?

The observer can only make a finite number of observations:

- after a few e-foldings, collisions with other bubbles will be causally inaccessible.

The observer counts vacua which are nucleated sufficiently soon after the nucleation of the reference bubble:

$$\epsilon_{F\to B}^{-1} << H_A^{-1}.$$

If H_A^{-1} is shorter than all decay times, he observes no vacua except his own.

In this limit, our measure seems to reduce to the single observer measure of Bousso.

Conclusions

- A measure based on the observables of a single reference observer is diffeomorphism invariant and independent of initial conditions.
- Roughly equivalent to a co-moving volume measure, but details differ once interesting features of moduli potential are accounted for.
- In particular, it appears to have some *final* state dependence.
- Can such measures can lead to phenomenologically viable predictions?
- How should we fold in observer considerations?