

• Superconductivity

(1) Condensation into state:

not occupied / occupied pairs of electrons of oppos. momentum (time-reversed)

(2) All pairs have same free parameter δ

$$\Psi = (\mu + \nu e^{i\delta} \sum_{\mathbf{k}} c_{\mathbf{k}}^+ c_{-\mathbf{k}}^+) |0\rangle$$

(4) Energy Gap Δ ($\sim 2kT_c$) for breaking of pair \rightarrow robust state

(3) S. current prop. to $\nabla\phi$; as any quantum state

(5) For tunnel junct; \rightarrow Show Pct's I-V

$$I \sim I_0 \sin \delta \quad (I_0 \propto A; \text{thick barrier})$$

(A s. current, I with $v=0$)

$$V = \frac{\Phi_0}{2\pi} \dot{\delta} = \frac{\hbar}{2e} \dot{\delta}$$

Later equation like Faraday's law

$$V = \dot{\Phi}; \text{ so } \Phi = \left(\frac{\hbar}{2e} \delta\right)$$

Can map δ with B field (Flux).

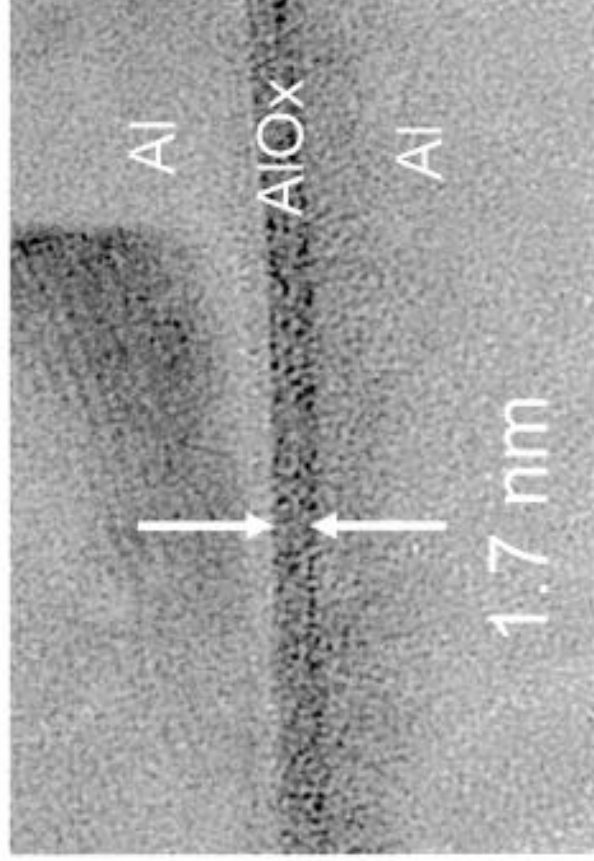
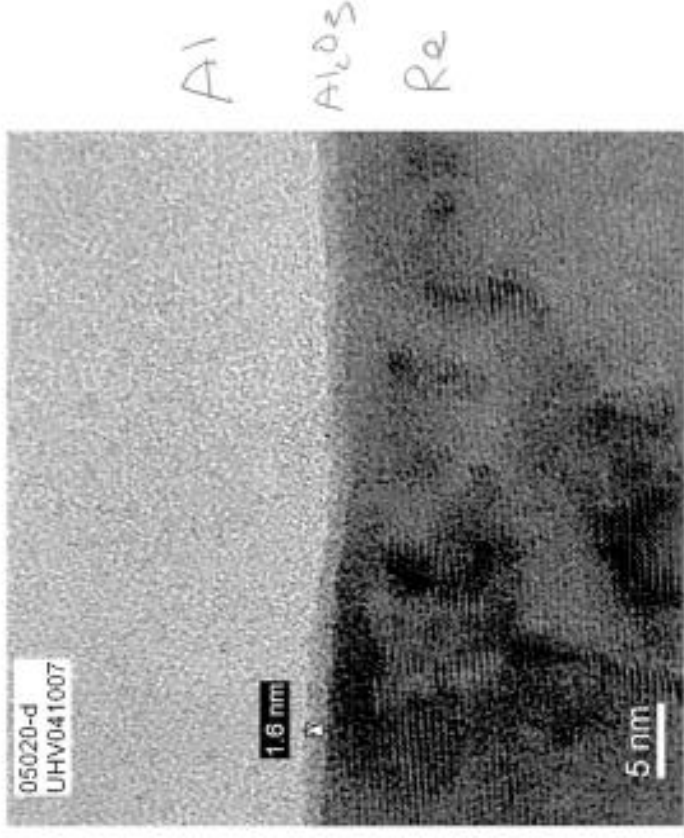
(6) In a loop, δ is 2π periodic, so see Device behavior periodic in Field; with applied Flux

$$\Delta B \cdot A = n(\Phi_0 = \hbar/2e) \quad \text{quantized flux from } I \text{ flow in loop.}$$

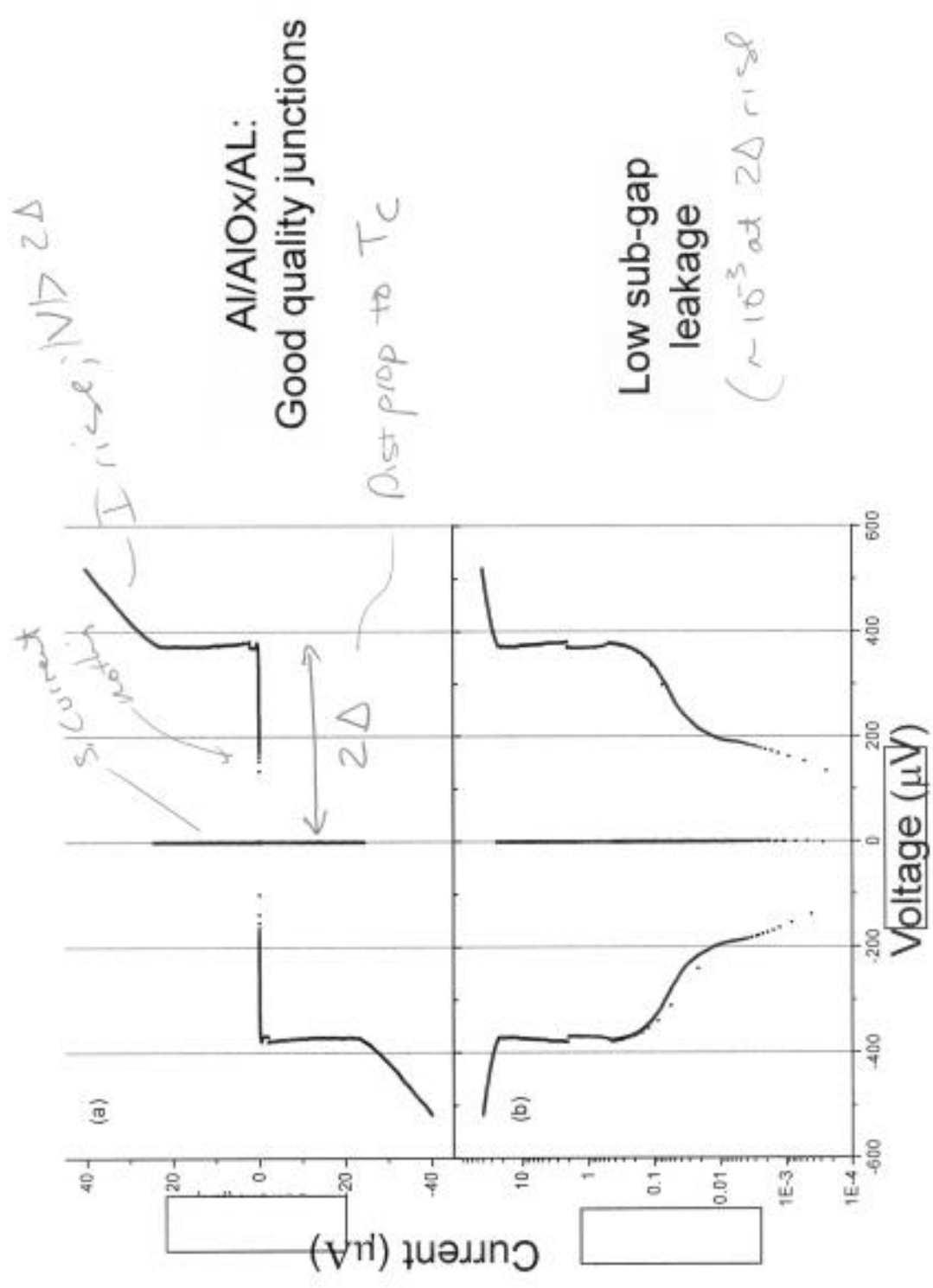
In tunnel junct. See S Current;

No I if $V < 2\Delta$ because for e's to tunnel, need to break CP's into e's $\rightarrow 2\Delta$ min.

Tunnel Junctions



Tunnel Junctions



Additional Discussion

(1) Understand phase parameter
in S.C.

(BCS_n) explicit focus of δ
"spin model"

(2) Josephson Effect

Proper 2nd order calculation of effect.

→ Understand where comes from (Not in textbooks)

→ Why S.C. has No resistance
→ An actual calculation!

(3) Exact Calc. of Josephson Effect

(Arbitrary coupling) via Andreev Bound states.

- S.C. Gap protects from Dissipation
- Understand subgap current

All with simple (as possible) Mathematics.
(Learn in 1st year of Grad. School).

understanding of the junction physics is thus needed so that nonideal behavior can be more readily identified, understood, and eliminated. Although we will not discuss specific imperfections of junctions in this paper, we want to describe a clear and precise model of the Josephson junction that can give an intuitive understanding of the Josephson effect. This is especially needed since textbooks do not typically derive the Josephson effect from a microscopic viewpoint. As standard calculations use only perturbation theory, we will also need to introduce an exact description of the Josephson effect via the mesoscopic theory of quasiparticle bound-states.

The outline of the paper is as follows. We first describe in Sec. 2 the nonlinear Josephson inductance. In Sec. 3 we discuss the three types of qubit circuits, and show how these circuits use this nonlinearity in unique manners. We then give a brief derivation of the BCS theory in Sec. 4, highlighting the appearance of the macroscopic phase parameter. The Josephson equations are derived in Sec. 5 using standard first and second order perturbation theory that describe quasiparticle and Cooper-pair tunneling. An exact calculation of the Josephson effect then follows in Sec. 6 using the quasiparticle bound-state theory. Section 7 expands upon this theory and describes quasiparticle excitations as transitions from the ground to excited bound states from nonadiabatic changes in the bias. Although quasiparticle current is typically calculated only for a constant DC voltage, the advantage to this approach is seen in Sec. 8, where we qualitatively describe quasiparticle tunneling with AC voltage excitations, as appropriate for the qubit state. This section describes how the Josephson qubit is typically insensitive to quasiparticle damping, even to the extent that a phase qubit can be constructed from microbridge junctions.

2 The Nonlinear Josephson Inductance

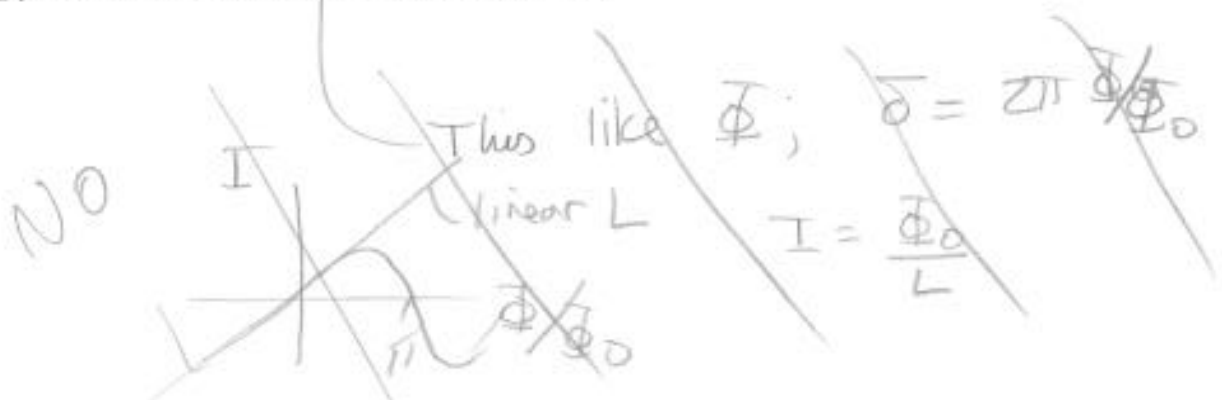
A Josephson tunnel junction is formed by separating two superconducting electrodes with an insulator thin enough so that electrons can quantum-mechanically tunnel through the barrier, as illustrated in Fig. 1. The Josephson effect describes the supercurrent I_J that flows through the junction according to the classical equations

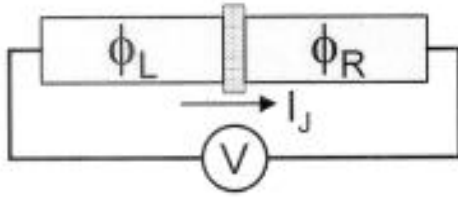
$$I_J = I_0 \sin \delta \quad (2.1a)$$

$$V = \frac{\Phi_0}{2\pi} \frac{d\delta}{dt} \quad (2.1b)$$

where $\Phi_0 = h/2e$ is the superconducting flux quantum, I_0 is the critical-current parameter of the junction, and $\delta = \phi_L - \phi_R$ and V are respectively the superconducting phase difference and voltage across the junction. The

② I-V relations,
will derive
later.





① thin oxide barrier

Fig. 1. Schematic diagram of a Josephson junction connected to a bias voltage V . The Josephson current is given by $I_J = I_0 \sin \delta$, where $\delta = \phi_L - \phi_R$ is the difference in the superconducting phase across the junction.

dynamical behavior of these two equations can be understood by first differentiating Eq. 2.1a and replacing $d\delta/dt$ with V according to Eq. 2.1b

$$\frac{dI_J}{dt} = I_0 \cos \delta \frac{2\pi}{\Phi_0} V. \quad (2.2)$$

With dI_J/dt proportional to V , this equation describes an inductor. By defining a Josephson inductance L_J according to the conventional definition $V = L_J dI_J/dt$, one finds

$$L_J = \frac{\Phi_0}{2\pi I_0 \cos \delta}. \quad (2.3a)$$

The $1/\cos \delta$ term reveals that this inductance is nonlinear. It becomes large as $\delta \rightarrow \pi/2$, and is negative for $\pi/2 < \delta < 3\pi/2$. The inductance at zero bias is $L_{J0} = \Phi_0/2\pi I_0$.

An inductance describes an energy-conserving circuit element. The energy stored in the junction is given by

$$U_J = \int I_J V dt \quad (2.4a)$$

$$= \int I_0 \sin \delta \frac{\Phi_0}{2\pi} \frac{d\delta}{dt} dt \quad (2.4b)$$

$$= \frac{I_0 \Phi_0}{2\pi} \int \sin \delta d\delta \quad (2.4c)$$

$$= -\frac{I_0 \Phi_0}{2\pi} \cos \delta. \quad (2.4d)$$

This calculation of energy can be generalized for other nondissipative circuit elements. For example, a similar calculation for a current bias gives $U_{\text{bias}} = -(I\Phi_0/2\pi)\delta$. Conversely, if a circuit element has an energy $U(\delta)$, then the current-phase relationship of the element, analogous to Eq. 2.1a, is

$$I_J(\delta) = \frac{2\pi}{\Phi_0} \frac{\partial U(\delta)}{\partial \delta}. \quad (2.5)$$

"Differential Inductance"
 Non linear L_J with $\cos \delta$ ($\delta \rightarrow \pi/2 @ I \rightarrow I_0$)

Energy stored in L_J

Useful relations used later

A generalized Josephson inductance can be also be found from the second derivative of U ,

$$\frac{1}{L_J} = \left(\frac{2\pi}{\Phi_0} \right)^2 \frac{\partial^2 U(\delta)}{\partial \delta^2}. \quad (2.6)$$

The classical and quantum behavior of a particular circuit is described by a Hamiltonian, which of course depends on the exact circuit configuration. The procedure for writing down a Hamiltonian for an arbitrary circuit has been described in detail in a prior publication [11]. The general form of the Hamiltonian for the Josephson effect is $H_J = U_J$.

3 Phase, Flux, and Charge Qubits

A Josephson qubit can be understood as a nonlinear resonator formed from the Josephson inductance and its junction capacitance. Nonlinearity is crucial because the system has many energy levels, but the operating space of the qubit must be restricted to only the two lowest states. The system is effectively a two-state system [12] only if the frequency ω_{10} that drives transitions between the qubit states $0 \longleftrightarrow 1$ is different from the frequency ω_{21} for transitions $1 \longleftrightarrow 2$.

We review here three different ways that these nonlinear resonators can be made, and which are named as phase, flux, or charge qubits.

The circuit for the phase-qubit circuit is drawn in Fig. 2(a). Its Hamiltonian is

$$H = \frac{1}{2C} \hat{Q}^2 - \frac{I_0 \Phi_0}{2\pi} \cos \hat{\delta} - \frac{I \Phi_0}{2\pi} \hat{\delta}, \quad (3.1)$$

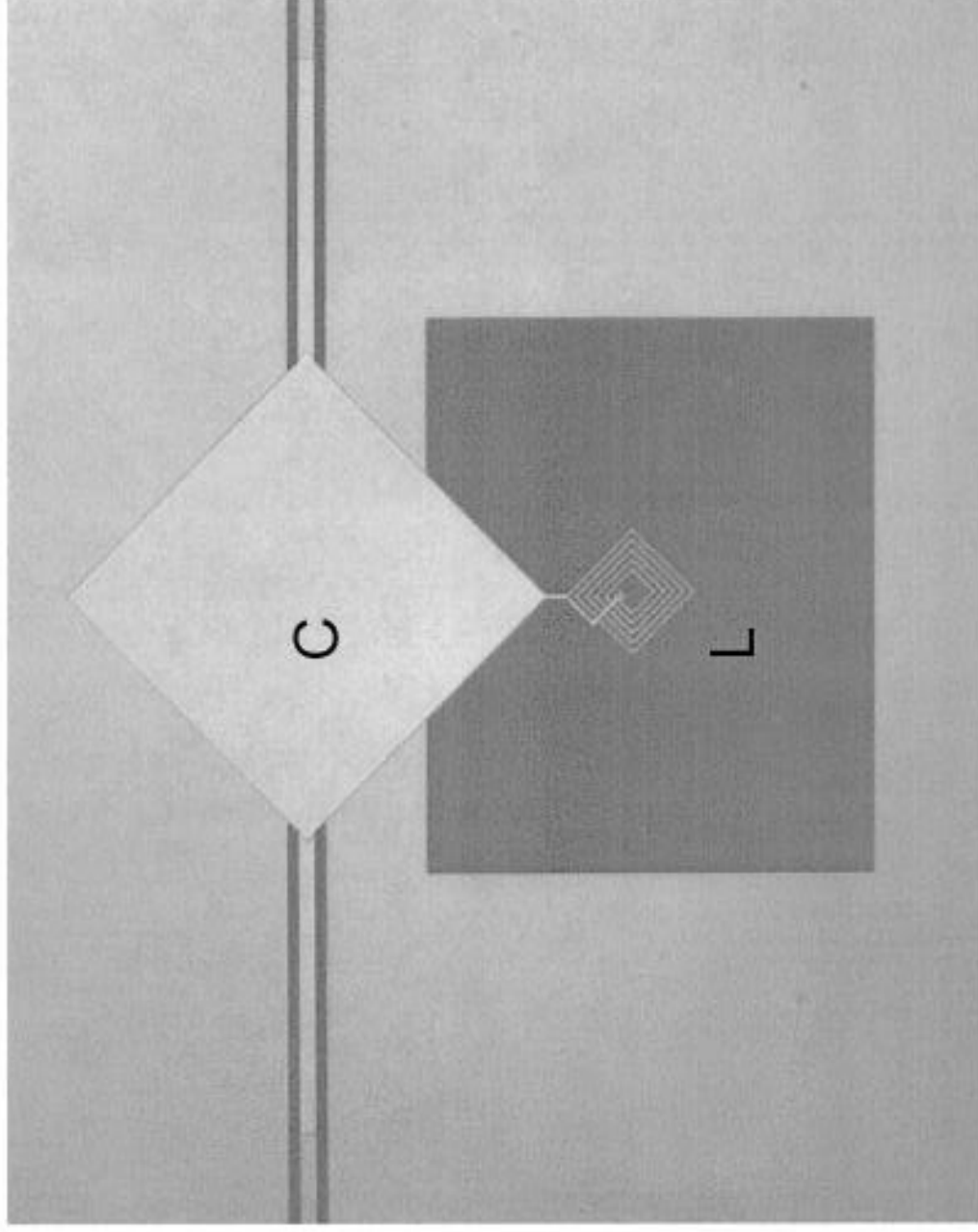
where C is the capacitance of the tunnel junction. A similar circuit is drawn for the flux-qubit circuit in Fig. 2(b), and its Hamiltonian is

$$H = \frac{1}{2C} \hat{Q}^2 - \frac{I_0 \Phi_0}{2\pi} \cos \hat{\delta} + \frac{1}{2L} \left(\Phi - \frac{\Phi_0}{2\pi} \hat{\delta} \right)^2. \quad (3.2)$$

The charge qubit has a Hamiltonian similar to that in Eq. 3.1, and is described elsewhere in this publication. Here we have explicitly used notation appropriate for a quantum description, with operators charge \hat{Q} and phase difference $\hat{\delta}$ that obey a commutation relationship $[\hat{\delta}, \hat{Q}] = 2ei$. Note that the phase and flux qubit Hamiltonians are equivalent for $L \rightarrow \infty$ and $I = \Phi/L$, which corresponds to a current bias created from an inductor with infinite impedance.

The commutation relationship between $\hat{\delta}$ and \hat{Q} imply that these quantities must be described by a wavefunction. The characteristic widths of this wavefunction are controlled by the energy scales of the system, the charging energy of the junction $E_C = e^2/2C$ and the Josephson energy $E_J = I_0 \Phi_0/2\pi$. When the energy of the junction dominates, $E_J \gg E_C$,

LC Microwave Resonator

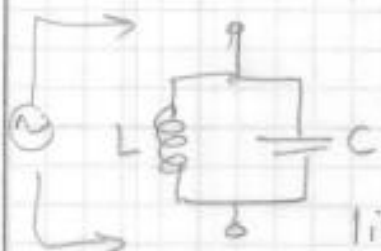


Circuit Atoms

- Most all qubits formed by considering 2 states ($|0\rangle + |1\rangle$ label) of more complex "atom"

- To understand Josephson, first look at LC osc.

$\omega_{res} = \frac{1}{\sqrt{LC}} ; Z_{res} = \omega_r L = \sqrt{\frac{L}{C}}$
 More useful class. descrip?



$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \quad [\hat{Q}, \hat{\Phi}] = i\hbar$$

like $[H_{HO} = \frac{p^2}{2m} + \frac{1}{2}k\hat{x}^2 \quad [\hat{x}, \hat{p}] = i\hbar]$

- Solve in standard way for Harm. Osc. ($\hat{p} \rightarrow \hat{p}; m \rightarrow C$
 $\hat{x} \rightarrow \hat{Q}; k \rightarrow \frac{1}{L}$)

$$a^+ |n\rangle = \sqrt{n+1} |n+1\rangle \quad H = \left(a^+ a + \frac{1}{2} \right) \hbar \omega$$

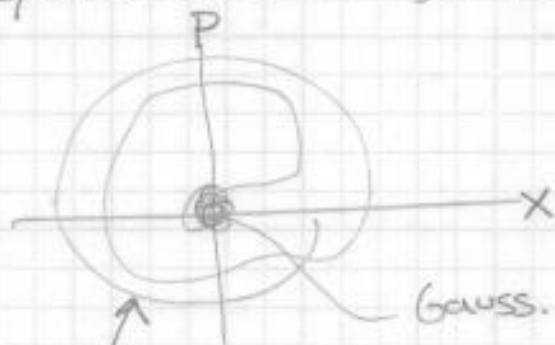
$$a |n\rangle = \sqrt{n} |n-1\rangle \quad \omega = \frac{1}{\sqrt{LC}} ; \text{osc freq}$$



- Problem(s) for qubit

(1) All energy level sep's same, can't define $|0\rangle |1\rangle$ and keep in that subspace.

(2) Response (to driving) is linear / classical



Gauss. 0nd state

Response to drive is 0nd state moves in a classical trajectory.

- Need Non-linearity for Qubit states



Circuit H / energies

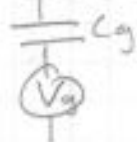
(1) (Paper by Devoret describes mathem. to determ. H , depends on wiring).

(2) Not spend time, but can understand basic energy terms:

(3) $C \parallel \rightarrow \frac{\Phi^2}{2C}$

$L \text{ (inductor)} \rightarrow \frac{\Phi^2}{2L}$

(4) biases:



$$(\Phi - CgVg)^2$$

offset to charge (new equilib. value)



$$\frac{(\Phi - \Phi_0)^2}{2L}$$

(5) J-Junction

$E = -E_J \cos \delta$ in δ -space
 (Josephson energy $E_J = \frac{I_0 \Phi_0}{2e}$)

(6) J-eg + Far Law $[\hat{\Phi}, \hat{Q}] = i\hbar$

$$[\frac{\hbar}{2e} \hat{\delta}, e \hat{q}] = i\hbar$$

$$[\hat{\delta}, \hat{q}] = 2i \quad \text{dim'less units}$$

$$\hat{q} = FT \hat{\delta} \text{ potential; } \frac{e^{i\hat{\delta}} + e^{-i\hat{\delta}}}{2}; \text{ chg. } \hat{q} \text{ by 2}$$

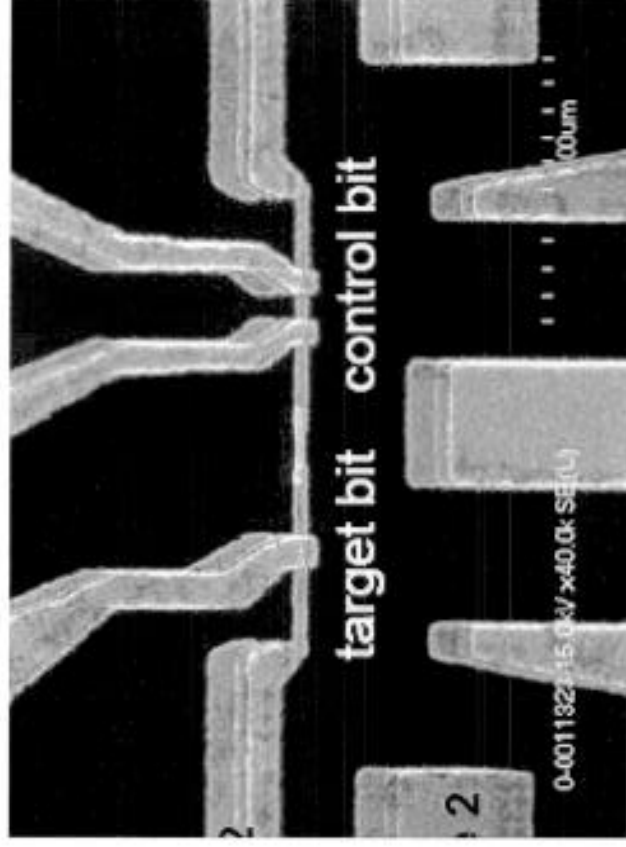
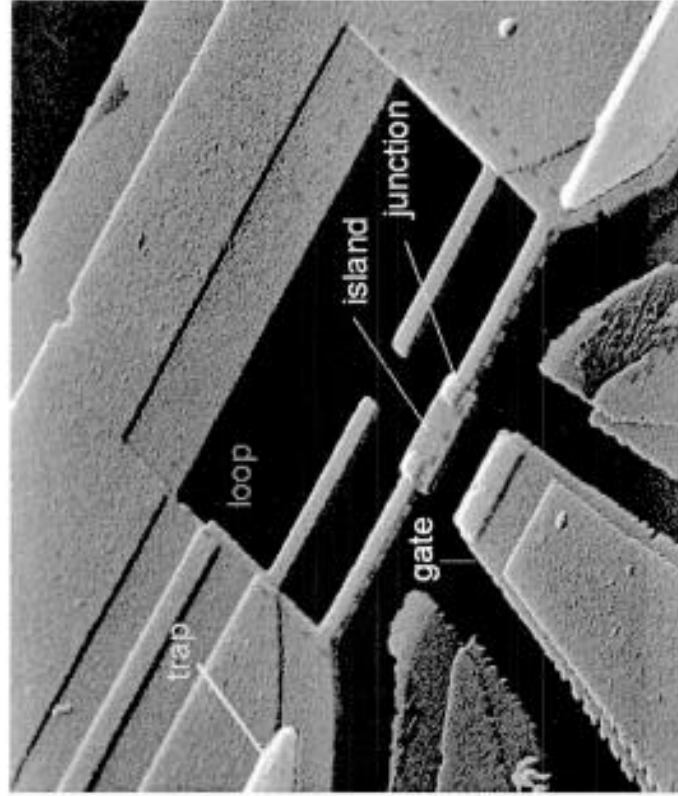
As expect, J-effect is trans 1 pair

$$E = -\frac{E_J}{2} [|q\rangle \langle q+2| + |q+2\rangle \langle q|] \quad \hat{q}\text{-space}$$

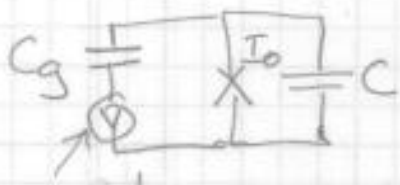
+ chg. by 1 CoP.

Charge Qubit

Made by evap Al thr a shadow mask
stencil made from e-beam resist.



Charge Qubit



Junct: Josephson I_0 + Cap. C

V_g control parameter

C energy

Josephson (CP-transfer) Energy

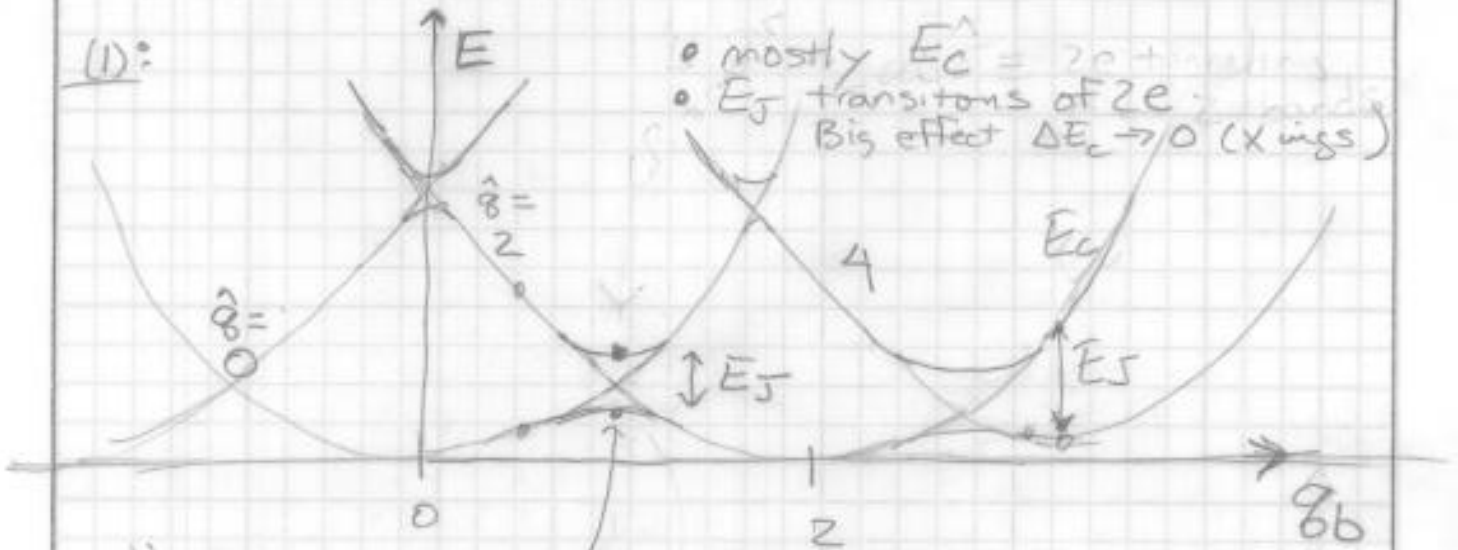
$$H = \frac{e^2}{2(C+C_g)} \left(\hat{q} - \frac{C_g V}{e} \right)^2 - \frac{I_0 \Phi_0}{2\pi} \cos \hat{\phi}$$

$$= E_c \left(\hat{q} - q_b \right)^2 - \frac{E_J}{2} \left[|\hat{q}\rangle \langle \hat{q}+2| + |\hat{q}+2\rangle \langle \hat{q}| \right]$$

To solve:

- (1) $E_c \gg E_J$; \hat{q} more classical coord. better basis (discrete values $\pm 2e$)
- $E_J \gg E_c$; $\hat{\phi}$ is better basis

(1):



Near Resonance ; basis $|\hat{q}=0\rangle, |\hat{q}=2\rangle$

$$H \approx 2E_c(1-q_b) \frac{\sigma_z}{2} - \frac{E_J}{2} \sigma_x$$

$$E = \pm \sqrt{\left(\frac{E_c}{2} \right)^2 + \left(\frac{E_J}{2} \right)^2}$$

"Avoided Crossing"

$$|0\rangle = \frac{1}{\sqrt{2}} (|\hat{q}=0\rangle + |\hat{q}=2\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}} (|\hat{q}=0\rangle - |\hat{q}=2\rangle)$$

Charge Noise Problem:

All fine, except nature not so kind!

Example of limitations to thy, from exp'l observation!

g_b random, sample-to-sample (have to tune, ok)

(Why so large?
Calc. later) \rightarrow g_b fluct's over time by 10^2 to 10^3 (charge noise)

\hookrightarrow Fluct's $E_{i0}(t)$, random Z oper, ϕ noise
 $\sim \pi$ s coherence time.

Big problem! (I orig. thought S.C. qubits
never would work)

(1) Solution (Saday):

Operate at $g_b = 1$, where $\frac{\partial E_{i0}}{\partial g_b} = 0$;

Insens. to noise (1st order, 2nd order signif!)

(2) Also, make band flatter

E_J big ; $E_J > E_C$ useful limit!

This is param's for more modern design

(can't really use this \hat{q} basis well)

since ψ is superp of many $|\hat{q}=0\rangle$,

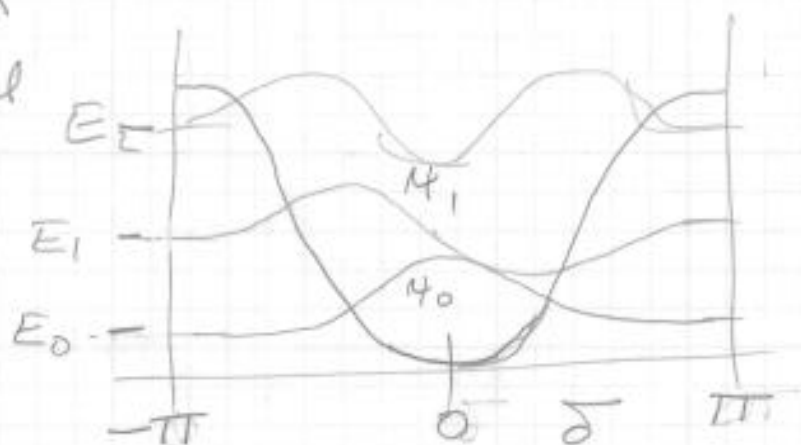
$|\hat{q}=2\rangle, |\hat{q}=4\rangle, \dots$ states)

Solution for $E_J > E_C$ ($\hat{\delta}$ basis)

(1) Charge quantiz. g : Periodic $\Psi(\hat{\delta} + 2\pi) = e^{-i2\pi g/2e} \Psi(\hat{\delta})$
 $\hat{\delta}$ from $-\pi$ to π .

(2) Solve S.E. in $\cos \hat{\delta}$ potential

(H.O.-like states, but non-linear)



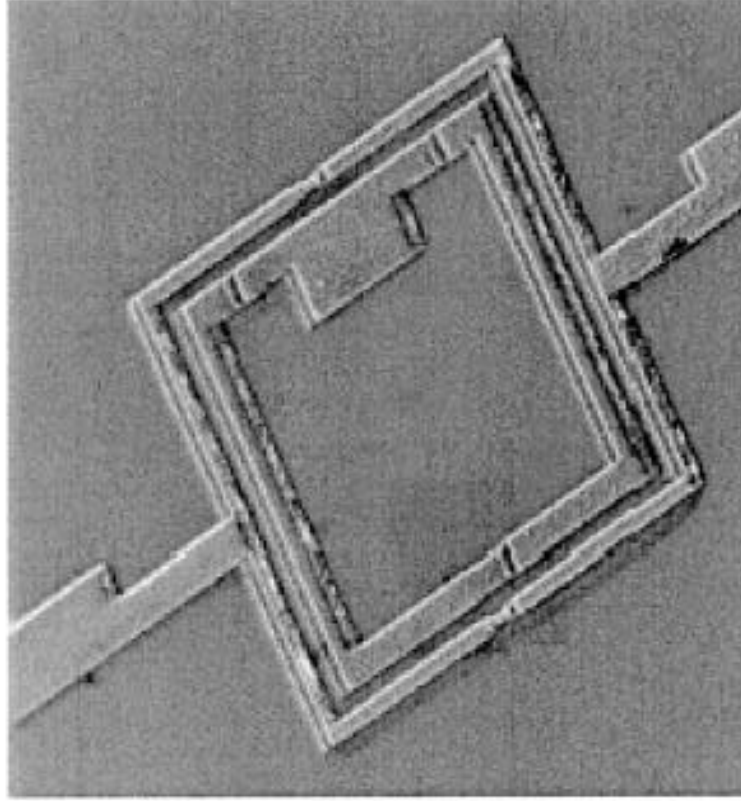
(3) See non-linear from non quad cos potential.

(4) Make $E_J \gtrsim 5E_C$ so Ψ is small at $0, 2\pi$, then small change with charge g_B

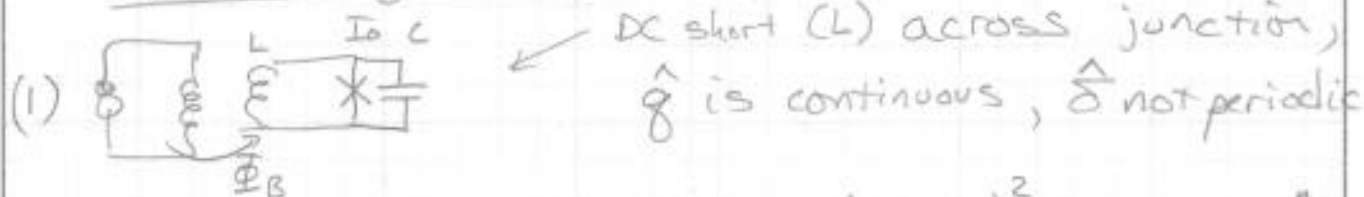
(5) Possible to get exp. small charge noise, but reasonable non-linear $\leftrightarrow z/0 \leftrightarrow$
 (Transmon circuit from Yale)

(If g does not affect the charge qubit, why still call it that?)

Flux Qubits



FLUX qubits



(2)
$$H = E_C \hat{q}^2 + \frac{(\Phi_B - \frac{\hat{\delta} \Phi_0}{2\pi})^2}{2L} - E_J \cos \hat{\delta}$$

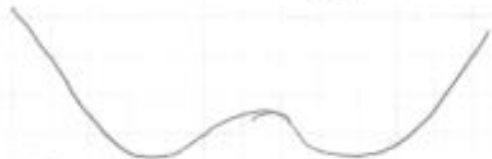
Parab. + cosine pot



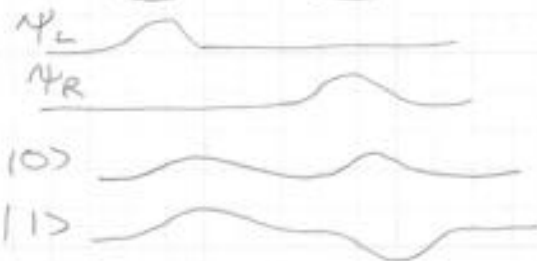
(3) =



(4) Idea: Make quartic potential, tunneling between 2 diff Φ states



tunnel \downarrow



(5) Why interesting?

E_{10} low because determ by tunneling

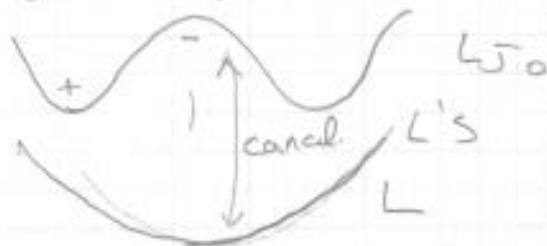
good \rightarrow E_{21} high, SO VERY nonlinear potential
forget about other states!

(6) How get highly non-linear (quartic) potential?

Cancel $-Lj_0 \rightarrow$ bias at

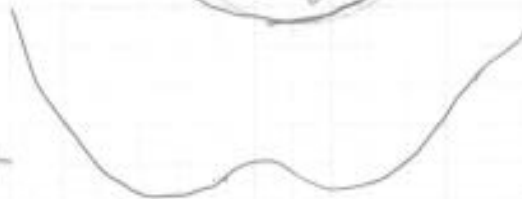
hard \rightarrow with L $\frac{\Phi_0}{2}$

(careful) design with



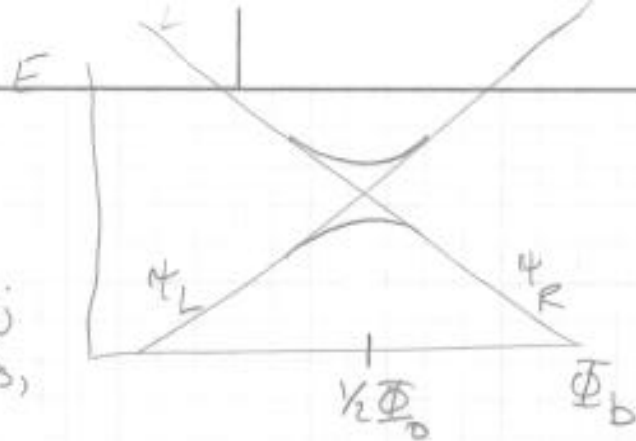
$$L \gtrsim Lj_0$$

\rightarrow Make C small so tunnels fast (wide ψ 's)



(7) In basis of $\psi_L, \psi_R \rightarrow$

Tunneling mixes states;
avoided crossing E_s ,
determ by param's
hard \rightarrow (expon. depend)



$$H = \alpha \frac{\Phi_0^2}{2L} \left(\frac{\Phi_0}{\Phi_0} - \frac{1}{2} \right) \sigma_z - \frac{E_s}{2} \sigma_x$$

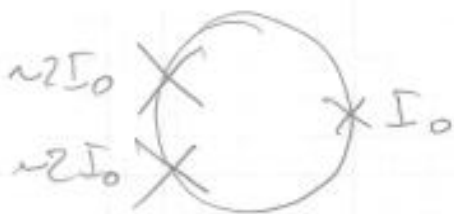
(ψ_L, ψ_R) basis

α, E_s calculated; depends on parameters

tunneling

(8) Nature: Flux noise small; $\sim 10^5 \Phi_0$, but gives decoher \sim ns if operate away from deg. pt. So operate at $\Phi_0 = \frac{1}{2} \Phi_0$.

(9) Real device; Hard to match $L, L_{50} = \frac{\Phi_0}{2\pi I_0}$
Solve with device design



\uparrow 2 juncts; $\frac{1}{2}$ induct; $\approx L_{50}$

• Good because I_0 's all scale with oxid. changes

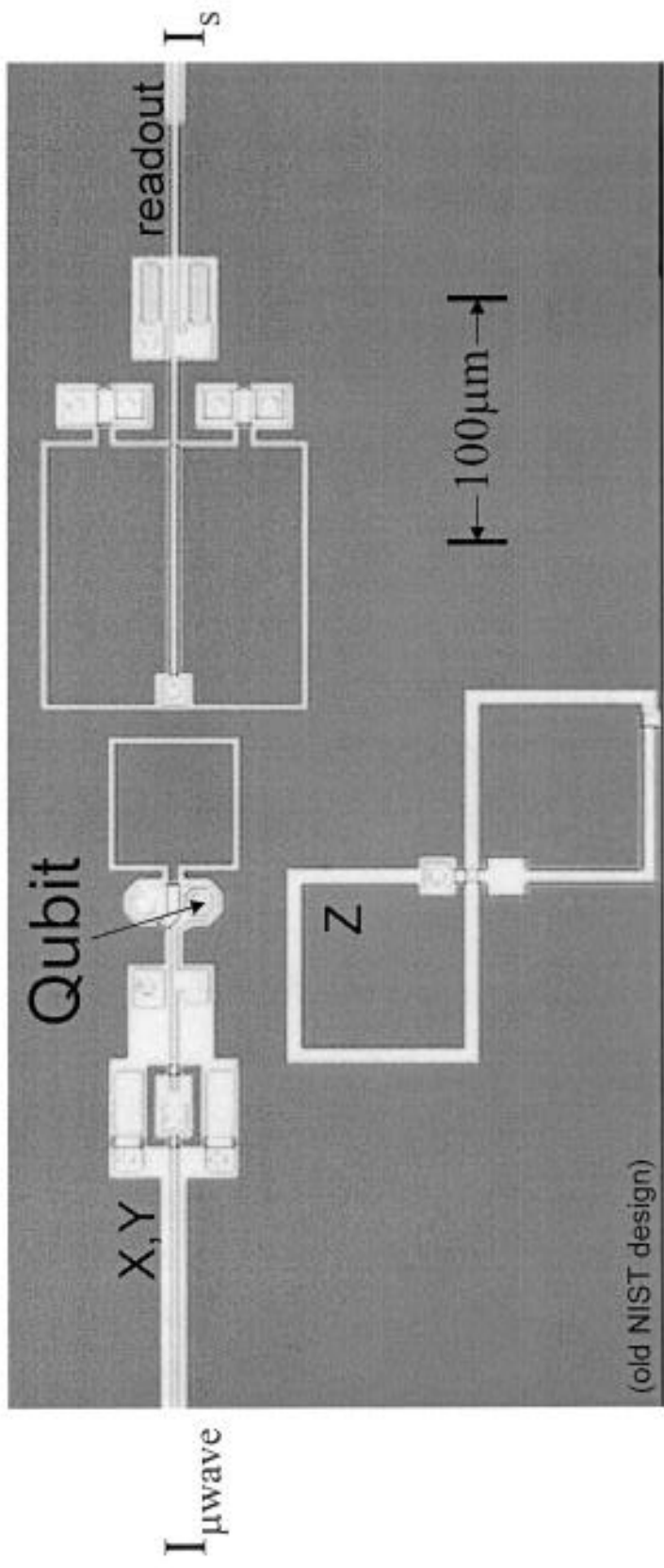
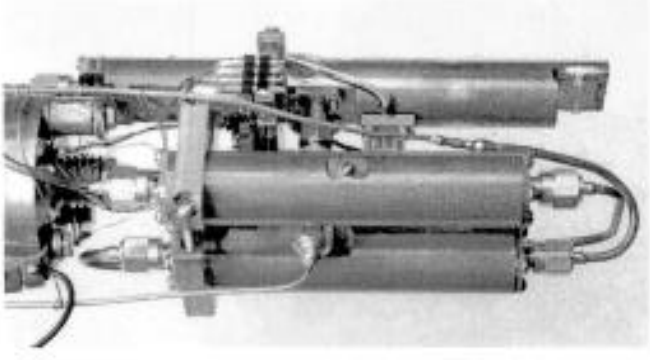
• In practice, solve Q.M. of entire system,

Φ qubit mode dominates

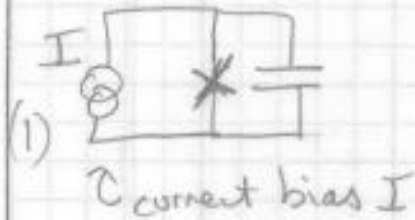
$2I_0 \rightarrow \sim 1.6 I_0$ for best design.

(Don't make C's too small or will see charge noise effects!)

Phase Qubits



Phase Qubits



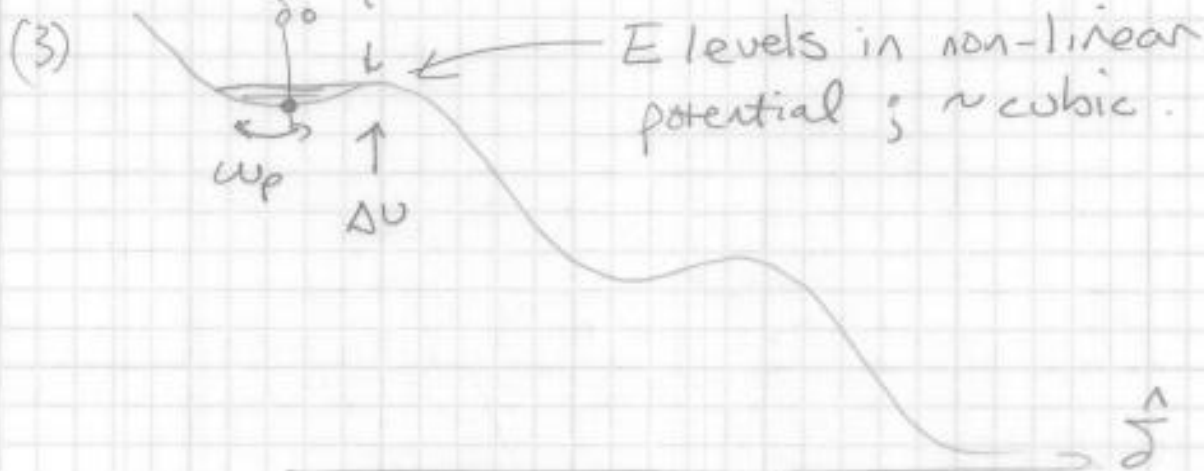
Large area junctions
 $I \rightarrow I_0$; $\sin \delta \rightarrow 1$ $\frac{1}{\cos \delta} \rightarrow \infty$
 for large non-linearity
 $\hat{\delta}, \hat{q}$ contin. var's (I gives orb charge)

(2)
$$H = \frac{\hat{q}^2}{2C} - \frac{I_0 \Phi_0}{2\pi} \cos \delta - \frac{I \Phi_0}{2\pi} \delta$$

C junct bias

$$\left[\int I V dt = \int \frac{I \Phi_0}{2\pi} \frac{d\delta}{dt} dt = \frac{I \Phi_0}{2\pi} \delta \right]$$

cosine potential + tilted by bias current



(4)
$$\omega_p = \frac{1}{\sqrt{L_j C}} = \frac{1}{\sqrt{L_0 C}} \sqrt{\cos \delta_0}$$

$I = I_0 \sin \delta_0$
 $\sin^2 \delta_0 = (I/I_0)^2$

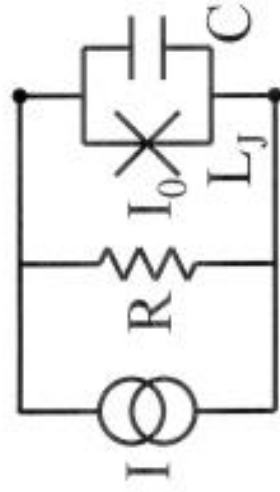
$= \frac{1}{\sqrt{L_0 C}} \left[1 - (I/I_0)^2 \right]^{1/4}$ ← slows with I

(5)
$$\Delta U \approx \frac{4\sqrt{2}}{3} \frac{I_0 \Phi_0}{2\pi} \left(1 + I/I_0 \right)^{3/2}$$
 ← fast with I

$\omega_p, \Delta U$ changes with I → tunable.

(6) No φ, Φ noise, so no degen pt fine (expl, more complicated)
 Nature!

Qubit: Nonlinear LC resonator



$$I = I_0 \sin \delta$$

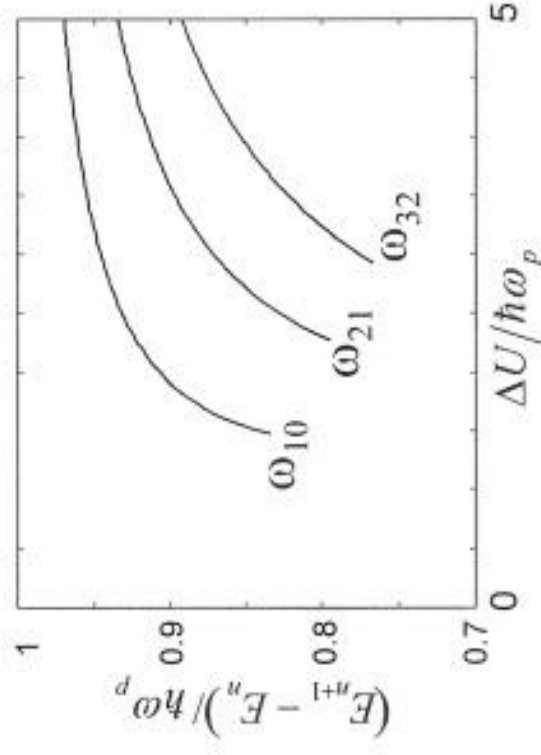
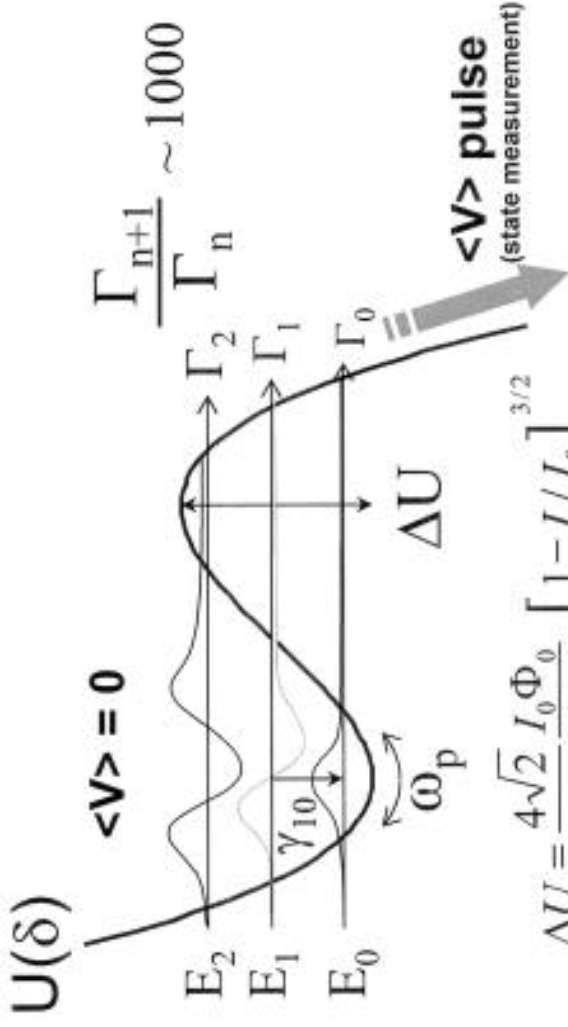
$$V = \frac{\Phi_0}{2\pi} \dot{\delta}$$

$$\dot{I}_J = I_0 \cos \delta \dot{\delta}$$

$$\equiv (1/L_J) V$$

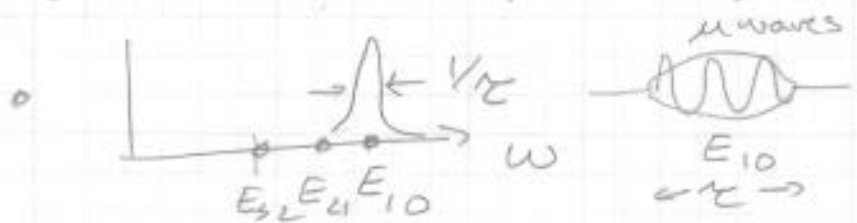
$$L_J = \Phi_0 / 2\pi I_0 \cos \delta$$

nonlinear inductor



- 1: Tunable well (with I)
- 2: Transitions non-degenerate
- 3: Tunneling from top wells
- 4: Lifetime from R

(7) Small Non-linearity ($\sim 5\%$)



See Pict

- Use $\sim 5-10$ ns μ wave pulses
- Practically, ok since
 - a) Hard to accurate & cheap shape faster pulses.
 - b) Rot. Wave Approx breaks down faster pulses (Logic errors then)

(8) Qubit control ; $I + \Delta I$
in $|0\rangle, |1\rangle$ of eigen states


$$H = \begin{pmatrix} 0 & 0 \\ 0 & E_{10} \end{pmatrix} - \frac{\Phi_0}{2I} \Delta I \begin{pmatrix} \langle 0 | \hat{\sigma}^x | 0 \rangle & \langle 0 | \hat{\sigma}^x | 1 \rangle \\ \langle 1 | \hat{\sigma}^x | 0 \rangle & \langle 1 | \hat{\sigma}^x | 1 \rangle \end{pmatrix}$$

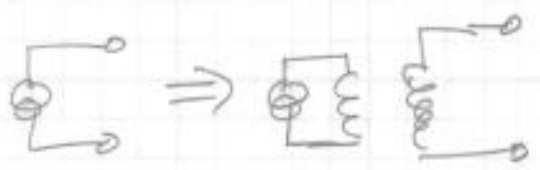
\leftarrow std H.O. M, E's \leftarrow not zero (non line)

$$= \frac{E_{10}}{2} \sigma_z - \sqrt{\frac{\hbar}{2\omega_0 C}} \Delta I \left(\sigma_x + \sqrt{\frac{E_{10}}{3\Delta U}} \sigma_z \right)$$

\uparrow H.O. coupl \uparrow tunable E_{10}

(9) LF (ΔI) give $\sigma_z ; Z$
HF (ΔI) gives $\sigma_x ; X, Y$ rotations

(10) Practical:  with large R does not exist!
(thy model only)



Transf I bias
Like Φ qubit; but not deg.

• Note: Now sens. to Φ noise
(100-200 ns decoh, can be made smaller!)



Experiments

Go through my standard qubit descr.

- 1) Init state
 - 2) Rotate
 - 3) Meas.
- easy
review
NEW, go slow!

4) Fabrication

Our /std S.C using special materials

Al, $T_c > 1K$ ok

Fabrication of small junction (e beam)

5) Meas. Setup in DR.

Low T

Filtering

Generation of microwave pulses.

- ## 6) Sequence to take data
- 0 or 1 ; 1000x for prob.

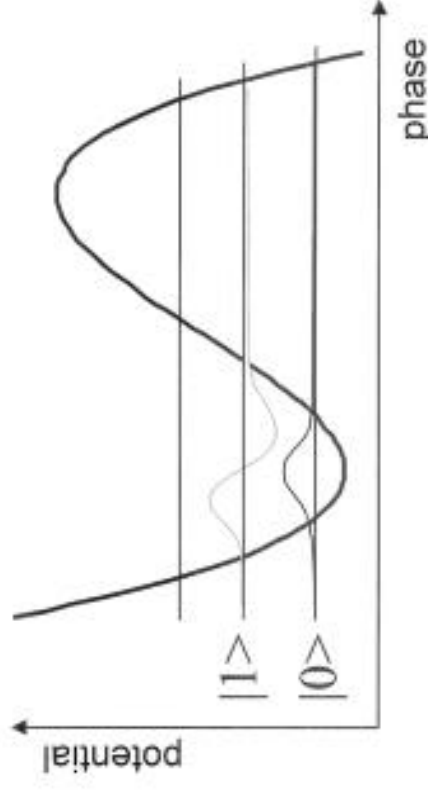
7) Spectr.

8) Rabi's ...

Josephson-Junction Qubit

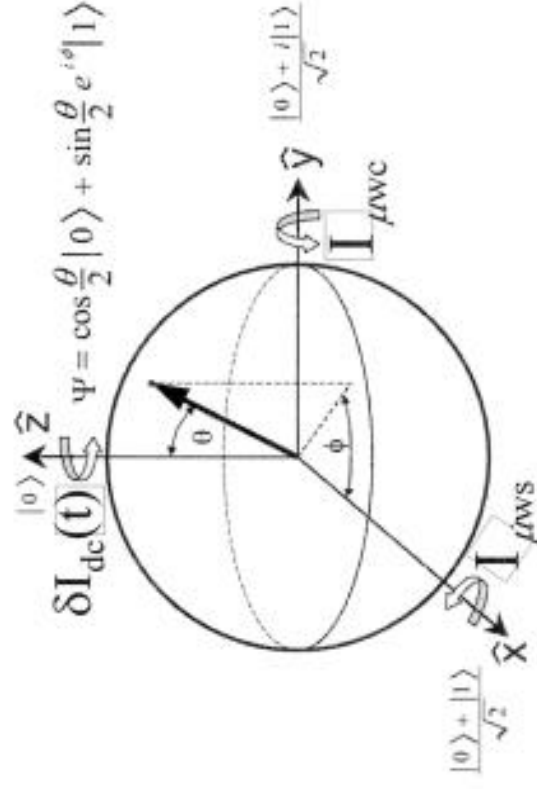
(1) State Preparation

Wait $t > 1/\gamma_{10}$ for decay to $|0\rangle$
 (20 mK ; $kT \ll \hbar\omega_{10}$; 6 GHz)



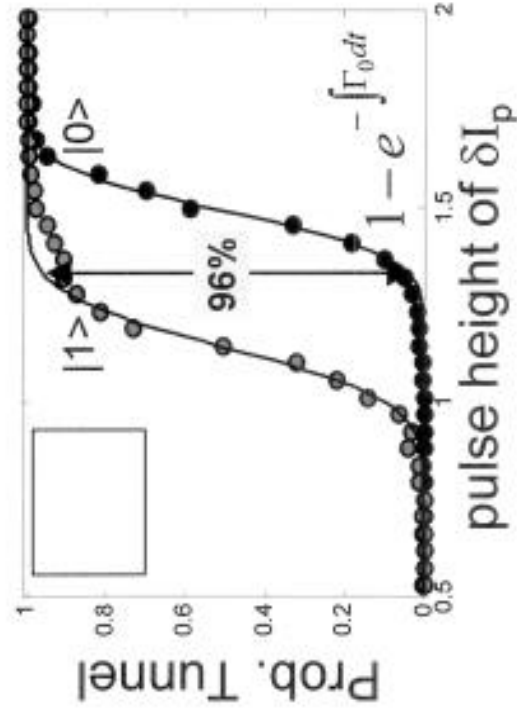
(2) Qubit logic with current bias

$$I = I_{dc} + \delta I_{dc}(t) + I_{\mu wc}(t)\cos\omega_{10}t + I_{\mu ws}(t)\sin\omega_{10}t$$



(3) State Measurement: $\Delta U(I_{dc} + \delta I_p)$

Fast single shot – high fidelity

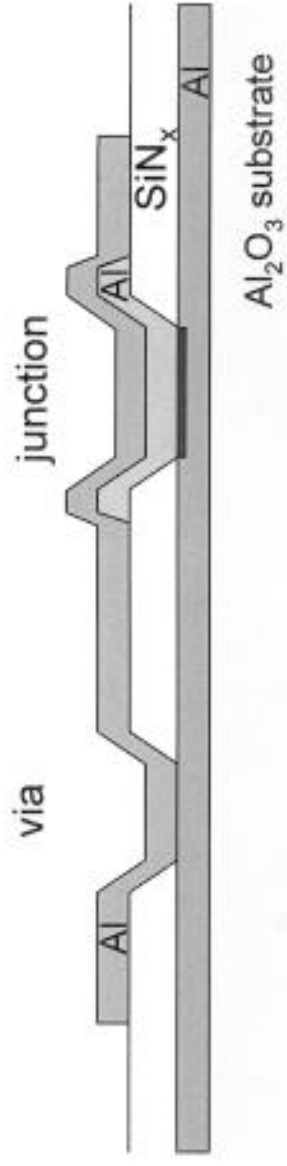
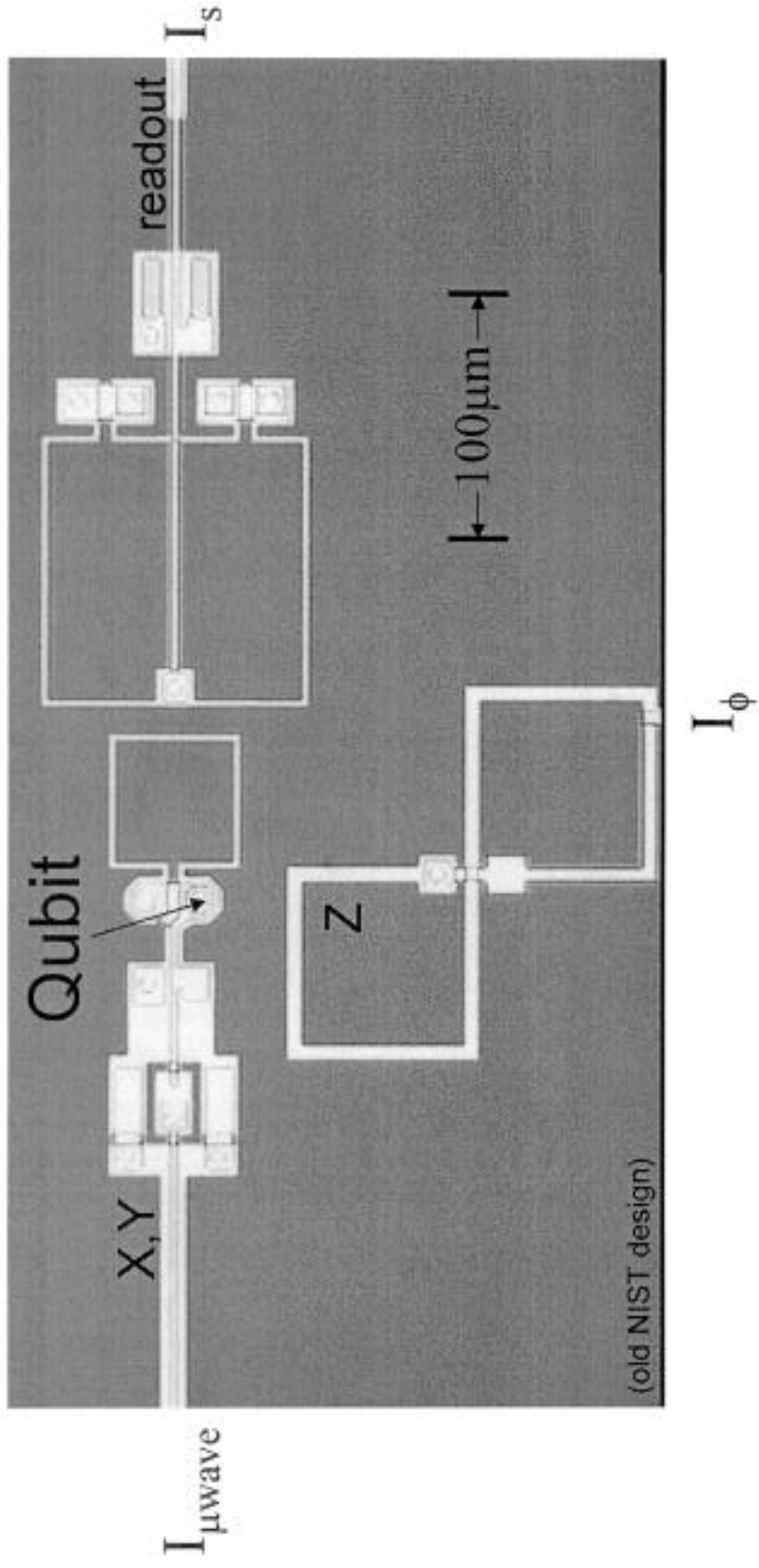


3 ns Gaussian pulse

$|1\rangle$: tunnel

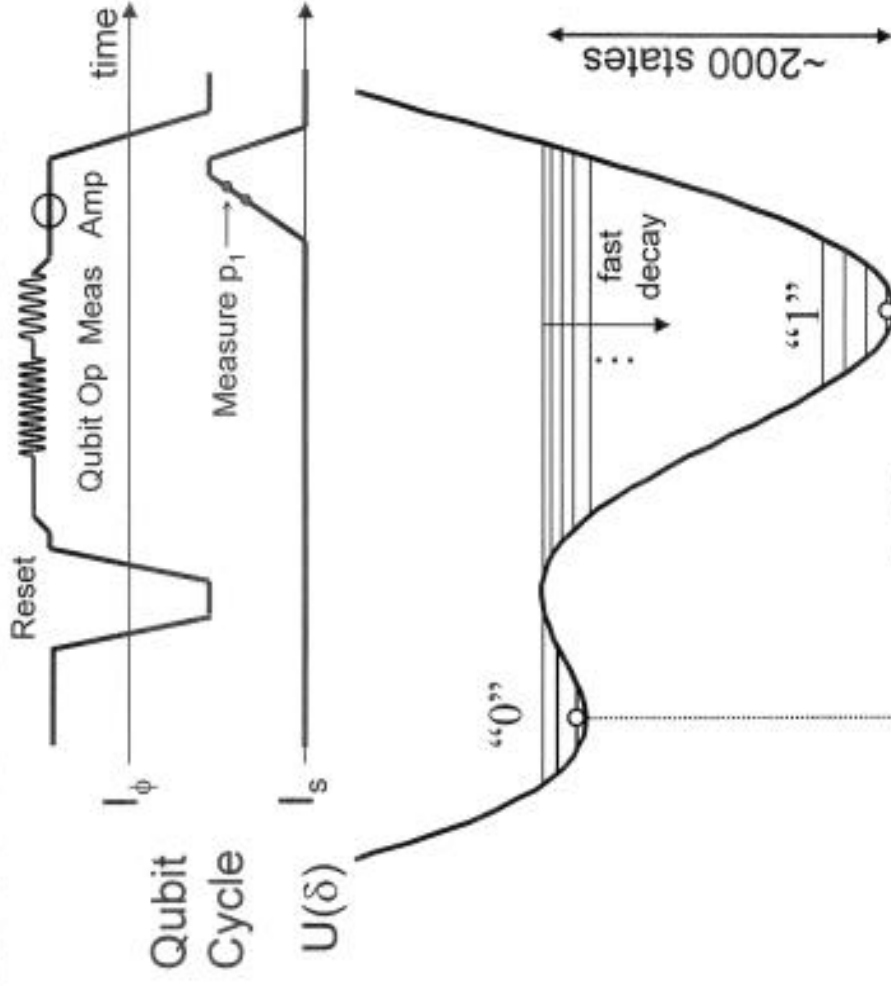
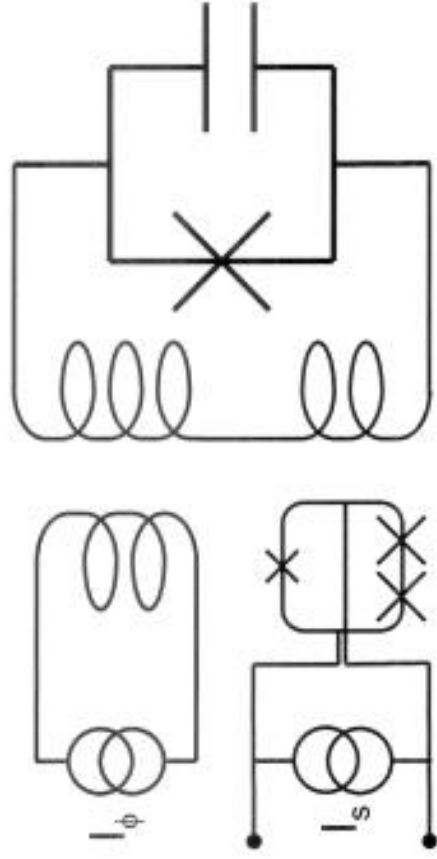
$|0\rangle$: no tunnel

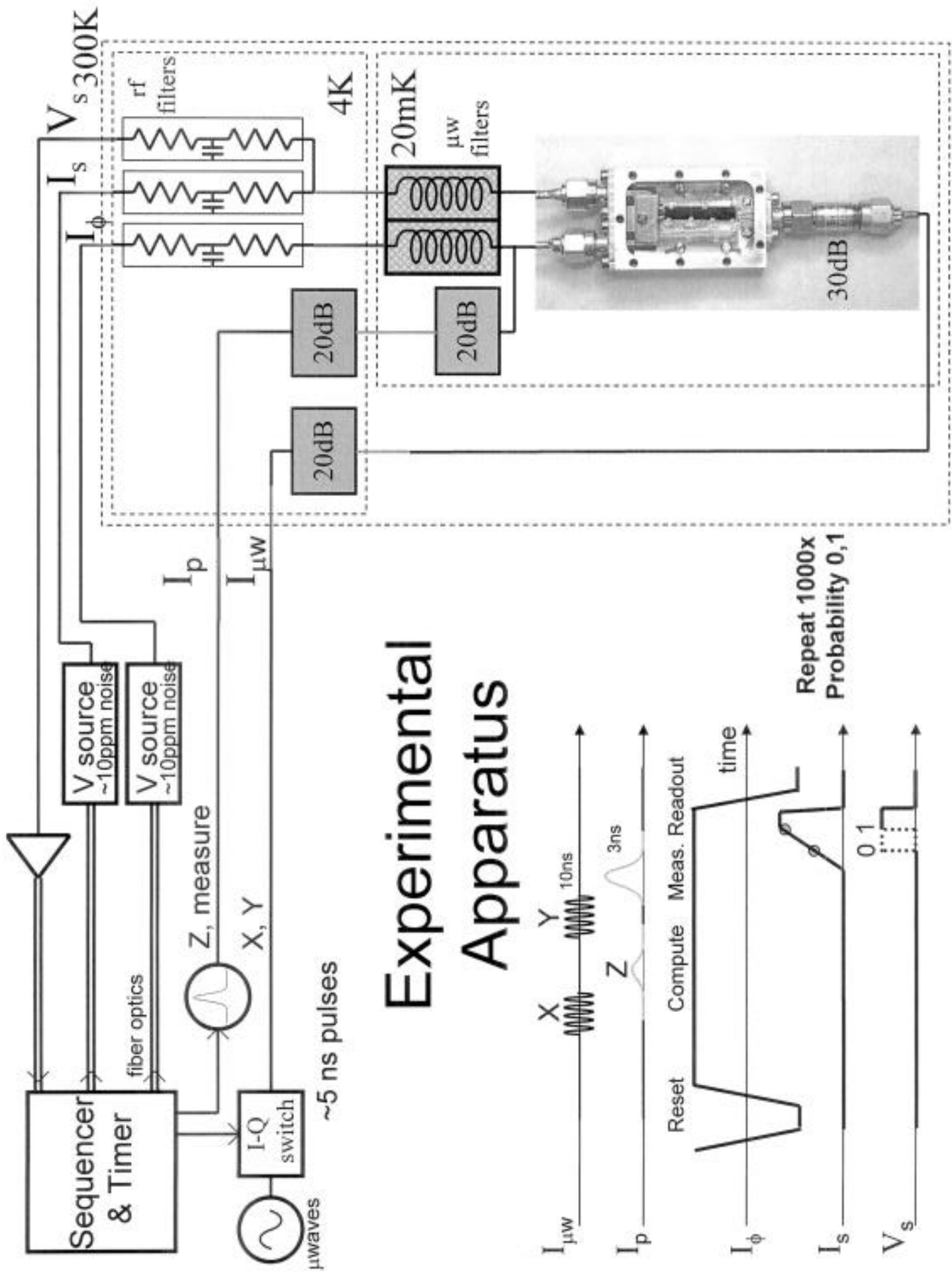
IC Fabrication



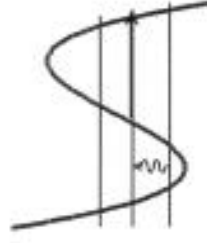
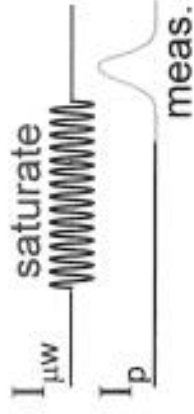
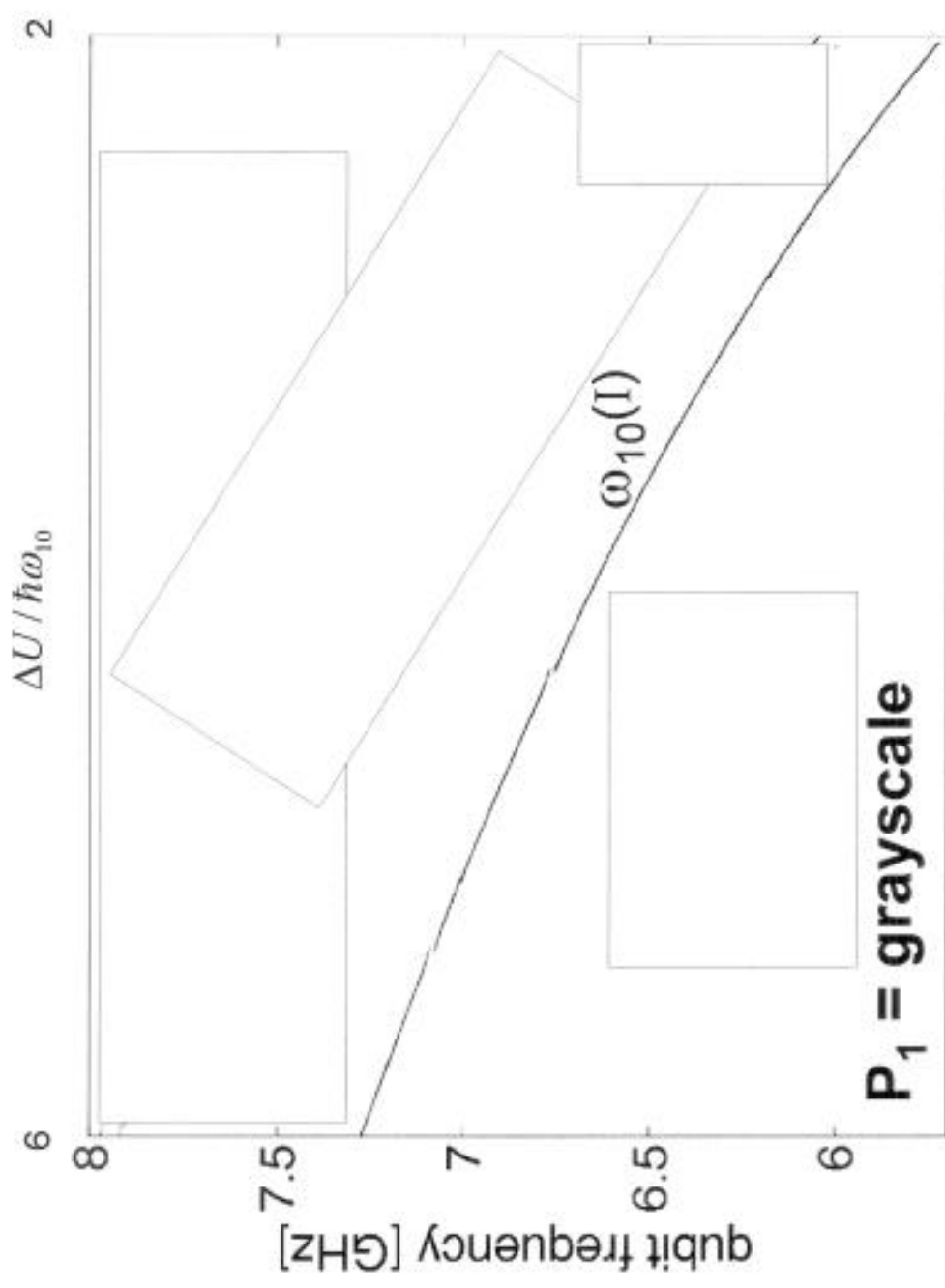
Al junction process
& optical lithography

Qubit operation → Measurement → Readout

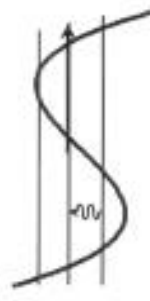




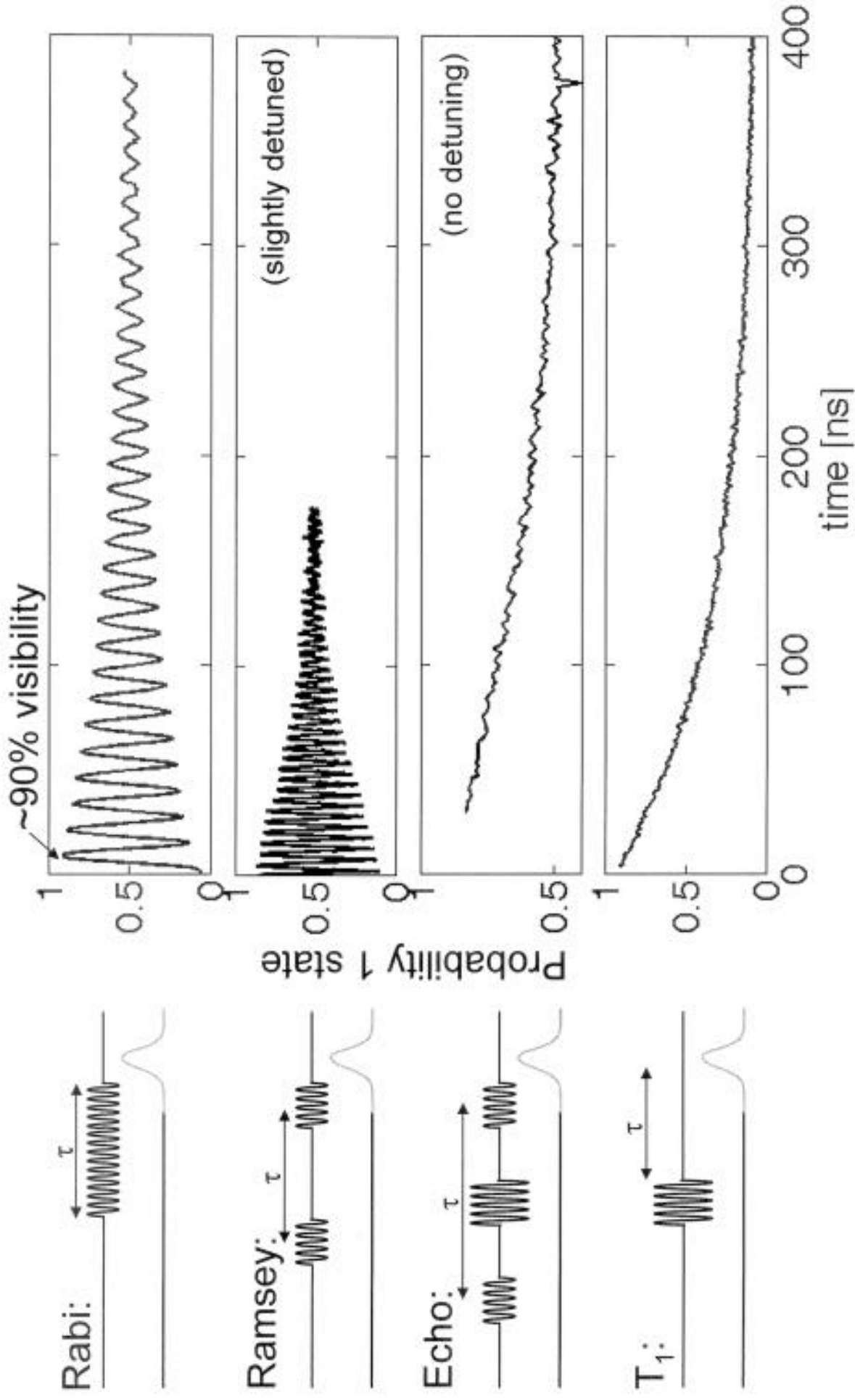
Spectroscopy



Bias current I (au)



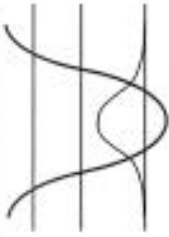
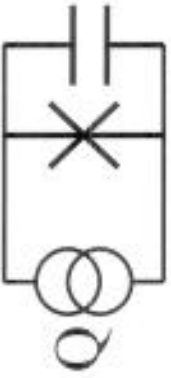
Qubit Fidelity Tests



Large Visibility! $T_1 = 110$ ns, $T_\phi \sim 85$ ns

Qubit Taxonomy

Charge



Circuit

Potential (in δ)

Area (μm^2) 0.01

$$\frac{E_J}{E_C} = \frac{I_0 \Phi_0 / 2\pi}{e^2 / 2C} \quad 1-10$$

Experimentally Important:

Small C

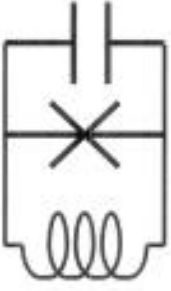
Non-linearity 50% - 5%

Impedance $1/\omega_f C$ 30 k Ω

Noise intrinsic degeneracy $Q_N - 1$ ns
1 μ s

↳ (more complex coupling)
(how true resonances allow)

Flux



0.1-1

10-10²

↳ Very sens. to parameters

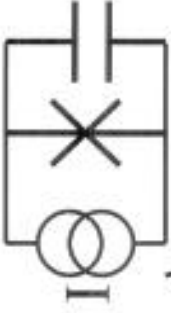
$$L \cong L_{J0} > 100\%$$

$$I \rightarrow I_0 \sim 5\%$$

1 k Ω

$\Phi_N - 10$ ns
1 μ s

Phase



10-100

10²-10⁴

50 Ω

$\Phi_N - 200+$ ns
(no design)
No Φ_N in the model,
 Φ_N from practical bias,
Not opt. yet, can be improved)