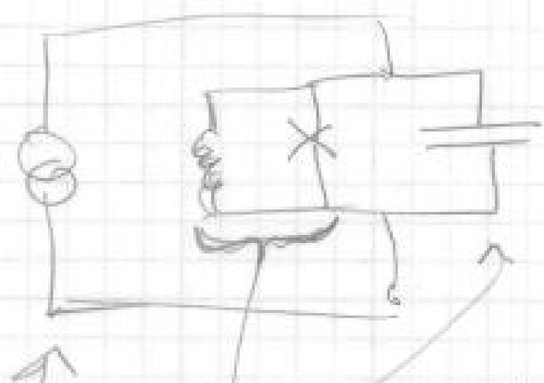
Decoherence

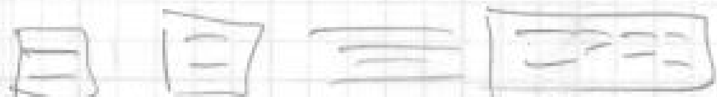
* Understand basic operation, see imperfect data, go deeper into importance issue of qu. coherence (gate errors).
  This is main issue facing qubits.
* Go into some detail
* What to worry about



1. S.C. + josephs. effect
   (where does non-linearity come from?)
2. Leads; dissip from circuit
   (+ ext'l noise).
   – Microwave Engineering –
3. Capacitor (2-level states)
   ↳ 1/F noise Q, Φ

Circuit Engineering + Decoherence

Gate errors need to be calculated!



qubits   bath (other qubit modes)

↳ coupling to bath, entagles qubits with other modes.
But since don't meas. bath, lose info about quantum state.

Nice general model – but how calculate anything?

(1) Need to model decoh. (bath) — Do this with circuit elements, particularly R's.

* No one bath, because it depends on how you build the circuit! (unlike atoms + vacuum) (+this interest in field!)

* Either measure or model circuit and R's, so need <u>general</u> theory of decoherence. (What <u>really</u> matters in design?!)

(2) Show Decoherence = Noise + Dissipation (Diss = Qu Noise)

Very general + powerful model!
"See" how to design qubits

But, will have to learn

- Circuit theory
- Microwave Eng.
- Noise

(classical) (+ qu)

Then Connection to Q.M.

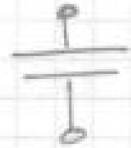
Circuits

① Basic circuit elements, linear response Z:

$\frac{V}{I} = R$ 

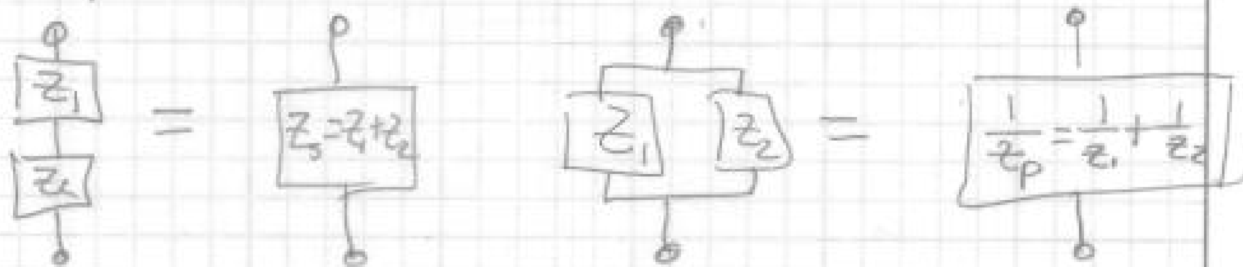
$\frac{V}{I} = Z = i\omega L$ 

$\left(v = L\frac{dI}{dt}\right)$

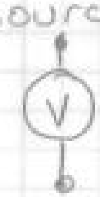
$Z = \frac{1}{i\omega C}$ 

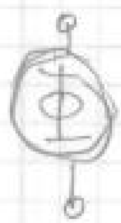
$\left(v = \frac{Q}{C}\right)$

② Response of multi-elements

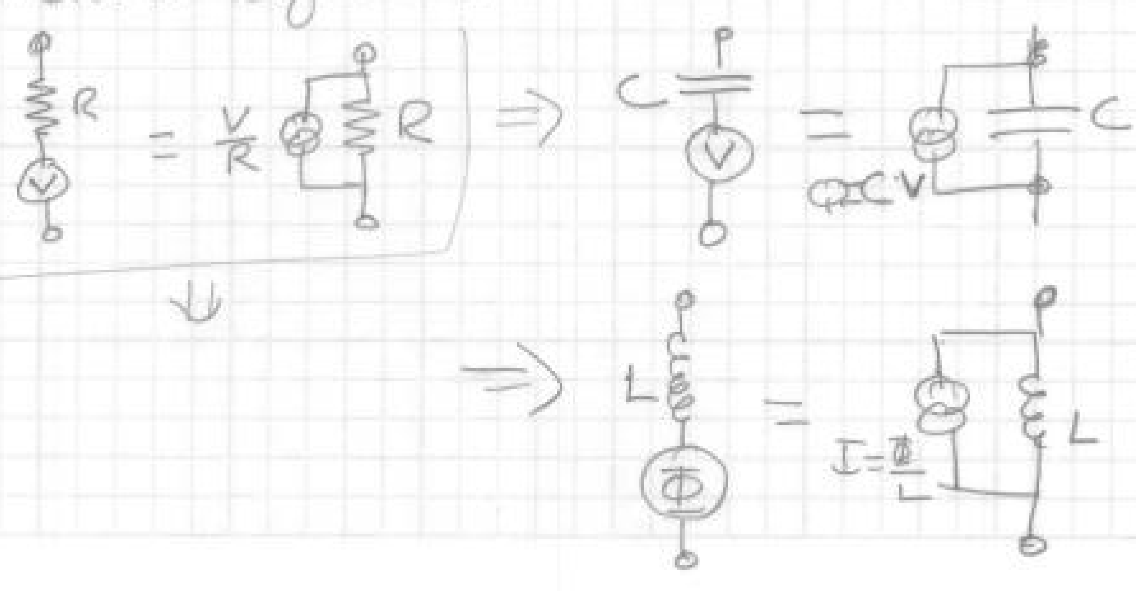


③ Sources:

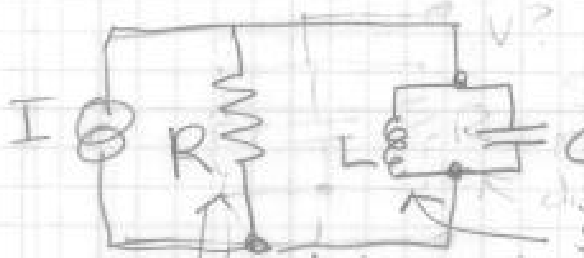
voltage  $R_{dyn} = 0$ current I  $R_{dyn} = \infty$

Flux  $= \int v\,dt$ charge. Q  $Q = \int I\,dt$

④ Thevenin Equiv's:



Resonator (Simple model of Qubit)



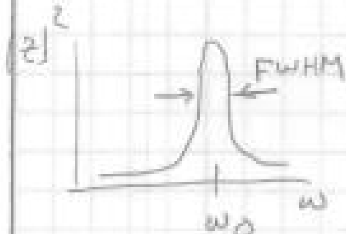
simple model of Loads.

$\boxed{\text{Freq. Domain (spectroscopy)}}$ $\frac{1}{Z} = \frac{1}{i\omega L} + i\omega C + \frac{1}{R}$

response $\frac{V}{I} = Z_t = \dfrac{1}{\underbrace{\frac{1}{i\omega L} + i\omega C}_{} + \frac{1}{R}}$ Freq. dependent, complex (ϕ shift — Input I to out V)

max when $= 0$ $\frac{1}{\omega L} = \omega C$

$\omega = \omega_0 + \Delta\omega$ reson. Freq. $\Rightarrow \omega_0^2 = \frac{1}{LC}$



$$\approx \frac{1}{\frac{1}{i\omega_0 L}\left(1 - \frac{\Delta\omega}{\omega_0}\right) + i(\omega_0 + \Delta\omega)C + \frac{1}{R}}$$

$$= \frac{1}{2i\Delta\omega C + \frac{1}{R}}$$

$$|Z_t|^2 \propto \frac{1}{(2\Delta\omega C)^2 + \left(\frac{1}{R}\right)^2} \propto \frac{1}{(\Delta\omega)^2 + \left(\frac{1}{2RC}\right)^2}$$

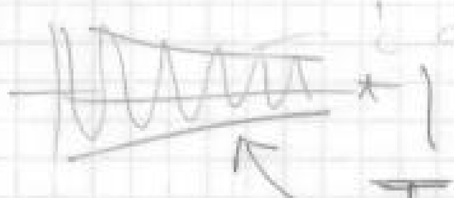
$$\text{FWHM} = 2\,\frac{1}{2RC} = \frac{1}{RC}$$

HM $\Rightarrow \Delta\omega = \frac{1}{2RC}$

$$Q \equiv \frac{\omega_0}{\text{FWHM}} = \frac{\omega_0 RC}{\cancel{RC}} = \frac{R}{(1/\omega_0 C)}$$

Ratio: $\leftarrow$ Dissip $\leftarrow Z$ resonator

$\boxed{\text{Time Domain}}$



$T_{1/e} = RC$

$1/e$ power decay time

So $Q \sim$ # oscillations before lose energy

Energy Decay (T_1) of Qubits

- This easy to understand from classical analysis (What know already!)

1) Imagine H.O. (resonator) weakly excited (spectroscopy or pulsed excitation)
   $|1\rangle$ state small ampl.
   ($|2\rangle$ state negligible)

2) Then only $1 \to 0$ transitions matter, just like a qubit

$$T_1 = \underbrace{RC}_{\text{Since H.O. \& Qu = Class.}} \left| \frac{\langle 0|\hat{\delta}|1\rangle_{qubit}}{\langle 0|\hat{\delta}|1\rangle_{H.O.}} \right|^2$$

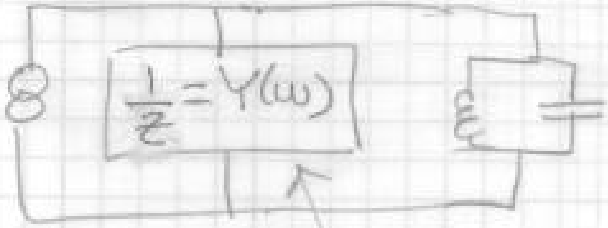
3) Qubit with correction

Corr. Fact. ≈ 1 mostly, (At least good approx.) Since slightly non-linear

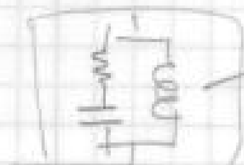
4) Proper calculations give this result.

5) Power of this calculation – Can do general models very easily

More General Model
(beyond dissip. = R)



Arbitrary admittance of loads.



complex circuit freq depend.

$$\frac{1}{Z_x} = \frac{1}{\underbrace{\frac{1}{i\omega L} + i\omega C}_{=0 \text{ @ reson.}} + i\,\mathrm{Im}\,Y(\omega) + \mathrm{Re}\,Y(\omega)}$$

(Dissipative part
Dispersive part)

Assuming effect of Y is small (pert. thy around ω_0)

(unperturbed resonance)

$$2\cancel{i}\,\delta\omega\, C + \cancel{i}\,\mathrm{Im}\,Y(\omega_0) = 0$$

shift in Res freq.

$$\delta\omega = \frac{\mathrm{Im}\,Y(\omega_0)}{2C} \qquad \left(+ \text{ small Lamb shift when do QM}\right)$$

$$\text{“}\frac{1}{R}\text{”} = \mathrm{Re}\,Y(\omega_0)$$

$$\left(T_1 = \frac{C}{\mathrm{Re}\,Y(\omega_0)} \right)$$

($Y(\underline{\underline{\omega_0}})$ matters
only near/at resonance for T_1)

Transmission Lines

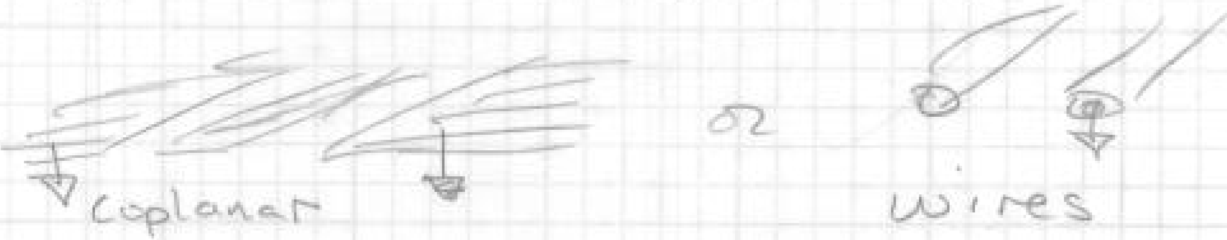
(Like discussing simple lumped circuits)

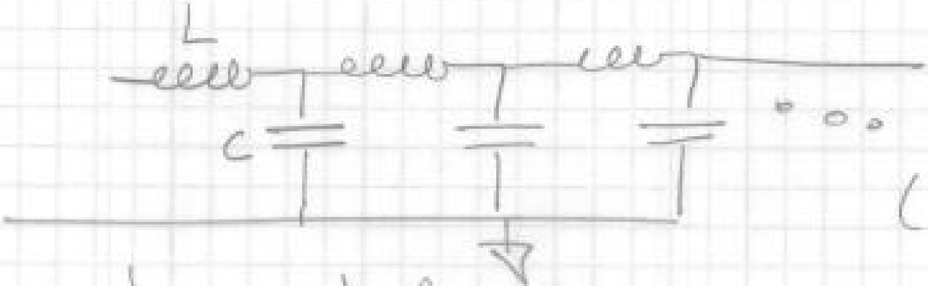
Lumped Circuit ; size $<< \lambda$ (wavelength)

At μwaves, $\lambda \sim 1$cm ; size effects?

t-lines show behavior (quant. + qual.)

(+ how to model a Resistor in Q.M.)





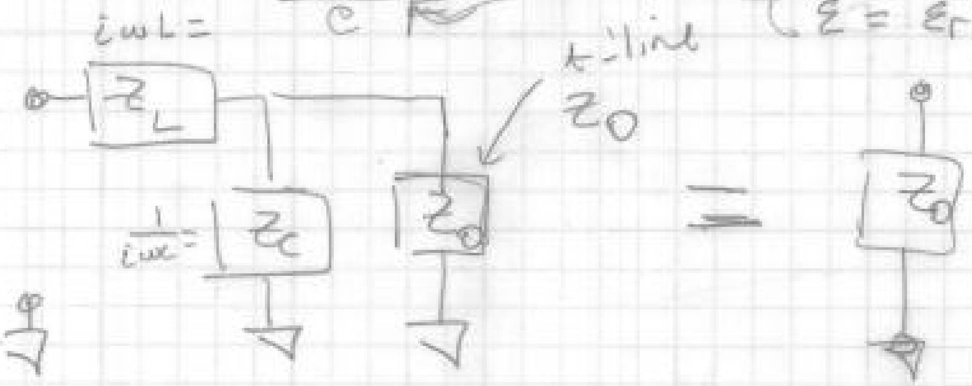
Lumped circuit

(Store En in B + E Fields)

$$L = g_L \mu \ell$$
$$C = \frac{g_C \varepsilon}{c} \ell$$

induct + Cap per unit length

($\mu \approx \mu_0$, $\varepsilon = \varepsilon_r \varepsilon_0$ ($\varepsilon_r \sim 10$ often))



transl. invariant (long length)

$$Z_0 = Z_L + \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_0}}$$

$$(Z_0 - Z_L) = \frac{Z_C Z_0}{Z_C + Z_0}$$

$$(Z_0 - Z_L)(Z_0 + Z_C) = Z_0 Z_C$$

$$Z_0^2 + \cancel{Z_0 Z_C} - Z_0 Z_L - Z_L Z_C = \cancel{Z_0 Z_C}$$

$Z_L \to 0$ $Z_C \to \infty$

$$Z_0^2 = Z_L Z_C$$

$$Z_0 = \pm\sqrt{Z_L Z_C} = \pm Z_0$$

$$Z_0 = \sqrt{i\omega L \frac{1}{i\omega C}}$$

$$= \sqrt{\frac{g_L \mu \cancel{\ell}}{g_c \varepsilon \cancel{\ell}}}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{g_L \mu_r}{g_c \varepsilon_r}}$$

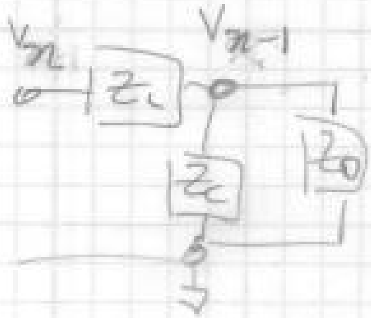
$$= 377\Omega \cdot \sqrt{} \quad \text{) Typical geometries}$$

$$\approx 50\Omega$$

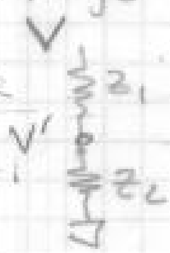
$\boxed{\text{(1) Transmission Line "Looks" like a } 50\Omega \text{ resistor}}$

Why? No dissipation $\boxed{\text{Very strange}}$

→ signal propogates down line, never reflecting since ∞ long!



) Voltage divider



$$V' = I \cdot Z_2 = \frac{V}{Z_1 + Z_2} Z_2$$

$$\frac{V'}{V} = \frac{Z_2}{Z_1 + Z_2}$$

$\frac{V_{n-1}}{V_n}$?

$$\frac{V_n}{V_{n-1}} = \frac{(Z_0 \| Z_c)}{Z_L + (Z_0 \| Z_c)}$$

$$= \frac{1}{1 + Z_L \left(\frac{1}{Z_0 \| Z_c}\right)}$$

$$= \frac{1}{1 + Z_L \left(\frac{1}{Z_0} + \frac{1}{Z_c}\right)}$$

$$= \frac{1}{1 + \frac{Z_L}{\sqrt{Z_L Z_c}} + \cancel{\frac{Z_L}{Z_c}}} \approx 1 - \sqrt{\frac{Z_L}{Z_c}}$$

$Z_L \to 0,\ Z_c \to \infty$

$$= \frac{1}{1 + \sqrt{Z_L / Z_c}}$$

For k segments:

$$\frac{V_n}{V_0} = \left(1 - \frac{n\sqrt{Z_L/Z_C}}{n}\right)^n$$

$$\simeq \exp\left[-n\sqrt{Z_L/Z_C}\right]$$

$$\sqrt{Z_L Z_C} = \sqrt{i\omega g_L \mu l \, i\omega g_C \varepsilon l}$$

$$= \pm i\omega l \underbrace{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r g_L g_C}}_{\frac{1}{v}\ \text{const.}}$$

$$= \pm \frac{i\omega l}{v}$$

$$\simeq \exp\left[\pm \frac{i\omega (nl)}{v}\right]$$

length

Velocity of light $\left(\frac{1}{\sqrt{\mu_0\varepsilon_0}} * \frac{1}{\sqrt{\cdots}}\right)$ $\frac{1}{\sqrt{\varepsilon}}$ free space

propogation of light up + down line (both directions).

$\boxed{\text{(2) Prop. both directions, } \approx \text{ speed of light}}$

$\boxed{(\propto 1/\sqrt{\varepsilon_r} \text{ as } g_L g_C = 1,\ \mu_r = 1)}$

Application, connect wires to qubit

← 1 meter — OR

$$\Rightarrow T_1 = \frac{C}{\mathrm{Re}\,Y(\omega)}$$

$$= \frac{C}{\mathrm{Re}\,(1/Z_0)}$$

$$\approx C \cdot 50\,\Omega$$

$$= 1\,\mathrm{pF} \cdot 50\,\Omega$$

$$= 50\,\mathrm{ps}$$

→ Short coh. time (even for big C)

$\boxed{\text{(3) (Long) Wires look like x-lines } (50\,\Omega)}$

$\boxed{\text{lots of dissip.}}$

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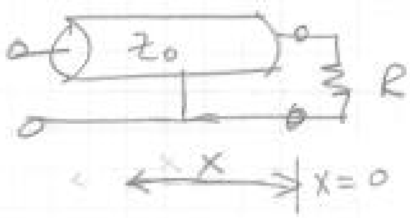
Termination of Lines

(1)

port 1



(2) 2 traveling waves $\begin{cases} \vec{V} = Z_0 \vec{I} e^{ikx} \\ \overleftarrow{V} = -Z_0 \overleftarrow{I} e^{-ikx} \end{cases}$ $\quad k = \frac{\omega}{v}$

(−) because I changes direction

(3) Given $\vec{V}$ (incident), what is $\overleftarrow{V}$ (reflected)

$$S_{11} = \overleftarrow{V} / \vec{V} \qquad \text{(Scatt. matrix)}$$

(4) At $x=0$, conserve I at node

$$\vec{I} + \overleftarrow{I} = \frac{\vec{V} + \overleftarrow{V}}{R}$$

$$+ \begin{cases} \vec{V} = Z_0 \vec{I} \\ \overleftarrow{V} = -Z_0 \overleftarrow{I} \end{cases}$$

$$\vec{V}/Z_0 - \overleftarrow{V}/Z_0 = \vec{V}/R + \overleftarrow{V}/R$$

$$\vec{V}\left(\frac{1}{Z_0} - \frac{1}{R}\right) = \overleftarrow{V}\left(\frac{1}{Z_0} + \frac{1}{R}\right)$$

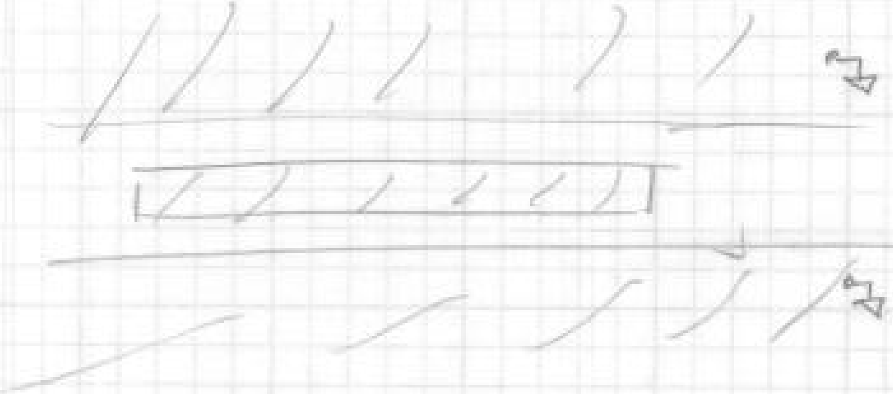
$$S_{11} = \overleftarrow{V}/\vec{V} = \frac{1 + Z_0/R}{1 - Z_0/R}$$

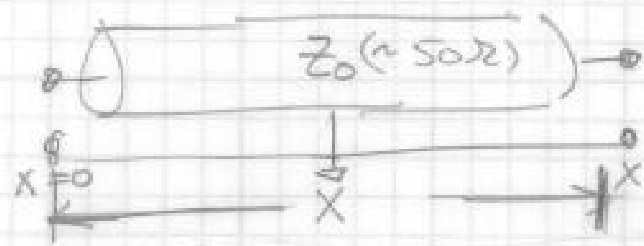
(5) $R = 0$; $S_{11} = -1$; reflects 

$R = 0$; $S_{11} = +1$; reflects 

* $R = Z_0$; $S_{11} = 0$; Matched ; term. resistor absorbes pulse.

Finite t-lines — Resonators





$\vec{V} = Z_0 \vec{I} e^{ikx}$

$\overleftarrow{V} = -Z_0 \overleftarrow{I} e^{-ikx}$

Boundry Condition

B.C. At $x=0$, $I_0 = \vec{I} + \overleftarrow{I} = 0$ (as open).

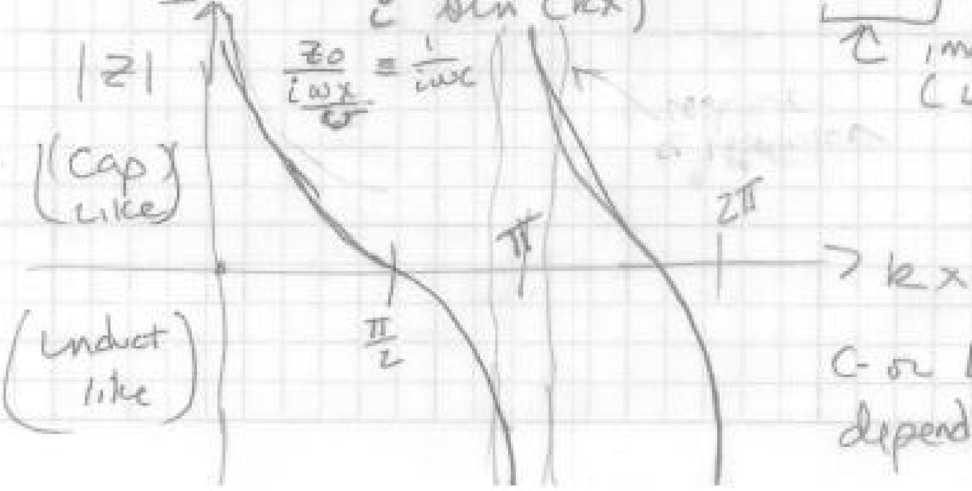
$\Rightarrow I(x) = I_0 \left(e^{ikx} - e^{-ikx}\right)$ (I_0 a parameter) (?)

$= 2 I_0 i \sin(kx)$

$V(x) = Z_0 I_0 \left(e^{ikx} + e^{-ikx}\right)$

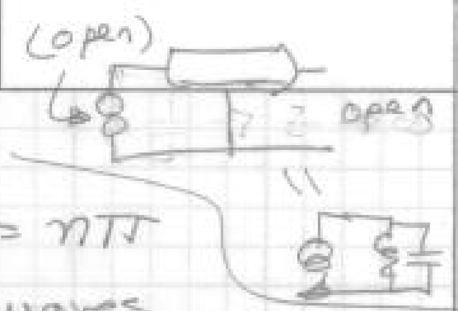
$= 2 Z_0 I_0 \cos(kx)$

$Z = \frac{V}{I} = \frac{Z_0}{i} \frac{\cos(kx)}{\sin(kx)} = \underbrace{-i Z_0}_{\text{imag. impedance (L or C, not R)}} \cot(kx)$



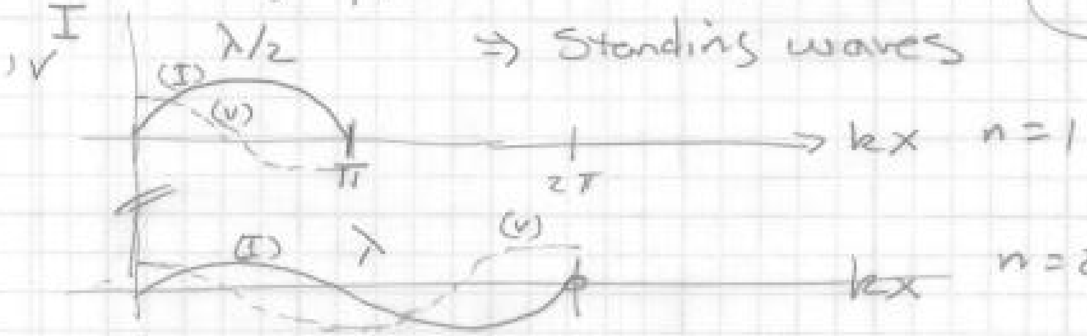
C- or L- like depending on length.

What is resonance condition?
($Z = \infty$, as for L/C)



For t-line, happens when $kx = n\pi$

$\Rightarrow$ Standing waves



Around resonance $kx = \alpha_0 + \Delta\alpha$, $\alpha_0 = n\pi$

$$\frac{\cos(\alpha_0+\Delta\alpha)}{\sin(\alpha_0+\Delta\alpha)} \approx \frac{\cos\alpha_0 - \cancel{\Delta\alpha\,\sin\alpha_0}}{\cancel{\sin\alpha_0} + \cos\alpha_0(\Delta\alpha)}$$

sin cos + cos sin

$$= \frac{1}{\Delta\alpha}$$

$$\boxed{\begin{gathered}\omega = \omega_{res} + \Delta\omega \\ \frac{\omega_{res}}{v}x = n\pi\end{gathered}}$$

$$Z = \frac{Z_0}{i\frac{\Delta\omega}{v}x}$$

$$= \frac{Z_0}{i\,\Delta\omega\left(\frac{n\pi}{\omega_{res}}\right)}$$

$$= \frac{1}{2i\Delta\omega\left(\frac{n\pi}{2\omega_{res}Z_0}\right)}$$

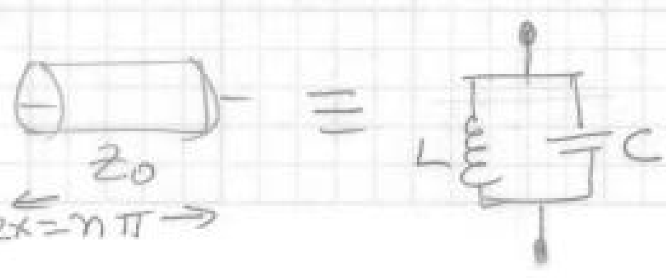
effective cap

$$C = \left(\frac{n\pi}{2Z_0}\right)\frac{1}{\omega_{res}}$$

(LC model) $$= \frac{1}{2i\Delta\omega C}$$

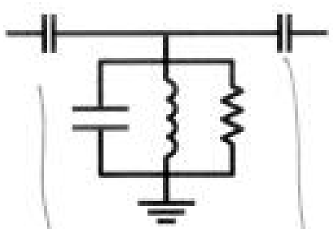
$$\omega_{res}^2 = \frac{1}{LC}$$

$$L = \frac{1}{C\omega_{res}^2} = \left(\frac{2Z_0}{n\pi}\right)\frac{1}{\omega_{res}}$$

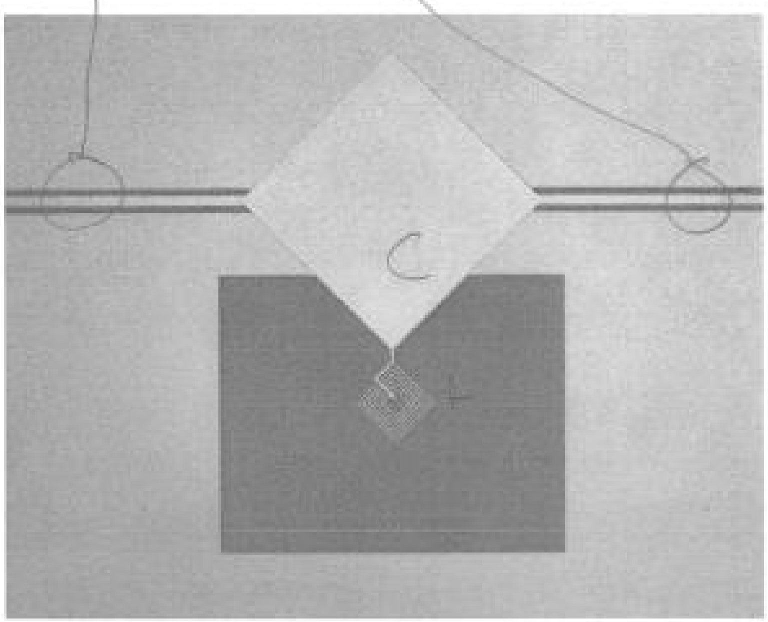


Resonators

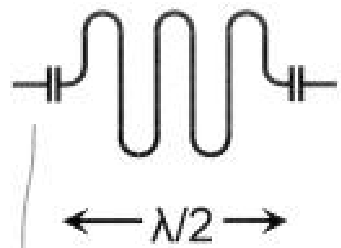
pancake (lumped element)



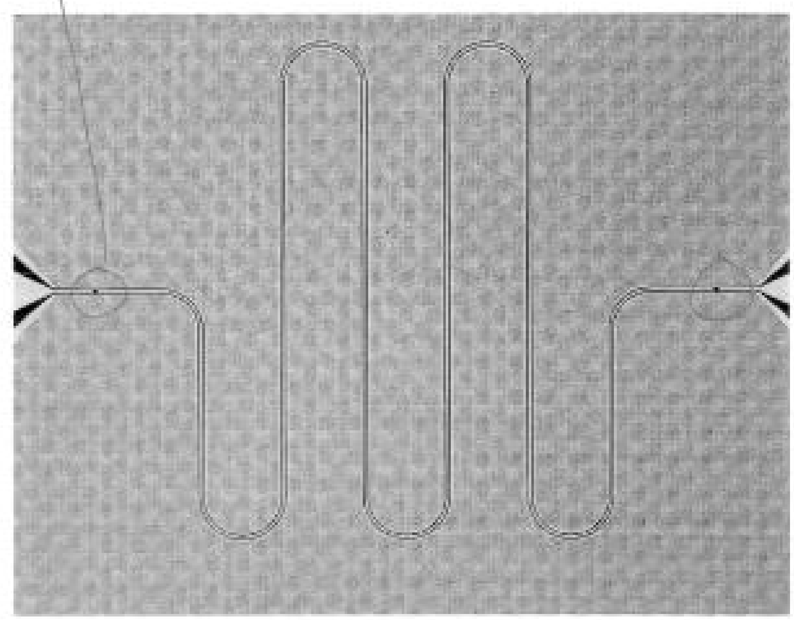
$C_c \sim 0.5\text{-}10$ fF
$C \sim 5$ pF
$L \sim 200$ pH
$f_0 \sim 5$ GHz
$Z_{res} \sim 10\ \Omega$



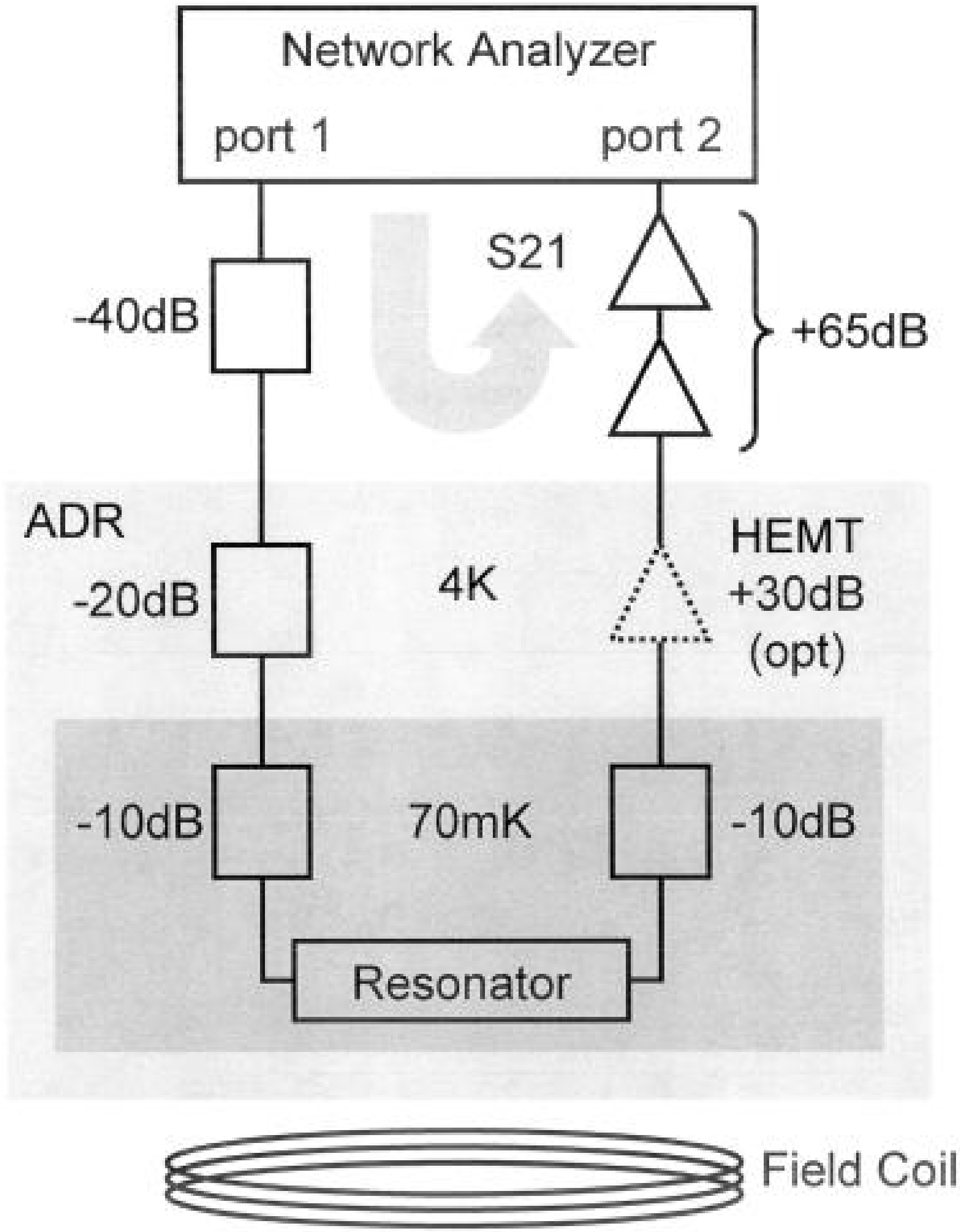
coplanar waveguide (CPW)

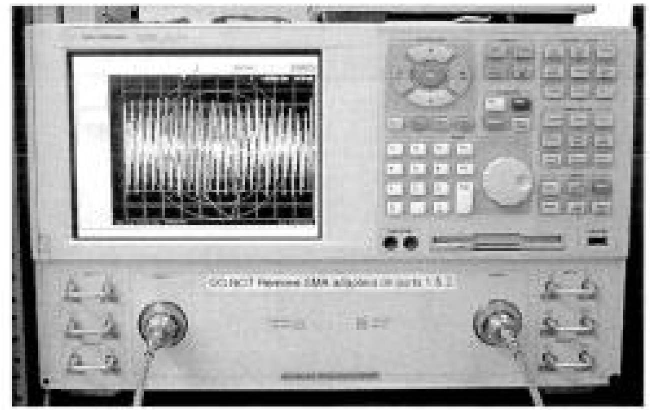


$C_c \sim 0.5\text{-}10$ fF
$C \sim 150$ pF/m
$L \sim 390$ nH/m
$f_0 \sim 6$ GHz
$Z_{res} \sim 50\ \Omega$

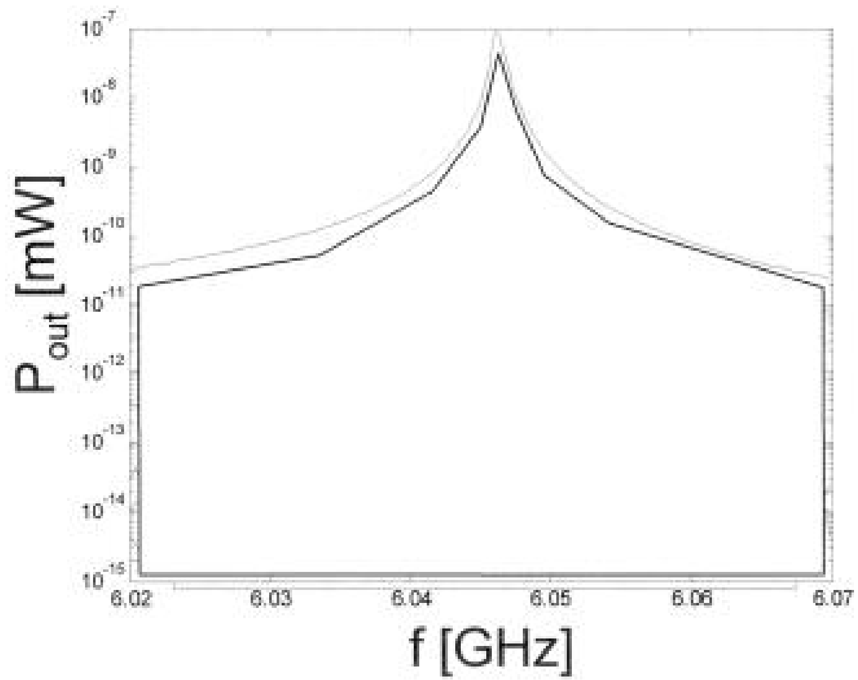


Experimental Setup





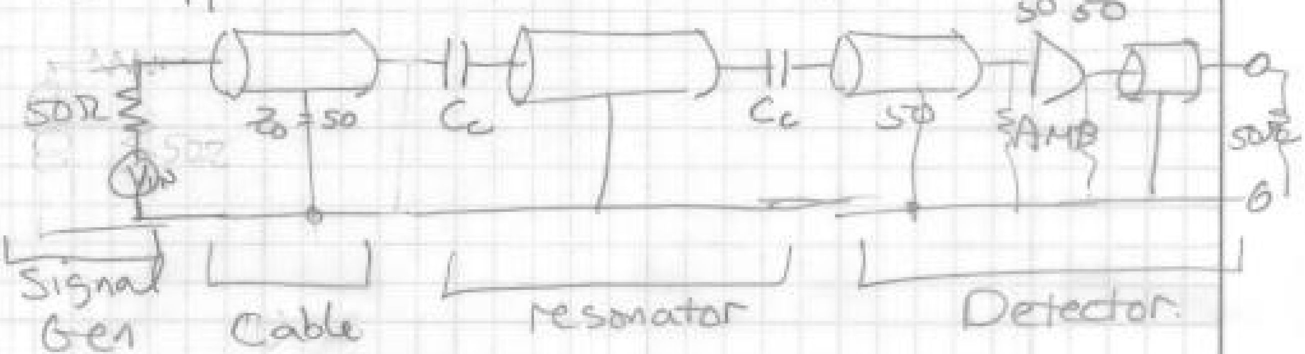
Microwave Resonator Measurements

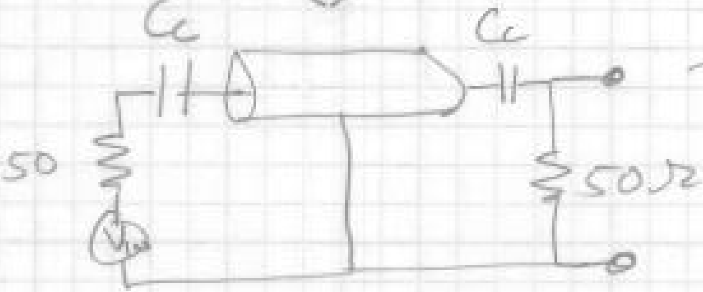


Let's do something Real circuit – (use these concepts!)

LC approx of transm. line; Real Circuit

show picts

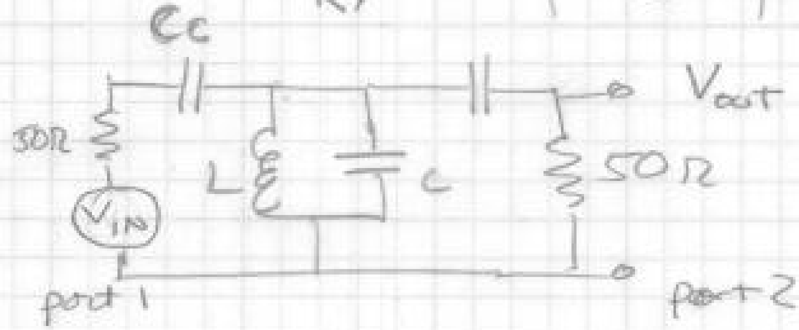




→ when multiply by gain of amp.

(neglect π phase shift of resonator).

Great Simplification!



What is $\frac{V_{out}}{V_{IN}} = S_{21}$? ← Scattering Matrix

Wave in port 1

Wave out port 2

$C_c \sim 10\,fF;\ \frac{1}{10^{10}\cdot 10^{-14}} \sim 10^4\,\Omega$

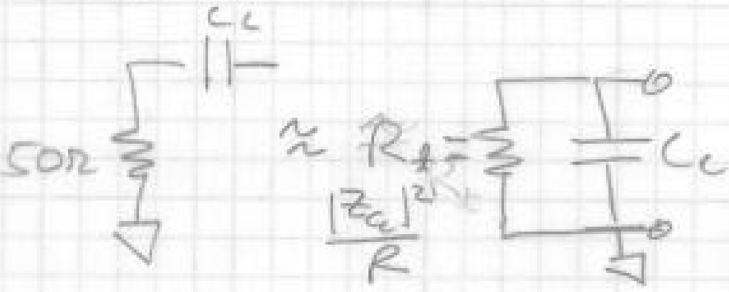
Circuit Analysis

Impt. Simplification $|Z_{cc}| = \left|\frac{1}{i\omega C_c}\right| >> R = 50\,\Omega$

(C_c couples resonator to outside world, but want to couple weakly so only slightly perturbs resonator)

2 steps:

(1) Z transf.

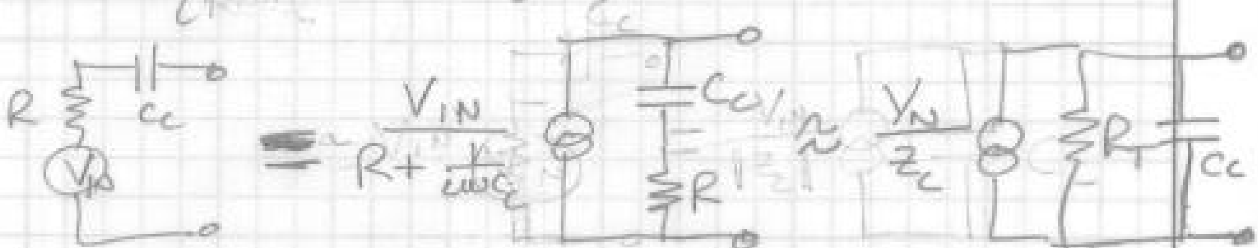


$$Z = R + \frac{1}{i\omega C_c}$$

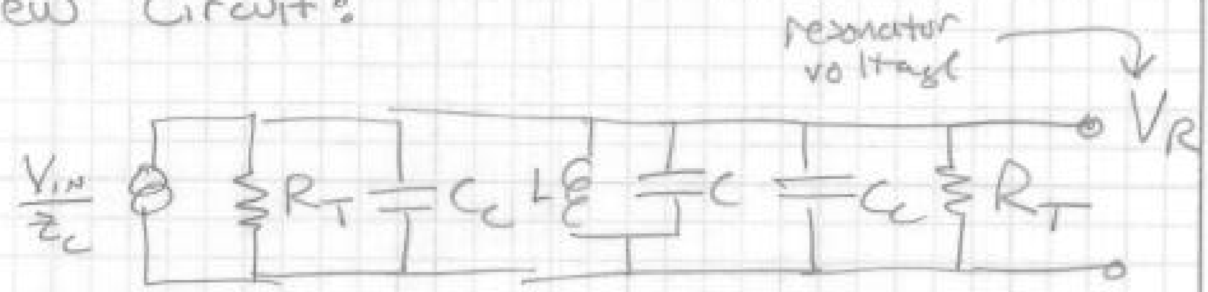
$$Y = \frac{1}{Z} = \frac{1}{R + \frac{1}{i\omega C_c}} = \frac{R - \frac{1}{i\omega C_c}}{R^2 + |Z_{cc}|^2} \approx \underbrace{\frac{R}{|Z_{cc}|^2}}_{\text{transf } R} + \underbrace{i\omega C_c}_{\text{same Cap}}$$

$$R_t = \frac{1}{Re(Y)} = \frac{|Z_{cc}|^2}{R}$$

(2) Thev. Equiv. Source:



New Circuit:

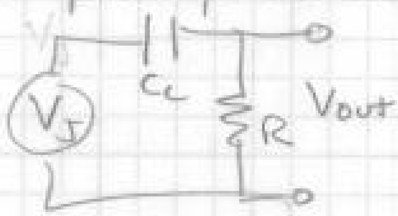


Just a L-C-R circuit, analyzed previously.

$$V_J = Z I \qquad \omega = \omega_{res} + \Delta\omega \; ; \; \omega_{res} = \frac{1}{\sqrt{L(C+\cancel{2C_C})}}$$

near resonance
$$\simeq \frac{1}{2i\,\Delta\omega\, C + \frac{1}{(R_T/2)}} \, \frac{V_{IN}}{Z_C}$$

What is preamp voltage (not just reson. V_J)?

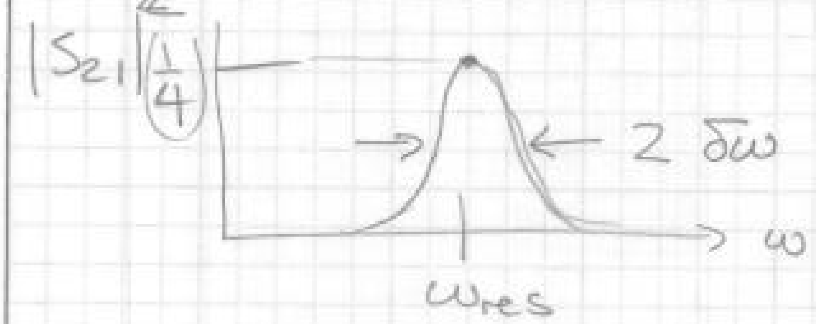


$$V_{out} = \frac{R}{R + Z_C} V_J \simeq \frac{R}{Z_C} V_J$$

$$S_{21} = \frac{V_{out}}{V_{IN}} = \frac{R}{Z_C^2} \, \frac{1}{2i\,\Delta\omega C + \frac{2}{R_T}}$$

$$= \frac{1}{2} \frac{R\, R_T}{Z_C^2} \, \frac{1}{i\,\Delta\omega C R_T + 1}$$

$$= -\frac{1}{2} \, \frac{1}{i\,\Delta\omega (C R_T) + 1}$$



On resonance, $\Delta\omega = 0$, $S_{21} = -\frac{1}{2}$

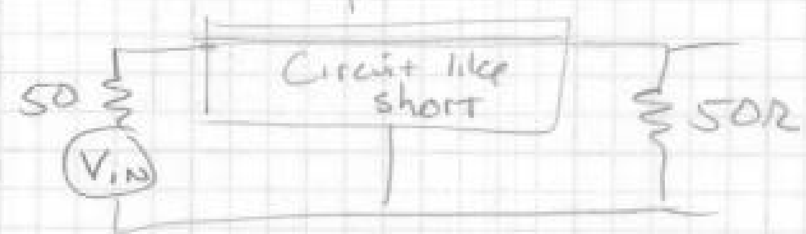
FWHM (power) when $\Delta\omega C R_T = 1$

$\Delta\omega = \delta\omega/2$

$$\delta\omega = \frac{2}{C R_T}$$

$$= \frac{1}{C(R_T/2)}$$; normal formula for Q.

Makes sense, on resonance, circuit sends power thr. resonator



$V_{out} = \frac{V_{IN}}{2}$

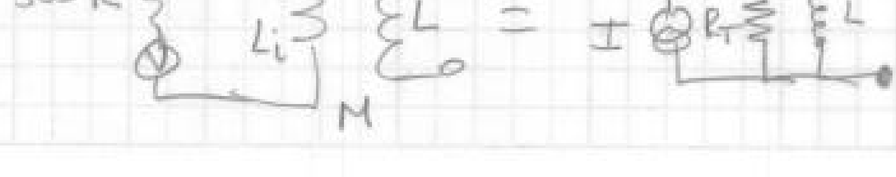
(but circuit phase shifts by π)

Summary: Small C_c impedance transforms

$$R \approx 50\,\Omega \text{ to } \frac{|Z_c|^2}{R} \sim 10^6\,\Omega$$

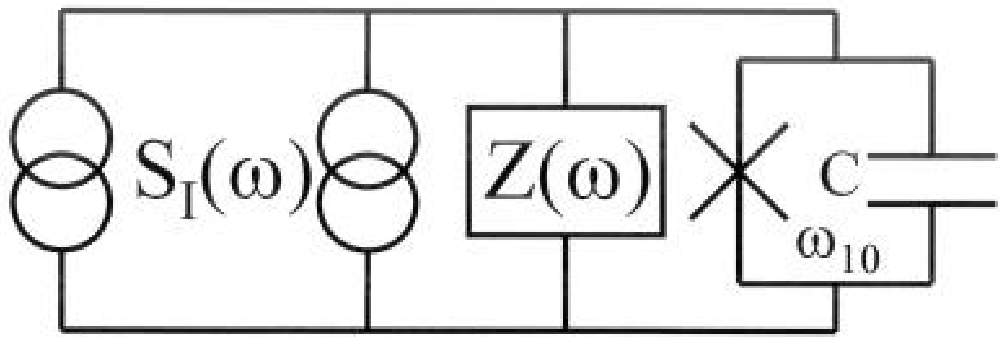
Thus resonator not damped much.

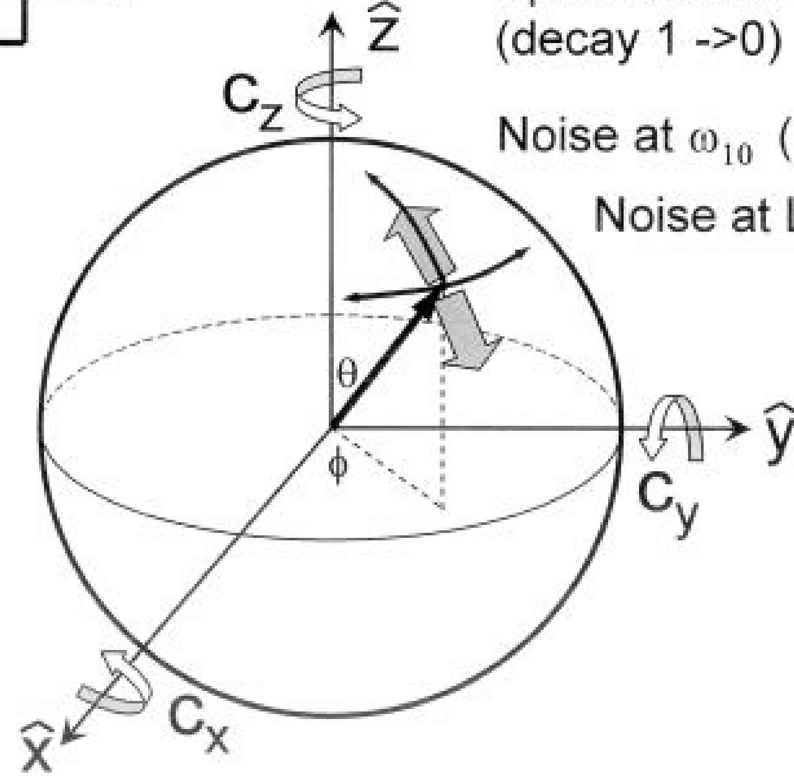
HW problem for making high-R I-source:



Josephson Junction Decoherence

$$H_{(2)} = \boldsymbol{\sigma}_x \bullet I_{\mu wc}(t) \bullet (\hbar / 2\omega_{10} C)^{1/2} / 2$$
$$+ \boldsymbol{\sigma}_y \bullet I_{\mu ws}(t) \bullet (\hbar / 2\omega_{10} C)^{1/2} / 2$$
$$+ \boldsymbol{\sigma}_z \bullet \delta I_{dc}(t) \bullet (\partial E_{10} / \partial I_{dc}) / 2$$





Spontaneous Emission
(decay 1 ->0)

Noise at ω_{10} (σ_x, σ_y op.'s)

Noise at LF (σ_z op.'s)

Decoherence arises from noise and dissipation

Decoherence from noise
(need more gen model than $1 \to 0$)

(1) Density Matrix Approach (standard, won't talk about)
- Ensemble avg, good for NMR, we use
- Only treats rates (like T_1) exp. decay).
- Not good for correl. noise (like we have)

relate results

(2) Noise in Bloch vector (most physical)
- Can treat correl noise
- Similar to treating noise in electrical circuits
- Full thy, use with "quantum Monte Carlo/jumps"
(Not go into here, but readily generalized
Just need to add T_1 energy decay)

Noise Theory:

If control signals change B.V, then
noise in control signal gives noise in B.V!

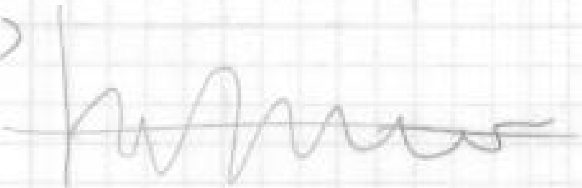
Spin:	Z	B_z	DC (low freq)
	X, Y	B_x or B_y	HF (transit. freq)
ϕ qubits:	Z	I	DC
	X, Y	I	HF

Concerned about noise in bias I.

Noise

Energy Decay From Dissip; $1 \to 0$. "T_1" state decay.
Can also have $1 \leftrightarrow 0$, dephasing from noise.
1st, need to describe noise.

$I(t)$



Random Fluctuations of I (or V)

→ Assume comes from sum of sine + cosine waves of random Ampl's + phases

$$I(t) = \sum_F S_F \sin(2\pi F t) + C_F \cos(2\pi F t)$$

Random

$$\langle I^2 \rangle = \frac{1}{2} \sum_F \{ (C_F)^2 + (S_F)^2 \}$$

↳ $\langle \sin^2 \rangle = 1/2$

Incoherent Sum (of Power) since {sin, cos $\perp$; Diff. F $\perp$}

$\{(S_F)^2 + (C_F)^2\}$ gives a spectral density, noise as a function of freq.

To meas.

$I(t)$ (old school) — [Bandpass filter at f, 1 Hz wide] (f) — [I^2 detector (power)] — $S_I(F)$ ↳ (I^2/Hz)

$I(t)$ (modern) — [Digitize] — [FFT] — [sum $|A_f|^2$ over 1 Hz] — $S_I(F)$

Note: With classical noise, only makes sense to discuss noise at (+) freq. only (No (−)f filters!).

$$\langle I^2 \rangle = \int_0^\infty S_I(f)\, df$$

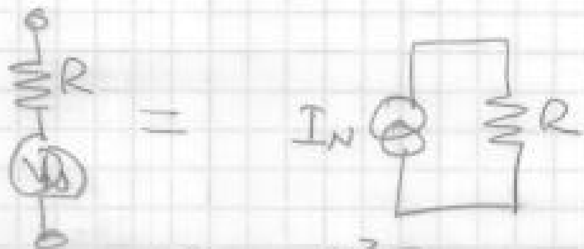
If want to know time correlation of noise, Wiener-Kinchine theorem (not derived here)

$$\langle I(t)\, I(0) \rangle = \int_0^\infty \underline{\underline{df}}\; S_I(f) \cos 2\pi f t$$

(t - Correl. = FT of $S_I(f)$)
($t=0$ is top eqn) Total Noise

<u>Example</u>: White noise ; $S_I(f) =$ const.

Resistor noise



$$S_{V_N} = R^2 S_{I_N}$$

$$S_{I_N} = \frac{4kT}{R} \quad \text{(classical)}$$

FT of const is δ function,

so noise <u>uncorrelated</u> in time.

[Computer simulation: Take each $I_N(t)$ randomly
or Random Ampls in f, then F.T.]

Phase Noise (Z)   consider $|0\rangle+|1\rangle$ state $|1\rangle e^{i\phi}$

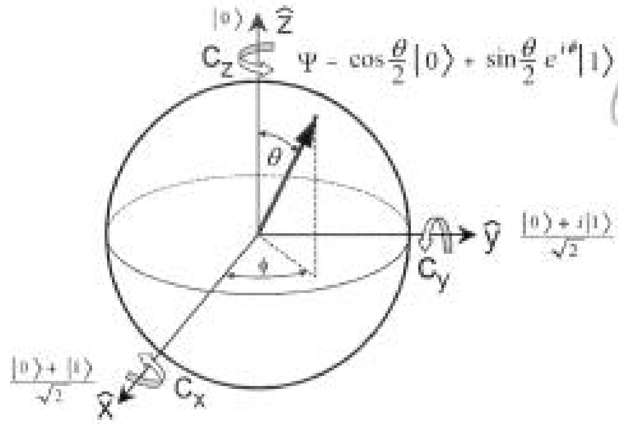


FIG. 2. Bloch sphere representation of the qubit state. Control vector $c=(c_x,c_y,c_z)$ rotates the Bloch vector around axis of direction $c_x\hat{x}+c_y\hat{y}+c_z\hat{z}$ with angle $|c|$.

and $I_{lf}(t)$. If these control currents have constant values over time Δt, we can define a control vector $\vec{c}=(c_x,c_y,c_z)$ with

$$\vec{c}=\left(I_{\mu wc}\sqrt{\frac{\hbar}{2\omega_{10}C}},I_{\mu ws}\sqrt{\frac{\hbar}{2\omega_{10}C}},I_{lf}\frac{\partial E_{10}}{\partial I_{dc}}\right)\frac{\Delta t}{\hbar}. \quad (6)$$

The control currents change the qubit state after time Δt according to the unitary transformation[1]

$$U=\exp[-iH_{(2)}\Delta t/\hbar], \quad (7a)$$

$$=\exp[-i\hat{\sigma}\cdot\vec{c}/2], \quad (7b)$$

$$=\hat{\sigma}_0\cos\frac{|\vec{c}|}{2}-i\frac{\hat{\sigma}\cdot\vec{c}}{|\vec{c}|}\sin\frac{|\vec{c}|}{2}, \quad (7c)$$

where $\hat{\sigma}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$ and $\hat{\sigma}_0$ is the identity matrix.

One way to visualize how $\vec{c}$ controls the qubit state is via the standard Bloch-sphere description. As illustrated in Fig. 2, the direction of the Bloch vector describes the qubit state according to $\Psi=\cos(\theta/2)|0\rangle+\sin(\theta/2)\exp(i\phi)|1\rangle$. The angle θ of the vector corresponds to the occupation amplitude of the state, whereas the angle ϕ gives the phase of the state. The probability of measuring the ground state is given by $\cos^2(\theta/2)$. Operations of $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ correspond to rotations of the state vector around the $\hat{x}$, $\hat{y}$, and $\hat{z}$ axis, respectively. In general, a control vector $\vec{c}$ rotates the Bloch vector around the $\vec{c}$ axis with angle $|\vec{c}|$. For example, a "$\pi/2$-pulse" with control vector $\vec{c}=(0,\pi/2,0)$ changes the state $|0\rangle$ to the state $(|0\rangle+|1\rangle)/\sqrt{2}$.

III. CALCULATION OF DECOHERENCE FOR AN ARBITRARY NOISE SOURCE

Because the bias current controls the qubit, noise in the bias current fluctuates the qubit state and causes decoherence. In this section we calculate how noise randomly rotates the Bloch vector around the three axes. Because we separated the effect of the bias current into low $I_{lf}(t)$ and microwave frequency $I_{\mu wc}(t)\cos\omega_{10}t+I_{\mu ws}(t)\sin\omega_{10}t$ components, the effect of noise can be separated likewise. Since the net effect of these rotations depends on the state of the qubit, we calculate how these fluctuations affect the measurement of the state for two typical experimental situations.

flucts in ϕ

Current noise at low frequency fluctuates the c_z component of the control vector, which randomly rotates the Bloch vector around the $\hat{z}$ axis due to $\hat{\sigma}_z$ operations. These random rotations produce noise in the phase ϕ of the qubit state. Since the phase is $\boxed{\phi(t)=\int_0^t dt\,\omega_{10}(t)}$, the phase noise after a time t is

$\omega_{10}(I(t))$

$$\phi_n(t)=\frac{\partial\omega_{10}}{\partial I_{dc}}\int_0^t dt'\,I_n(t'). \quad (8)$$

Physically, phase noise arises from noise current flowing through the nonlinear inductance of the junction that in turn causes ω_{10} to vary.

$\langle\phi_n\rangle=0$

The magnitude of the phase noise is described by its mean-squared value $\langle\phi_n^2(t)\rangle$. This quantity is calculated with the noise power of I_n, described as the spectral density $S_I(f)$. It is defined as the mean-squared amplitude of the current noise at frequency f per 1 Hz bandwidth. The time average of the correlation function is computed with the noise power by

$\langle I^2\rangle=\int_0^\infty df\,S_I(f)$

$$\langle I_n(t)I_n(0)\rangle=\int_0^\infty df\,S_I(f)\cos 2\pi ft. \quad (9)$$

Using Eq. (8) the mean-squared phase noise is

W.K

$$\langle\phi_n^2(t)\rangle=\left(\frac{\partial\omega_{10}}{\partial I_{dc}}\right)^2\left\langle\int_0^t dt'\,I_n(t')\int_0^t dt''\,I_n(t'')\right\rangle \quad (10a)$$

using

$$=\left(\frac{\partial\omega_{10}}{\partial I_{dc}}\right)^2\int_0^\infty df\,S_I(f)\int_0^t dt'\int_0^t dt''\,\mathrm{Re}\,e^{i2\pi f(t'-t'')} \quad (10b)$$

sp. out

$$=\left(\frac{\partial\omega_{10}}{\partial I_{dc}}\right)^2\int_0^{\omega_{10}/2\pi} df\,S_I(f)\,W_0(f), \quad (10c)$$

where $W_0(f)$ is a spectral weight function given by

t', t'' complex conj.

$$W_0(f)=\left|\int_0^t dt'\,e^{i2\pi ft'}\right|^2 \quad (11a)$$

$$=\frac{\sin^2(\pi ft)}{(\pi f)^2}. \quad (11b)$$

$\to \frac{(\pi f t)^2}{(\pi f)^2} \to t^2$

The phase noise integral is cutoff for frequencies greater than $\omega_{10}/2\pi$. For these frequencies, the noise current primarily flows through the junction capacitance, not the junction, and thus does not significantly modulate ω_{10}. Furthermore, noise *at* ω_{10} should not be included because it is accounted for in stimulated transitions, as computed below. Integrating the noise to a cutoff frequency $\omega_{10}/2\pi$ is a good approximation because for most circuit impedances a change in this cutoff frequency only logarithmically affects the phase noise [see Eq. (26)].

Area $\propto t$

White noise

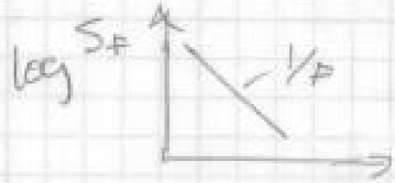
$$\langle\phi_N^2\rangle = \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 S_F^0 \int_0^\infty \frac{d(\pi t f)}{(\pi t)} \frac{\sin^2(\pi f t)}{(\pi f t)^2}\frac{1}{t^2}$$

$$= \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 S_F^0 \frac{t}{\pi}\underbrace{\int_0^\infty \frac{\sin^2 x}{x^2}\,dx}_{=\pi}$$

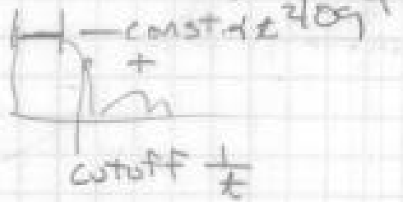
$\langle\phi_N^2\rangle \propto t$; like diffusion! $= \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 S_F^0\, t$

1/F noise

(more physical, q, Φ, I_0 flucts)



$$\langle\phi_N^2\rangle = \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 \int_0^\infty df\, \frac{S_I^*(1\,Hz)}{f}\, \frac{\sin^2(\pi f t)}{(\pi f)^2}$$



$$\simeq \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 S_I^* \int_0^{\sim 1/t} \frac{df}{f}\, t^2$$

HF cutoff

$$\simeq \left(\frac{\partial\omega_{10}}{\partial I}\right)^2 S_I^*(1\,Hz) \ln\left(\frac{0.401}{f_m t}\right) t^2$$

$$\equiv \left(t/t_{1/f}\right)^2$$

time of exp

$\delta\omega_{10}$

$\langle\phi_N^2\rangle \propto t^2$, makes sense!

LF ($\frac{1}{f}$) noise ; bias fluctuates very slowly, different $\omega_{10} + \delta\omega_{10}$ (LF) each experiment , $\phi_N \propto \int_0^t \delta\omega_{10}\,dt \propto t$

so $\phi_N^2 \propto t^2$.

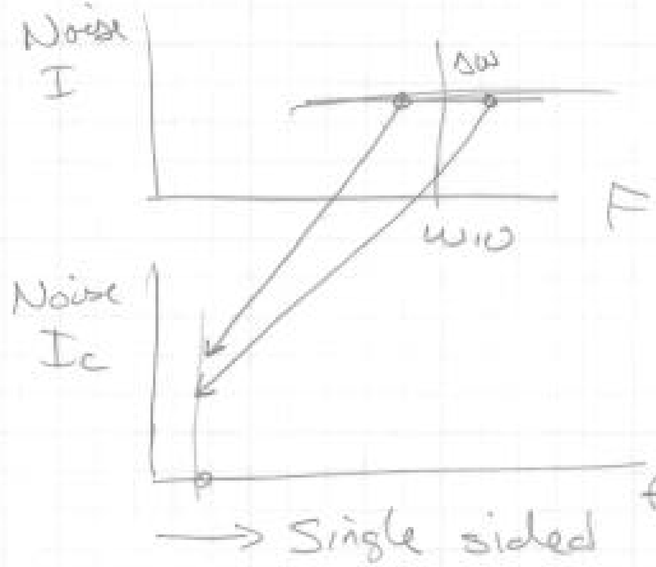
X, Y errors from H.F. Noise

$$I = I_{LF}(t) + I_c(t)\cos\omega_{10}t + I_s(t)\sin\omega_{10}t$$

Z X Y

$$H = \frac{\sigma_z}{2}\left(\frac{\partial\omega_{10}}{\partial I}\right)I_{LF}(t) + \frac{\sigma_x}{2}\frac{1}{\sqrt{2E_0C}}I_c(t) + \frac{\sigma_y}{2}\frac{1}{\sqrt{2E_0C}}I_s(t)$$

Need to express H.F. noise around ω_{10}
as noise in $I_c(t)$, $I_s(t)$



Noise at $\pm\Delta\omega$ mixes to same freq. $\Delta\omega$

$$S_{I_c} = S_{I_s} = 2\,S_I(\omega_{10}/2\pi)$$

since assume noise const. around $\omega_{10}/2\pi$

Noise Mixing

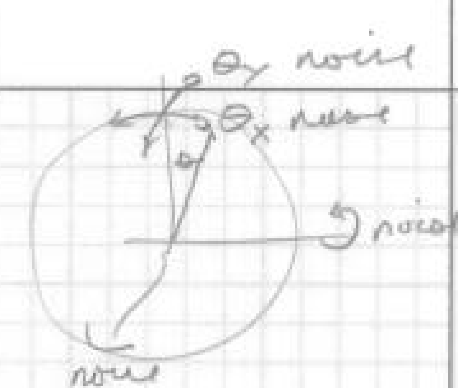
$$I_N = \sum_{\pm\Delta\omega} c_{\Delta\omega} \cos(\omega_{10} t + \Delta\omega t) + s_{\Delta\omega} \sin(\omega_{10} t + \Delta\omega t)$$

Noise from +- $\Delta\omega$ around ω_{10}

$$= \sum_{\pm\Delta\omega} c_{\Delta\omega} [\cos(\omega_{10} t)\cos(\Delta\omega t) - \sin(\omega_{10} t)\sin(\Delta\omega t)] + s_{\Delta\omega}[\sin(\omega_{10} t)\cos(\Delta\omega t) + \cos(\omega_{10} t)\sin(\Delta\omega t)]$$

$$= \cos(\omega_{10} t) \sum_{\Delta\omega} \{(c_{+\Delta\omega} + c_{-\Delta\omega})\cos(\Delta\omega t) + (s_{+\Delta\omega} - s_{-\Delta\omega})\sin(\Delta\omega t)]\} \; + \; \sin(\omega_{10} t)\sum_{\Delta\omega} \ldots$$

Noise adds from +$\Delta\omega$ and - $\Delta\omega$



$\underline{\Theta \text{ errors from}}$ noise in μw drive

I_c, I_S white noise

$$\langle \theta_x^2 \rangle = \frac{1}{2\pi\omega_{10}C} 2S_I(\omega_0/2\pi)\, t$$

$$= \langle \theta_y^2 \rangle$$

(just like noise in z, change constants)

Total tilt θ of B.V. is (Starting from $|0\rangle$ state (at top))

$$\langle \theta^2 \rangle = \langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle$$

$$= \frac{1}{2\pi\omega_0 C} 4 S_I\, t$$

Ampl $|0\rangle$ = $\cos(\theta/2)$

$$P_0 = \langle \cos^2(\tfrac{\theta}{2}) \rangle \simeq \langle \left[1 - \tfrac{1}{2}(\tfrac{\theta}{2})^2\right]^2 \rangle$$

$$\simeq \langle 1 - \cancel{2}\tfrac{1}{\cancel{2}}(\tfrac{\theta}{2})^2 \rangle$$

$$= 1 - \frac{1}{4}\langle \theta \rangle^2$$

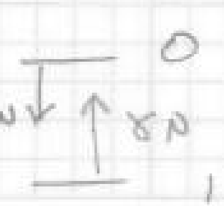
$$= 1 - \underbrace{\frac{1}{2\pi\omega_{10}C} S_I\left(\frac{\omega_{10}}{2\pi}\right) t}$$

$\hookrightarrow$ Represents noise transition rate γ_N of $0 \rightarrow 1$

Likewise, if start at bottom $|1\rangle$;

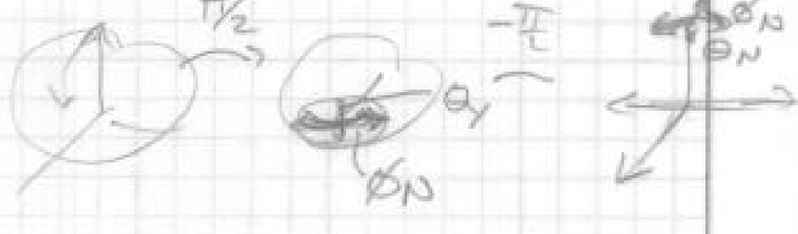
γ_N = same as above

transition picture



ϕ_N and Ramsey Sequence

How to meas ϕ_N? — γ_N meas. from $|0\rangle$ or $|1\rangle$ state



With ramsey seq.

$\langle\phi_N^2\rangle$ seen as noise in angle θ_x

$\langle\theta_y^2\rangle$ " " " " angle θ_y

$\langle\theta_x^2\rangle$ not seen!

$$P_0 = 1 - \frac{1}{4}\langle\theta_y^2\rangle - \frac{1}{4}\langle\phi_N^2\rangle$$

$$= 1 - \underbrace{\frac{1}{2}}\gamma_N t - \frac{1}{4}\langle\phi_N^2\rangle$$

$\frac{1}{2}$ as bis because only 1 component of μwave noise.

Beyond small angle expansion of θ^2 –

Many modes of noise $\Rightarrow$ Gaussian Distribution

$$\frac{dp(x)}{dx} = \frac{e^{-x^2/2\langle x^2\rangle}}{\sqrt{2\pi\langle x^2\rangle}}$$

$$\text{so}\quad P_0 = \int_{-\infty}^{\infty} d\theta\, \frac{e^{-\theta^2/2\langle\theta^2\rangle}}{\sqrt{2\pi\langle\theta^2\rangle}}\cos^2(\theta/2)$$

$$= \frac{1}{2} + \frac{1}{2}\exp\left[-\langle\theta^2\rangle/2\right]$$

$\frac{1}{2}$ also found in transition picture

$0 \leftrightarrow 1$ transitions cause avg to $1/2$ and dephasing

γ_N
$\langle \theta^2 \rangle \gg \langle \theta_y^2 \rangle$

$$P_0 = \frac{1}{2} + \frac{1}{2} \exp\left[- \frac{1}{2\hbar\omega_0 C} 2 S_I t/2 \right]$$

$$= \frac{1}{2} + \frac{1}{2} \exp\left[- \gamma_N t \right]$$

ϕ_N

$$P_0 = \frac{1}{2} + \frac{1}{2} \exp\left[- \langle \phi_p^2 \rangle / 2 \right]$$

$(1/f) \rightarrow$

$$\cancel{= \frac{1}{2} + \frac{1}{2} \exp\left[- \left(\frac{\partial \omega_{10}}{\partial I}\right)^2 S_I^* \ln(\;\;) \left(\frac{1}{2}\right) t^2 \right]}$$

$$\equiv \frac{1}{2} + \frac{1}{2} \exp\left[- \frac{1}{2} (t/t_{1/f})^2 \right]$$

Combining both (as non-correlated)

$$P_0 \simeq \frac{1}{2} + \frac{1}{2} \exp\left[- \gamma_N t \right] \exp\left[- \frac{1}{2} \left(\frac{t}{t_{1/f}}\right)^2 \right]$$

Multiply decay from both sources.

Dissipation and Noise

(1) Up to now, treated separately! But can be combined!

Idea: Classical current $\sim \cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$

has $\pm\omega t$ freq components

Quantum states differ by $e^{-i\omega_0 t}$ factor,

so $\pm\omega$ represents $\pm$ energy transitions

* Thus, dissip repres. by dif'l noise $\pm$ freq's!

(2) How treat dissip in Q.M. since Q.M. reversible?

Std. - Fermi Golden Rule

Small coupling to many states

Energy/State never comes back.

- Example was in t-line, where line $\to \infty$ means energy reflected back to end

* Model for a resistor:

Finite length x; can describe by bath of L-C modes

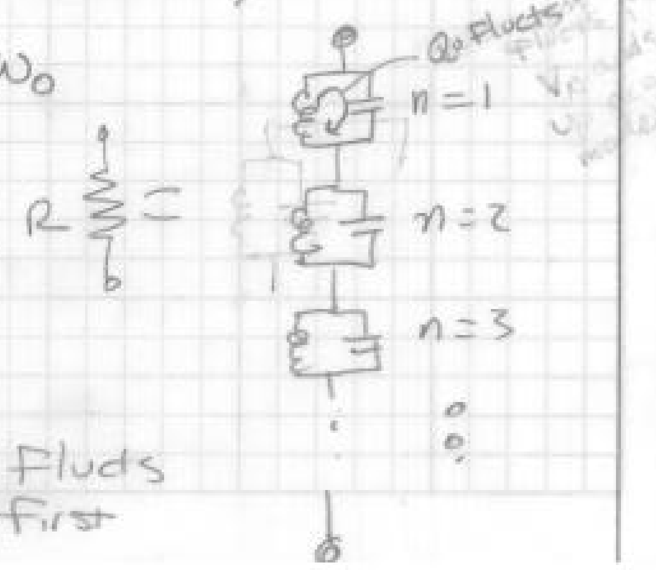
let $x \to \infty$ for ∞ density of modes.

reson: $\omega_n = n\left(\frac{\pi v}{x}\right) \equiv n\omega_0$

$C_n = n\frac{\pi}{2Z_0}\frac{1}{\omega_n}$

$= \left(\frac{\pi}{2Z_0}\right)\frac{1}{\omega_0}$

$=$ const.



With series circuit, V_N flucts add up $\to$ Calc. that first

To compute noise($\pm\omega$); look at correlator

now operators

$$\langle \hat{V}(t)\hat{V}(0)\rangle = \frac{\langle \hat{Q}(t)\hat{Q}(0)\rangle}{C^2}$$

$$\hat{Q}(t) = \frac{\sqrt{2\hbar\omega_n C}}{2}\left(a e^{-i\omega_n t} + a^+ e^{i\omega_n t}\right)$$

(For 1 mode res. freq. ω_n)

$$= \frac{2\hbar\omega_n C}{4C^2}\left\langle\left(a e^{-i\omega_n t} + a^+ e^{i\omega_n t}\right)(a - a^+)\right\rangle$$

$$= \frac{\hbar\omega_n}{2C}\left[\langle a a^+\rangle e^{-i\omega_n t} + \langle a^+ a\rangle e^{i\omega_n t} + \underbrace{\langle a a\rangle}_{0} \ldots - \underbrace{\langle a^+ a^+\rangle}_{0}\right]$$

$$\langle a^+ a\rangle \neq \langle a a^+\rangle = \langle a^+ a\rangle + 1$$

Something interesting $\rightarrow$ non classical $\pm\omega$ spectral weight!

For H.O. state (bosonic mode), temp. T

$$\langle a^+ a\rangle_T = \frac{1}{e^{\hbar\omega_n/kT} - 1} \rightarrow 0 \text{ with } T \rightarrow 0$$

$$\langle a a^+\rangle_T = \frac{\cancel{1} + (e^{\hbar\omega_n/kT} \cancel{-1})}{e^{\hbar\omega_n/kT} - 1} \rightarrow 1 \text{ with } T \rightarrow 0$$

$$= \frac{1}{1 - e^{-\hbar\omega_n/kT}}$$

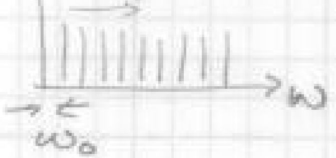
Only $e^{-i\omega_{10} t}$ freq. content

$$\langle \hat{V}(t)\hat{V}(0)\rangle = \sum_n \frac{\hbar\omega_n}{2\left(\frac{\pi}{2Z_0}\frac{1}{\omega_0}\right)}\left[\frac{1}{e^{\hbar\omega_n/kT}-1}e^{-i\omega_n t} - \frac{1}{e^{-\hbar\omega_n/kT}-1}e^{i\omega_n t}\right]$$

total (sum over all modes)

const

$$= \frac{Z_0}{\pi}\underbrace{\sum_n \omega_0}_{\int_0^\infty d\omega}\left[\frac{\hbar\omega_n}{e^{\hbar\omega_n/kT}-1}e^{-i\omega_n t} + \frac{-\hbar\omega_n}{e^{-\hbar\omega_n/kT}-1}e^{i\omega_n t}\right]$$



$+\omega_n \rightarrow -\omega_n$

$$= 2Z_0\int_{-\infty}^{\infty} d\left(\frac{\omega}{2\pi}\right)\left[\frac{\hbar\omega}{1-e^{-\hbar\omega/kT}}e^{+i\omega t}\right]$$

Z_0 = resistance

↳ Now 2-sided integral

* Now F.T. both sides

$$S_V^{QU}\left(\frac{\omega}{2\pi}\right) = 2Z_0\frac{2\hbar\omega}{1-e^{-\hbar\omega/kT}}$$

+ and − frequencies!

General case for Z

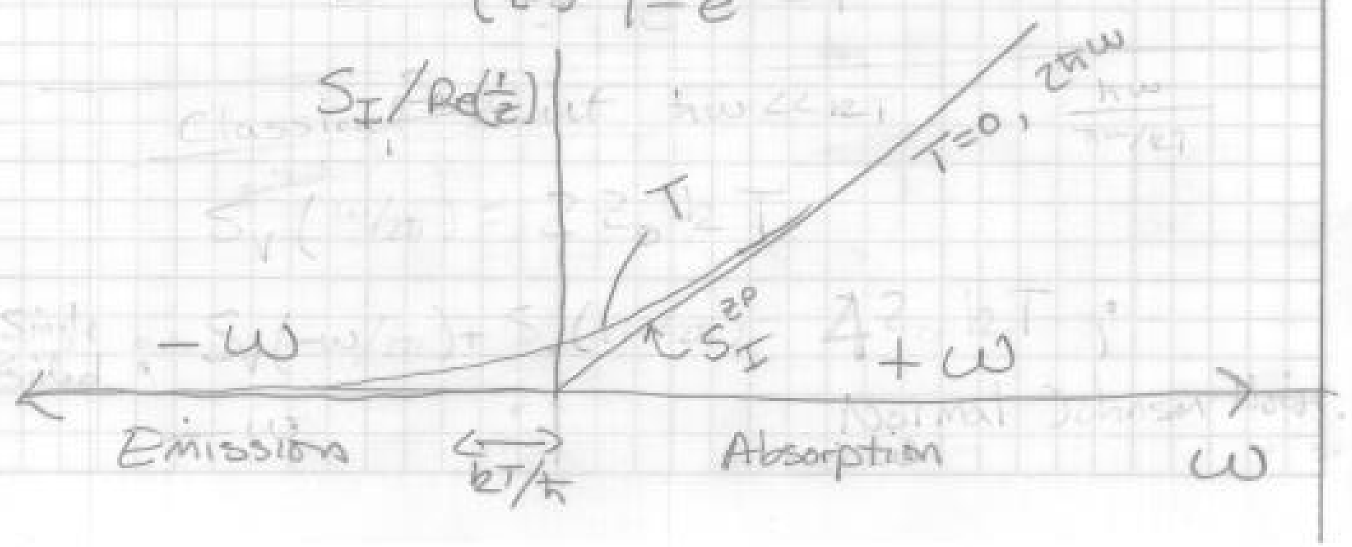
$$= 2\,\mathrm{Re}\{Z\}\frac{2\hbar\omega}{1-e}$$

Thev. equiv

$$S_I^{QU} = S_V/|Z|^2$$

$$\mathrm{Re}\frac{Z}{|Z|^2} = \mathrm{Re}\left(\frac{Z}{ZZ^*}\right) = \mathrm{Re}\left(\frac{1}{Z}\right)$$

$$= \mathrm{Re}\left\{\frac{1}{Z}\right\}\frac{2\hbar\omega}{1-e^{-\hbar\omega/kT}}$$



Physics of Spectrum.

(0) Absorbtion $= +\omega$, Emission $= -\omega$ $\Rightarrow$ $\hat{V}(t) \propto \underbrace{a}_{\text{emit photon}} e^{-i\omega_{n}t} + \underbrace{a^{+}}_{\text{absorb photon}} e^{i\omega_{n}t}$

① Emission $= -\omega =$ Blackbody Formula (1D)

② $S^{ZP}(\omega/2\pi) = S_I^{QU}(+\omega/2\pi) - S_I^{QU}(-\omega/2\pi)$ Diff. of spectr.

$$= 2\,\mathrm{Re}\left(\tfrac{1}{Z}\right)\left[\frac{\hbar\omega}{1-e^{-\hbar\omega/kT}} - \frac{-\hbar\omega}{1-e^{+\hbar\omega/kT}}\right] \quad +?$$

$$= 2\,\mathrm{Re}\left(\tfrac{1}{Z}\right)\hbar\omega\,\underbrace{\left[\langle a a^{+}\rangle - \langle a^{+} a\rangle\right]}_{=1}$$

$$= \mathrm{Re}\left(\tfrac{1}{Z}\right) 2\hbar\omega$$

(Same calc., only use $+\omega$)

$$\gamma_1 = \gamma_N \left(S_I \Leftarrow S_I^{ZP}(\omega_{10}/2\pi) \right)$$

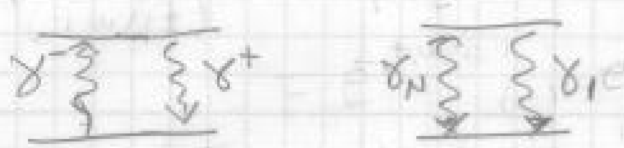
$$= \frac{1}{\cancel{2\hbar\omega_{10}}C}\,\cancel{2\hbar\omega_{10}}\,\mathrm{Re}\left(\tfrac{1}{Z}\right)$$

$$= \frac{1}{C}\mathrm{Re}\left(\tfrac{1}{Z}\right)$$; Dissipation Formula Derived previously

* Z.P. flucts. drive $1\to 0$ transition (dissipation)

$$\begin{cases} S_I^{QU}(-\omega/2\pi) & \text{— Classical Noise} \quad 0 \leftrightarrow 1 \; (\gamma_N) \\ S_I^{ZP}(+\omega/2\pi) & \text{— Z.P. Noise} \quad 1 \to 0 \; (\gamma_1) \end{cases}$$

②′ Two ways to picture noise



+/− rates ; Class + Decay

$\gamma^+ = \gamma_1 + \gamma_N$

$\gamma^- = \gamma_N$

③ $$\frac{S_I^{QU}(-\omega/2\pi)}{S_I^{QU}(+\omega/2\pi)} = e^{-\hbar\omega/kT}$$ 2 transition rates give Boltz factor correctly

④ $S_I^{+}(\omega/2\pi) = S_I^{QU}(-\omega/2\pi) + S_I^{QU}(\omega/2\pi)$ Single-sided Spectrum

(do math) $= 2\hbar\omega \coth(\hbar\omega/2kT)\,\mathrm{Re}\left(\tfrac{1}{Z}\right)$

$\to 4kT\,\mathrm{Re}\left(\tfrac{1}{Z}\right)$ $\hbar\omega \ll kT$

Johnson/Nyquist Noise of R

Measurement of Quantum Noise

Voltage state of CBJJ : HF noise mixed to LF

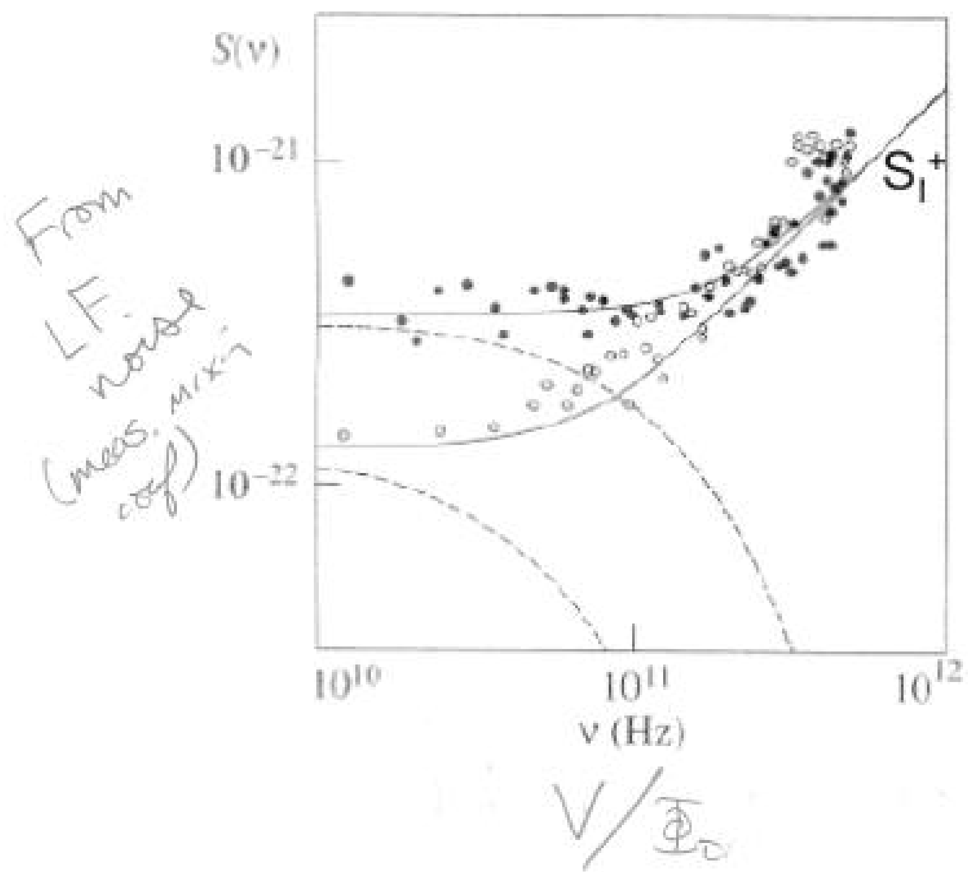


Fig. 1.2 Measured spectral density of noise current in the experiment of *Koch et al.* at 4.2 K (solid circles) and 1.6 K (hollow circles). The solid lines are the prediction of (1.2.5), while the dashed lines correspond to the Planck spectrum (1.2.3)

Like X, Y op's, Z op's need double-sided spectrum. To express as show previously (with 1-sided integral),

Use S_I^{+} to integrate both $\pm$ freqs.

$$\langle \phi_N^2 \rangle = \left(\frac{\partial \omega_{10}}{\partial I}\right)^2 \int_{-\infty}^{\infty} df\, W_\phi(f)\, S_I^{\Phi\upsilon}(\omega_{10}/2\pi)$$

$$= \left(\frac{\partial \omega_{10}}{\partial I}\right)^2 \int_{0}^{\infty} df\, W_\phi(f)\, S_I^{+}(\omega_{10}/2\pi)$$

cotanh( ) Formula, ZP noise included.

Using Noise

Decoherence = noise

→ How do I isolate my qubit from external world (which can drive noise into it?).

So far, shown **HOW** to **calc.** decoherence.

circuit $\xrightarrow{\text{calc.}}$ Decoh.

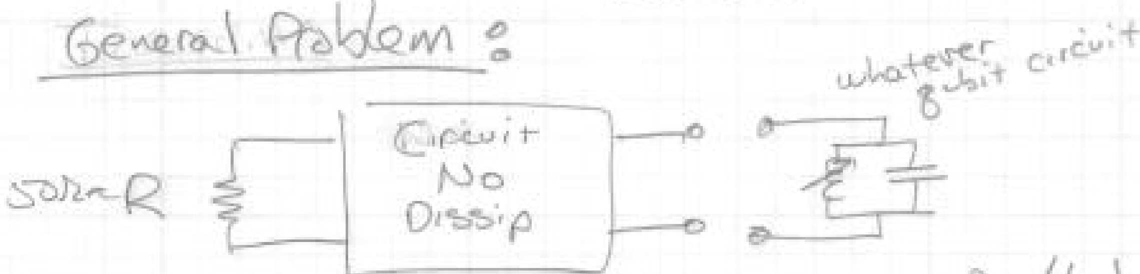
→ Really interested in *inverse* problem, principles behind designing a *good* circuit

external world

(50Ω @ Microwave)

Solution: Build a circuit to reduce (atten.) current noise!

General Problem:

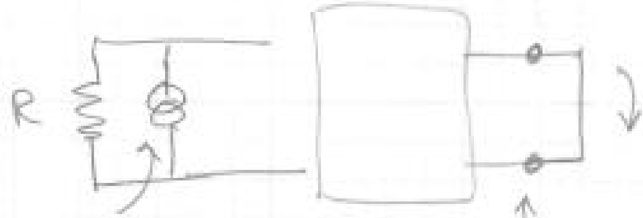


← what is $Re\left(\frac{1}{Z}\right)$ looking into leads

→ Tells us how external circuit loads/decoh qubit.

Solution:

(I)



Calculate S_{I_N} @ output

$S_I \propto \frac{1}{R}$
(input)

II 2-term dissip. part: $\mathrm{Re}(Y)$

$$\frac{1}{R_{out}} = \frac{1}{R} \; \frac{S_{I_N}}{S_I} \quad \begin{matrix} \leftarrow \text{output noise} \\ \leftarrow \text{input noise} \end{matrix}$$

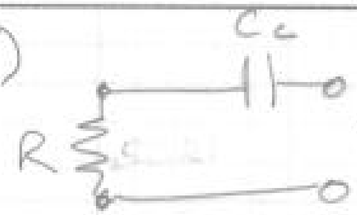
R_{out} is transformed <u>up</u> by (current)^2 ratio!

(This is, in fact, a classical concept).

There are 3 example circuits that are used in qubit devices —

(It's the simplicity of this calcul. that is so useful - can calculate with normal Z methods used previously; intuition is the key!)

(1)



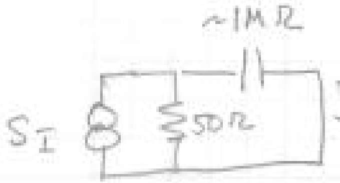
$\leftarrow Re\left(\frac{1}{z}\right)$

$$z = R + \frac{1}{i\omega C_c}$$

$$\frac{1}{z} = \frac{R - \frac{1}{i\omega C_c}}{R^2 + \left(\frac{1}{\omega C_c}\right)^2}$$

normal calc.

$$\frac{1}{R_{out}} = Re\left(\frac{1}{z}\right) \approx R(\omega C_c)^2$$



most flow thr R

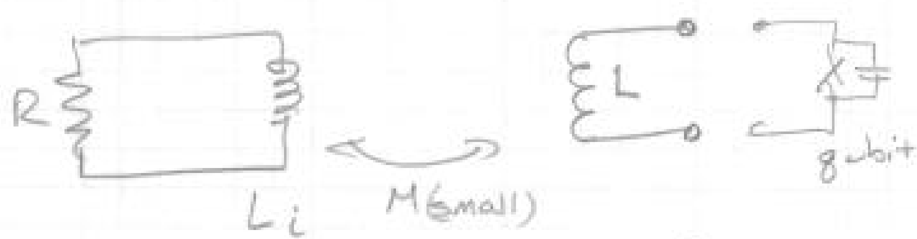
$$S_{IN} = \frac{R^2}{\left(\frac{1}{\omega C_c}\right)^2} S_I$$

$$\frac{1}{R_{OUT}} = R(\omega C_c)^2$$

$$=$$

$$\frac{1}{R_{OUT}} = \frac{1}{\cancel{R}} \frac{R^2}{\left(\frac{1}{\omega C_c}\right)^2}$$

(2)



↳ assume $\omega L_i \ll R$

$$\overline{\Phi_N^2} = M^2 S_I$$

$$I_N = \Phi_N / L$$

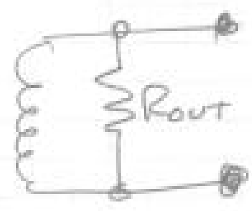
$$S_{IN} = \left(\frac{M}{L}\right)^2 S_I$$

$$\frac{1}{R_{out}} = \left(\frac{M}{L}\right)^2 R$$

⤳ R_{out} transformed up by $\left(\frac{L}{M}\right)^2$

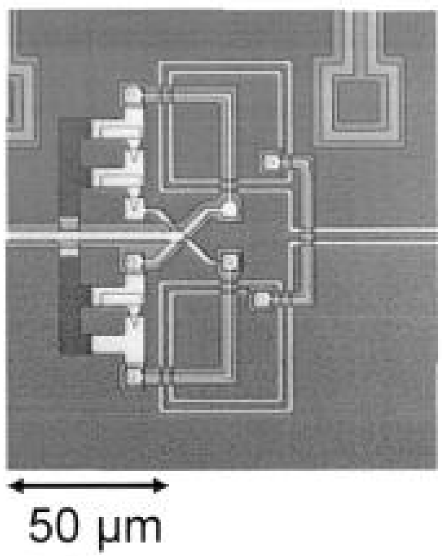
$50\,\Omega \rightarrow (100)^2 50$

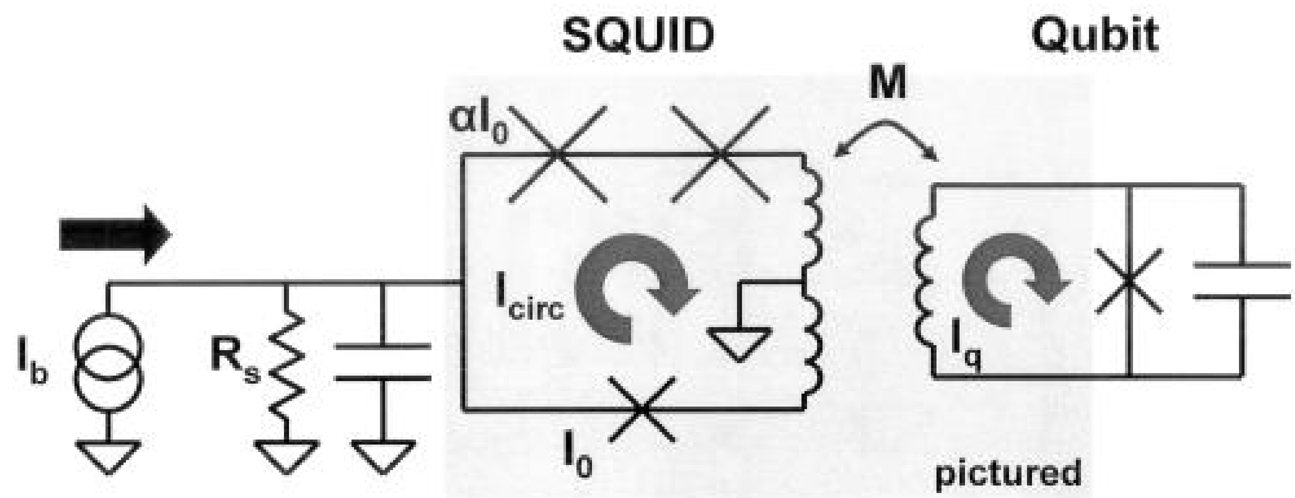
$\approx \frac{1}{2} M\Omega$



equiv. circuit

Variable Coupling SQUID Readout

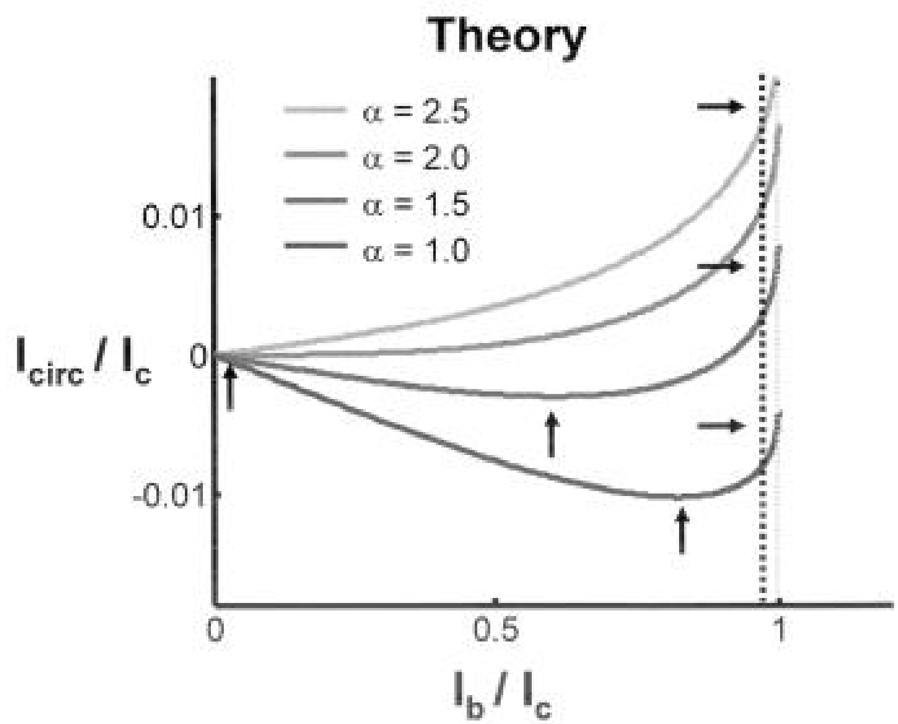




Squid is a variable transformer

Operation: squid balanced
I_b chosen so I_{circ} is flat
insensitive to flux

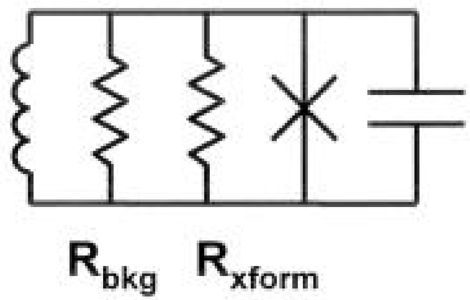
Measurement: squid unbalanced
ramp $I_b \rightarrow I_c$, I_{circ} large
sensitive to flux



Variable Impedance Transformer

Qubit dissipation from:

- x-formed SQUID shunt R_s
- unkown "background" dissipation



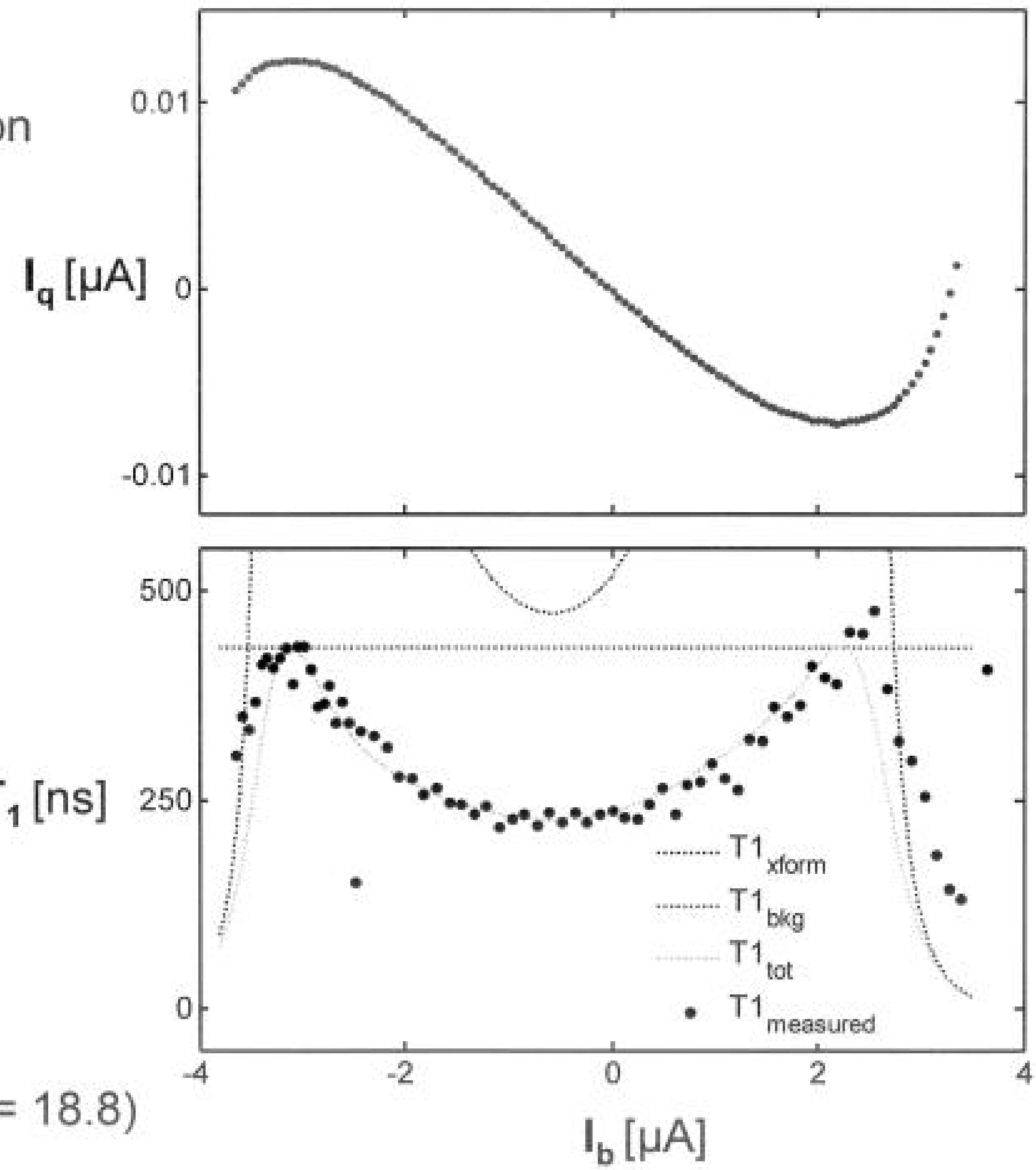
$$R_{xform} = \frac{R_s}{(dI_q / dI_b)^2}$$

$$R_{tot} = R_{xform} \,||\, R_{bkg}$$

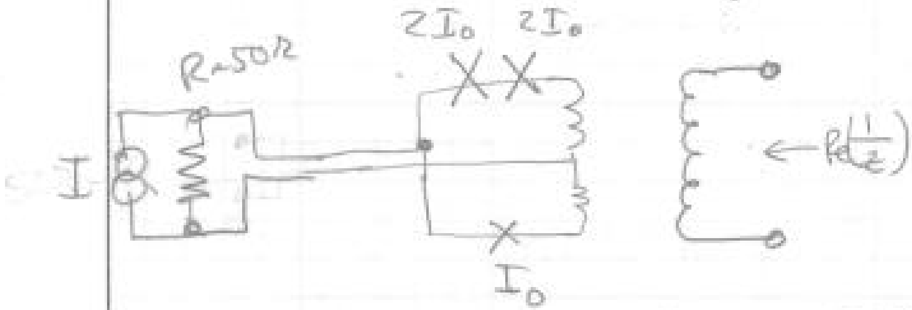
$$T_1 = R_{tot} C$$

Best Fit to Data:

- $T1_{bkg}$ = 433 ns
- R_s = 12.2 Ω (expected 30 || 50 = 18.8)



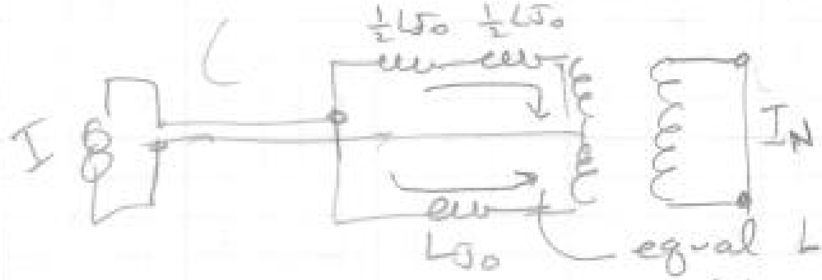
(3) Tunable coupling (SQUID measmt.)



What is $\mathrm{Re}\left(\frac{1}{Z}\right)$ vs I?

At $I=0$ $\quad L = \dfrac{\Phi_0}{2\pi I_0 \cos\delta}$; $L = L_{J0}$ bottom

$L = \frac{1}{2} L_{J0}$ top



equal L's ⇒ equal currents
no flux couplg!

so $\dfrac{dI_N}{dI} = 0$; $\quad \mathrm{Re}\left(\frac{1}{Z}\right) = 0$

(∞ imped. transf.)

SQUID + Loop are
decoupled

At large I; current in lower branch reaches it's I_0 sooner, so $\cos\delta \to 0$ faster. More L_J reduces I relative to top, producing imbalance and thus Φ coupling.

coupling
turns
on.

$$\frac{1}{R_{out}} = \left(\frac{\partial I_N}{\partial I}\right)^2 \frac{1}{R}$$

↑ Calc. or measure.

→ Show exp'l data.

→ Go thr. all qubits + show transf structures

(1) Our Data for R transf.

(2) ϕ qubit; talk about 3 transformers

(3) Q qubit

a) C_g for μ waves.

a) Outside noise shunted by
cap.
Large JJ. (10x I_0, smaller L)

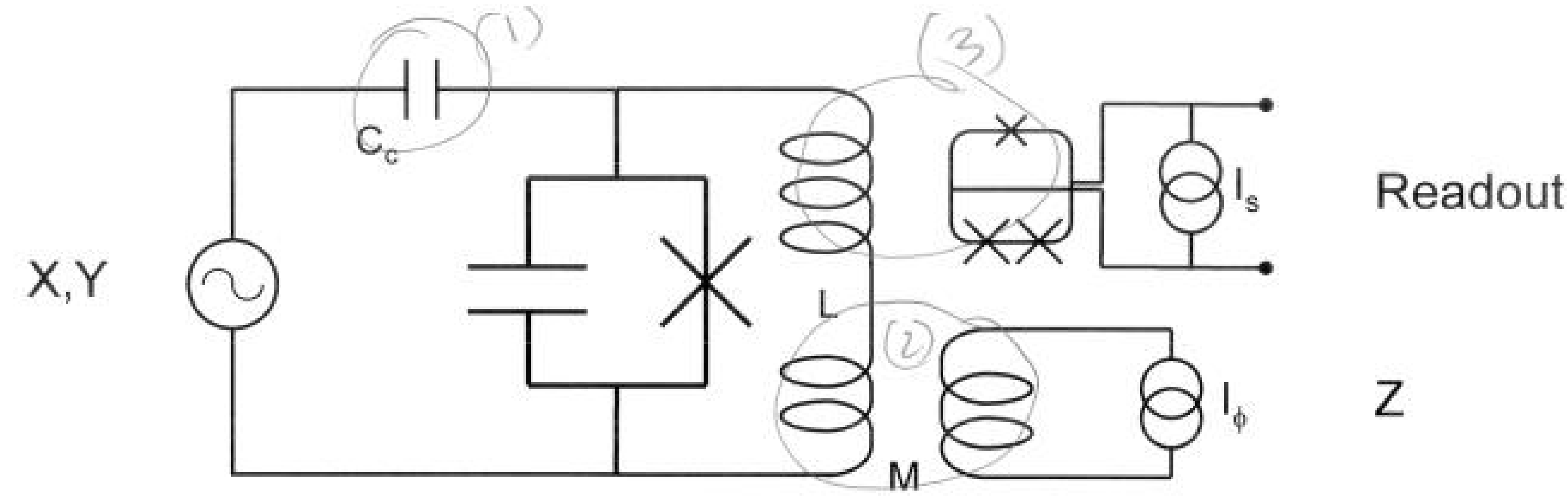
b) Φ transf. to loop.

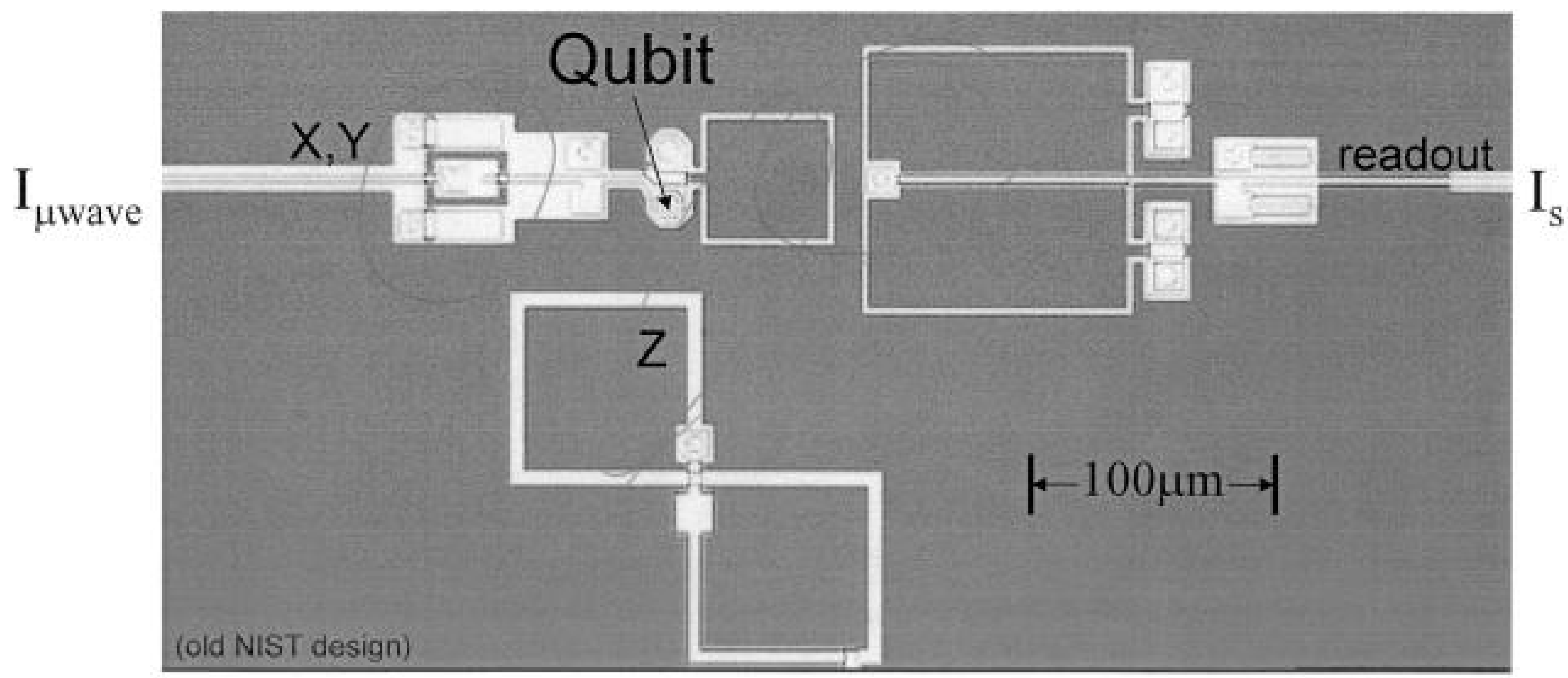
(4) Φ qubit

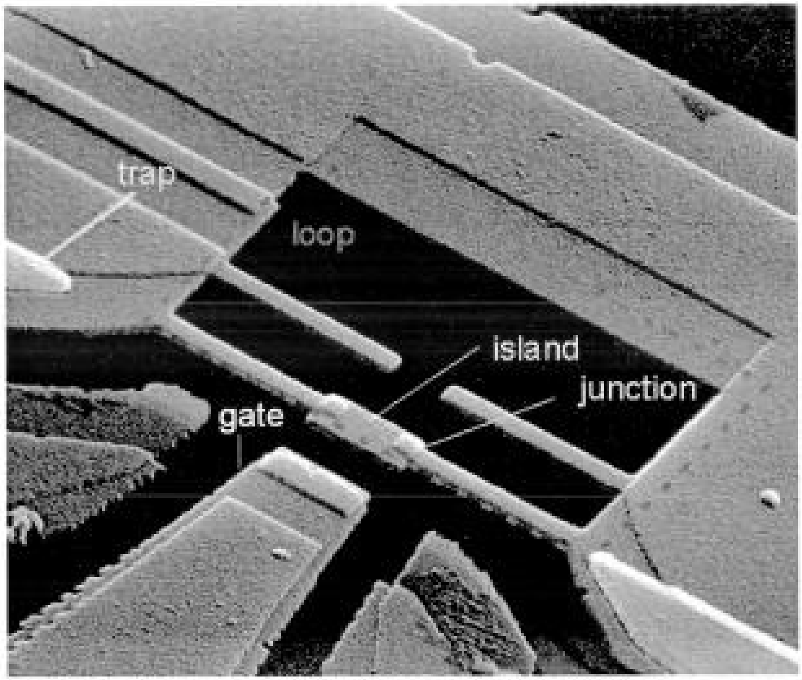
a) Φ coupling μwave to SQUID loop.

b) I_b, C shunts out some I
symmetry; I not couple to
flux (circ. current in loop).
(Φ bias changes this coupling,
like ϕ qubit).

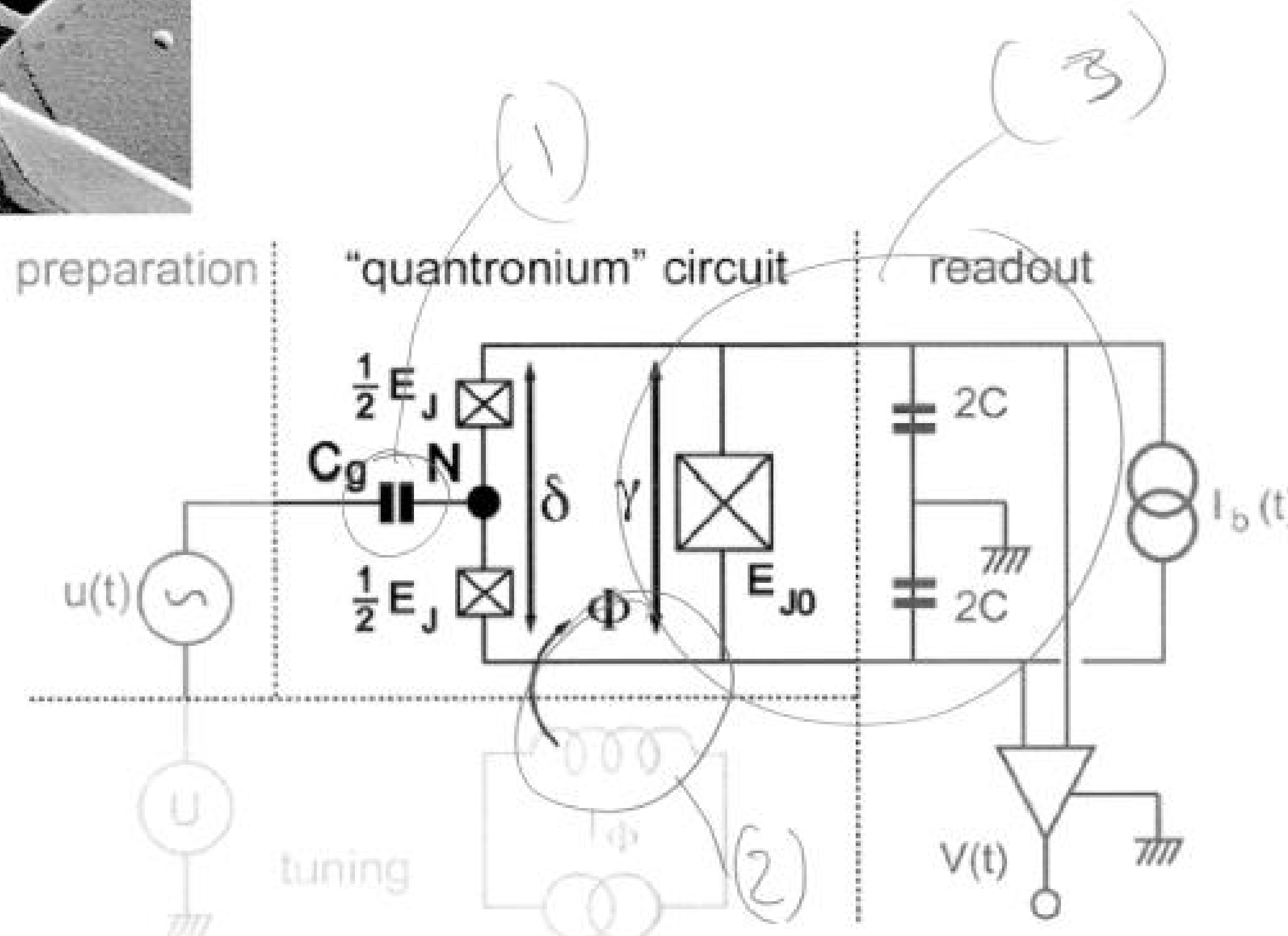
R Transformation : Phase Qubits







R Transformation : Charge Qubits



R Transformation: Flux Qubits