Erratum: Measurement of energy decay in superconducting qubits from nonequilibrium quasiparticles

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In Eq. (C7) of Ref. [1], the coupled differential equations for the change in the number n_i of quasiparticles at energy bin i are given as

$$\frac{d}{dt}n_i = G^s_{ji}n_j - \sum_j G^s_{ij}n_i - \sum_j (1+\delta_{ij})G^r_{ij}n_jn_i, \tag{1}$$

where $\Gamma^s_{i\to j} = \sum_j G^s_{ij}$ is the scattering rate from energy bin i to energy bin j and $\Gamma^r_{i,j} = \sum_j G^r_{ij}n_j$ is the recombination rate of quasiparticles in energy bins i and j. In this erratum, we show that these equations should instead be

$$\frac{d}{dt}n_i = \sum_j G^s_{ji}n_j - \sum_j G^s_{ij}n_i - \sum_j 2G^r_{ij}n_jn_i. \tag{2}$$

The factor of 2 in the recombination term is due to treating a quasiparticle in energy bin i recombining with one in j separately from one in j recombining with one in i. This arises from the definition of the recombination rate. To see this more clearly, consider a population of quasiparticles with density $n = n_{qp}/n_{cp}$ and a recombination matrix element Γ. Then the recombination rate is Γn^2, so the loss rate of quasiparticles is $2\Gamma n^2$ due to the loss of a pair of quasiparticles for each recombination event.

Now suppose that this population is split evenly into two energy bins, A and B, with nearly identical energies, so the recombination matrix element for all recombination events is Γ. Then, the loss rate for each bin should be Γn^2. According to Eq. (1), but including a symmetric loss for A and B, the loss rates for each possible recombination event are:

Recombination Pair	A-A	B-B	A-B	B-A	Total
Energy Bin A	$2\Gamma(n/2)^2$	0	$\Gamma(n/2)^2$	$\Gamma(n/2)^2$	Γn^2
Energy Bin B	0	$2\Gamma(n/2)^2$	$\Gamma(n/2)^2$	$\Gamma(n/2)^2$	Γn^2

Note that if recombination process B-A is not considered, the total loss rate in each channel would instead be $(3/4)\Gamma n^2$, not the expected Γn^2. Because of symmetry, it is easier to account for this by using a loss rate of $2\Gamma(n/2)^2$ in the first bin of the interaction and zero in the second bin, giving rise to Eq. (2) as indicated below:

Recombination Pair	A-A	B-B	A-B	B-A	Total
Energy Bin A	$2\Gamma(n/2)^2$	0	$2\Gamma(n/2)^2$	0	Γn^2
Energy Bin B	0	$2\Gamma(n/2)^2$	0	$2\Gamma(n/2)^2$	Γn^2

Note that similar behavior can be seen for subdividing the original energy bin into three or four equal bins.

[1] M. Lenander *et al.*, Phys. Rev. B **84**, 024501 (2011).