Emulation of a Quantum Spin with a Superconducting Phase Qudit

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In quantum information processing, qudits (d-level systems) are an extension of qubits that could speed up certain computing tasks. We demonstrate the operation of a superconducting phase qudit with $d = 5$, showing how to manipulate and measure the qudit state, including simultaneous control of multiple transitions. The qudit is used to emulate the dynamics of single spins with principal quantum number $s = 1/2$, 1 and $3/2$, allowing a measurement of Berry's phase and the even (odd) parity of integer (half-integer) spins under 2π-rotation. This extension of the two-level qubit to a multi-level qudit holds promise for more complex quantum computational architectures, and for richer simulations of quantum mechanical systems.

Quantum computers are typically thought of as being composed of qubits, or two-level quantum systems (*1*). However, one can also use qutrits (three-level systems) or, more generally, qudits (d-level systems), which can simplify some quantum computations (*2, 3*) and improve robustness in quantum cryptography (*4*). The advantages of qudits are also evident when one considers using a quantum computer not to perform computations, but rather to "emulate" another quantum system by directly implementing an analogous physical Hamiltonian. This requires a map between the Hilbert space and unitary operators of the emulator and the target system. If the target system contains parts with $d > 2$ levels, then it maps much more naturally to a set of qudits, making a qudit emulator potentially more efficient.

We demonstrate the operation of a superconducting phase qudit, with full unitary control and measurement of the state (*5, 6*). This device, one of a family of superconducting quantum information processing devices (*7*), is typically operated as a qubit (*8, 9*) by restricting it to the two lowest-energy eigenstates. By relaxing this restriction, we can operate it as a qudit where the number of levels d can be chosen as desired, in this case up to $d = 5$.

Emulation of spin, or intrinsic angular momentum, naturally calls for qudits with $d > 2$. A spin state is described by two quantum numbers (*10*), the principal quantum number $s = 0$, 1/2, 1, 3/2,... and the azimuthal quantum number m, limited to the $d = 2s + 1$ values $m = s$, $s - 1$,..., $-s$. For a given s, the general spin states $|\psi\rangle = \sum_m c_m |s, m\rangle$ span a d-dimensional Hilbert space, so while qubits can be used to model spin-1/2 physics, a qudit allows one to model spins $s \geq 1$ ($d \geq 3$).

When rotated about a closed path (Fig. 1), a spin state $|s, m\rangle$ acquires a phase factor $\exp(-im\Omega)$ where Ω is the solid angle enclosed by the path, as predicted by Berry (*11, 12*). For a 2π-rotation ($\Omega = 2\pi$), integer spins are unchanged while half-integer spins are multiplied by -1. This parity difference leads to the symmetric (anti-symmetric) statistics of bosons (fermions) under exchange, as described by the spin-statistics theorem

(*13, 14*). The effect of 2π-rotations was first observed on spins $s = 1/2$ via neutron interferometry (*15, 16*) and later for $s = 1$ and $s = 3/2$ in nuclear magnetic resonance (*17*). In superconducting qubits, the spin-1/2 parity (*18*) and Berry's phase (*19*) have been measured. Here we measure Berry's phase and spin parity for spin-1/2, spin-1 and spin-3/2 at all solid angles using our qudit emulation (*20*).

Our flux-biased phase qudit (Fig. 2A) is a nonlinear resonator formed by a Josephson junction, inductor and capacitor. Applied magnetic flux produces a cubic potential as a function of the junction phase δ, with barrier height ΔU that can be tuned to change the number of energy levels in the well (Fig. 2B). The cubic anharmonicity is crucial for qubit operation (*21*), allowing microwaves at frequency $\omega_{10} = (E_1 - E_0)/\hbar$ to drive transitions between $|0\rangle$ and $|1\rangle$, while minimizing "leakage" to $|2\rangle$ and higher (*22*). For measurement, a brief current pulse $I^{(1)}_{\text{meas}}$ is applied to lower the barrier and cause $|1\rangle$ (but not $|0\rangle$) to tunnel out of the well. An on-chip superconducting quantum interference device (SQUID) detects this tunneling (*23*).

For qudit operation, anharmonicity is again crucial as it ensures that all transition frequencies $\omega_{n,n-1} = (E_n - E_{n-1})/\hbar$ are unique, allowing frequency-selective control of all qudit states. In the present sample, transition frequencies are ~ 6 GHz, separated from each other by ~ 200 MHz (Fig. 2B). This separation is large enough that transitions can be selectively driven using fast pulses (compared to the state lifetimes), but small enough that the total bandwidth required is within that of our microwave control (*24*). This selective control of transitions between neighboring levels allows for the construction of arbitrary unitary gates on the d-level qudit manifold (*25*).

To measure the qudit, $d-1$ pulse amplitudes $I^{(1)}_{\text{meas}} > I^{(2)}_{\text{meas}} > ... > I^{(d-1)}_{\text{meas}}$ are chosen (Fig. 2C), with each pulse $I^{(n)}_{\text{meas}}$ adjusted so the upper states $|n\rangle$, $|n+1\rangle$,... tunnel out of the well, while the lower states $|n-1\rangle$, $|n-2\rangle$,... do not. Each tunneling measurement is repeated $\sim 10^3$ times on identically-prepared qudit states to obtain the cumulative tunneling probability $P_{\geq n}$. From these, we obtain the individual occupation probabilities $P_n = P_{\geq n} - P_{\geq n+1}$, which are the diagonal elements $P_n = \rho_{nn}$ of the qudit density matrix $\rho_{mn} = \langle m| \hat{\rho} |n\rangle$.

Arbitrary unitary gates combined with state measurement make this system a universal single qudit (*5, 25*). By applying an appropriate set of unitaries before measurement, one could for example reconstruct the entire qudit density matrix, similar to previously demonstrated single- and coupled-qubit state tomography (*8, 9*). Qudit-qudit coupling (*25*) is also possible, but beyond the scope of this work.

The qudit is calibrated one transition at a time from the ground state upwards. First, as for qubit operation, the system is initialized in $|0\rangle$ and a standard protocol (*22*) is used to find $I^{(1)}_{\text{meas}}$ and ω_{10}, and to calibrate a π-pulse $|0\rangle \to |1\rangle$. Next, this π-pulse is applied to initialize the system in $|1\rangle$, and the protocol is repeated to find $I^{(2)}_{\text{meas}}$ and ω_{21}, and to calibrate a π-pulse $|1\rangle \to |2\rangle$. This process is repeated as desired, in this case up to $|d-1\rangle = |4\rangle$. Each π-pulse has a 16 ns envelope (*26*), with the amplitudes scaled to equalize the rotation rates (Fig. 3A-C), thus calibrating the transition matrix elements $\delta_{n,n-1} = \langle n| \hat{\delta} |n-1\rangle$. The measured lifetimes of the excited states are $T_1 = 610$ ns, $T_2 = 320$ ns, $T_3 = 220$ ns, $T_4 = 170$ ns, in good agreement with the $T_n = T_1/n$ scaling seen in harmonic oscillators (*27, 28*) due to the weak anharmonicity.

Evolution of the qudit state is best described in the basis of moving eigenkets $|n'\rangle = \exp(-iE_n t/\hbar) |n\rangle$. In this basis, microwaves at $\omega_{n,n-1}$ appear as off-diagonal elements in the Hamiltonian

$$H' = \begin{pmatrix} 0 & A^*_{10} & 0 & 0 \\ A_{10} & 0 & A^*_{21} & 0 \\ 0 & A_{21} & 0 & A^*_{32} \\ 0 & 0 & A_{32} & 0 \end{pmatrix}, \tag{1}$$

where the $A_{n,n-1}$ are arbitrary complex numbers giving the amplitude and phase of the microwaves at $\omega_{n,n-1}$, and we have made the usual rotating-wave approximation by discarding off-resonant terms. The calibration shown in Fig. 3A-C ensures that the $A_{n,n-1}$ are calibrated relative to each other.

To emulate a spin rotation, the applied qudit Hamiltonian should be the appropriate rotation generator. The generators of rotation about X for $s = 1/2$, 1 and 3/2 are (*10*)

$$
\begin{aligned}
X^{(1/2)} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
X^{(1)} &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
X^{(3/2)} &= \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix},
\end{aligned} \tag{2}
$$

where the largest element in each matrix has been normalized to 1. Generators of Y-rotation are similar, but with imaginary off-diagonal terms. These operators all have the form Eq. 1 of microwave drive Hamiltonians, allowing us to use microwaves to emulate spin rotations about X, Y or any other axis in the X-Y plane.

The evolution of the qudit state under emulated spin rotation is shown in Fig. 3 for spin-1 (D) and spin-3/2 (E). In both cases, the ground state $|0\rangle$ is reserved as a phase reference, so the spin is mapped to $|1\rangle \equiv |s, s\rangle$, $|2\rangle \equiv |s, s-1\rangle$,..., $|1+2s\rangle \equiv |s, -s\rangle$. The spin starts in $|1\rangle$, rotates to $|3\rangle$ (spin-1) or $|4\rangle$ (spin-3/2), then back to $|1\rangle$ and so on. While the state populations evolve in a complicated fashion, the expectation value $\langle \hat{Z} \rangle = \sum_m m P_{|s,m\rangle}$ evolves sinusoidally (dashed line), as expected for a rotating spin. Compared to spin-1/2 (A-C), the rotation is slowed by a factor $\sqrt{2}$ (spin-1) or 2 (spin-3/2), in agreement with direct exponentiation of the matrices in Eq. 2.

Next, these emulated spin rotations are used to measure Berry's phase, as described in Fig. 1. The phase measurement is made using Ramsey interference with $|0\rangle$ as a reference.

First, a $\pi/2$-pulse is applied to prepare the superposition $(|0\rangle + |1\rangle)/\sqrt{2}$. Then two emulated π-pulses are applied with angle Θ between their rotation axes, rotating the spin component $|1\rangle \equiv |s, s\rangle$ about a closed path and giving the state $(|0\rangle + \exp(-is\Omega)|1\rangle)/\sqrt{2}$. Finally, a second $\pi/2$-pulse is applied to detect the phase of $|1\rangle$. As the rotation axis ϕ of the latter $\pi/2$-pulse is varied, P_1 traces out a sinusoid—a Ramsey fringe—whose phase corresponds to the acquired spin phase.

The result of this experiment is shown in Fig. 4. For spin-0 no π-rotations are performed (the rotation generator is $X^{(0)} = 0$) so the Ramsey fringes are stationary. For $s = 1/2$, 1, and 3/2, the Ramsey fringes shift by -2π, -4π, and -6π, respectively, as Θ increases from 0 to 2π (Ω changes by 4π), in agreement with the predicted Berry phase factor $\exp(-is\Omega)$. The slices at $\Theta = \pi$ ($\Omega = 0$) and $\Theta = 0$ ($\Omega = 2\pi$) clearly show the parity difference between integer spins $s = 0$, 1 and half-integer spins $s = 1/2$, 3/2, with in- and out-of-phase Ramsey fringes, respectively.

The Ramsey fringes show reduced contrast when higher qudit states are used in the sequence, due largely to the reduced lifetimes $T_n \approx T_1/n$ of the higher states. In addition, using higher states leads to imperfections in the microwave control due to the large bandwidth required and the effect of off-resonant terms dropped from Eq. 1. Ongoing work to reduce decoherence in superconducting quantum circuits (*29*) will improve the state lifetimes, and the off-resonant terms could be taken into account to improve the fidelity of qudit operation (*21*).

We have shown that the superconducting phase qubit can be extended to operate as a qudit up to $d = 5$ levels. The qudit state can be readily manipulated and measured using our existing control electronics, allowing us to perform non-trivial qudit protocols to emulate spins $s = 1/2$, 1 and 3/2. We reproduced the quantum phase acquired by each spin under closed-path rotation, in particular the even (odd) parity of integer (half-integer) spins under 2π-rotation. This demonstration opens possibilities for using phase qudits in quantum information processing.

References and Notes

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Fig. 1. Effect of rotation on a spin. The spin begins in the up state $|\uparrow\rangle = |s, +s\rangle$. After two π-rotations (blue, red) with angle Θ between the rotation axes (dotted arrows), the spin returns to $|\uparrow\rangle$ with a phase factor depending on Θ and s. In (**A**), the second rotation reverses the first, giving a phase factor 1 which leaves the spin state unchanged. In (**B**), both rotations are about the same axis. The spin traces out a great circle and acquires a phase factor $\exp(-i2\pi s)$. For integer spins (bosons) this has no effect, but for half-integer spins (fermions) this gives a factor of -1. In the general case (**C**), the acquired phase factor is $\exp(-is\Omega)$, where $\Omega = 2\alpha$ is the enclosed solid angle.

Fig. 2. Operation and measurement of a superconducting phase qudit. (**A**) Schematic of qudit circuit and control electronics. Current I_{dc} biases the junction, microwave drive $I_{\mu\mathrm{w}}$ manipulates the qudit state, and an on-chip SQUID detects tunnelling events for readout. (**B**) The potential energy as a function of junction phase δ forms a well with several energy levels. The frequencies $\omega_{n,n-1} = (E_n - E_{n-1})/\hbar$ are distinct, allowing transitions to be driven independently. (**C**) For measurement, a brief current pulse $I_{\mathrm{meas}}^{(n)}$ is applied to lower the potential energy barrier, causing states $|n\rangle$, $|n+1\rangle$,... to tunnel out of the well. For each n, this is repeated $\sim 10^3$ times to obtain a probability.

Fig. 3. Manipulation of the qudit state with microwaves. (**A**) Left, the pulse sequence used to calibrate microwaves $|1\rangle \rightarrow |2\rangle$. A π-pulse excites the qubit to $|1\rangle$, then a second pulse drives Rabi oscillations between $|1\rangle$ and $|2\rangle$. Right, occupation probabilities P_n plotted versus the integrated area of the Rabi pulse, where π corresponds to a standard 16 ns pulse envelope with amplitude adjusted to create a *pi*-rotation (swap) from $|1\rangle$ to $|2\rangle$. (**B, C**) Similar calibration for $|2\rangle \rightarrow |3\rangle$ and $|3\rangle \rightarrow |4\rangle$. (**D**) Left, the pulse sequence for generating spin-1 rotations by simultaneously driving $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$. Right, a plot of P_n versus the integrated pulse area of the single drives. The spin-1 π-rotation (from $|1\rangle$ to $|3\rangle$) is $\sqrt{2}$ longer than a spin-1/2 rotation. The dashed line shows $\langle \hat{Z} \rangle$, which

varies sinusoidally. (**E**) Generation of spin-3/2 rotations by driving three transitions, with the outer drives scaled by $\sqrt{3}/2$. The π-rotation is 2 times longer than a spin-1/2 rotation. The dashed line shows $\langle \hat{Z} \rangle$.

Fig. 4. Measurement of spin parity. The upper panels show the microwave control sequence: the central π-pulses implement a closed-path spin rotation, while the outer $\pi/2$-pulses use Ramsey interference to detect the phase shift of $|1\rangle$. For spin-0 no π-rotations are applied. The middle panels show P_1 in color as a function of the angle Θ between the π-pulse rotation axes and the angle ϕ between the $\pi/2$-pulse rotation axes. The lower panels show Ramsey fringe slices at $\Theta = \pi$ ($\Omega = 0$ or no rotation, black) and $\Theta = 0$ ($\Omega = 2\pi$ or one full rotation, gray), giving the relative phase shift due to a 2π-rotation. In phase fringes (integer spins **A**, **C**) indicate a relative phase factor of 1, while out-of-phase fringes (half-integer spins **B**, **D**) indicate a relative phase factor of -1.