Homework 2: Classical Cosmology

Due Mon Jan 21 2013

You may find Hogg astro-ph/9905116 a useful reference for what follows. Ignore radiation energy density in all problems.

Problem 1. Distances.

a) Compute and plot for at least three sets of cosmological parameters of your choice the following quantities as a function of redshift (up to z=10): age of the universe in Gyrs; angular size distance in Gpc; luminosity distance in Gpc; angular size in arcseconds of a galaxy of 5kpc in intrinsic size. Choose one of them to be the so-called concordance cosmology $(\Omega_m, \Omega_\Lambda, h) = (0.3, 0.7, 0.7)$, one of them to have non-zero curvature and one of them such that the angular diameter distance becomes negative. What does it mean to have negative angular diameter distance? [10 pts]

b) Consider a set of flat cosmologies and find the redshift at which the apparent size of an object of given intrinsic size is minimum as a function of Ω_Λ. [10 pts]

2. The horizon and flatness problems

1) Compute the age of the universe t_{CMB} at the time of the last scattering surface of the cosmic microwave background (approximately $z = 1000$), in concordance cosmology. Approximate the horizon size as ct_{CMB} and get an estimate of the angular size of the horizon on the sky. Patches of the CMB larger than this angular scale should not have been in causal contact, but nonetheless the CMB is observed to be smooth across the entire sky. This is the famous "horizon problem". Try for other standard cosmologies, allowing for curvature and photon content and show that this is a generic problem of classic Big Bang cosmology. [10 pts]

2) Several studies find that the universe at present time is very close to be Eucledian. For example Suyu et al. 2012 astroph/1208.6010 reports $1-\Omega_0 = 0.003^{+0.005}_{-0.006}$. Work out what curvature that corresponds to at $z = 1000$ and discuss in the context of the flatness problem. [10 pts]

3. Redshifts

a) The redshift of all cosmological sources drifts systematically. Using the Friedmann equations, calculate the observed rate of change of redshift for an object with no peculiar velocity. What fractional precision in observed frequency is necessary to detect this drift in redshift within a decade? *Note:* Ignore the contaminating effect of peculiar velocities (of order $\sim 500\mathrm{km\,s^{-1}}$) and peculiar accelerations due to mass inhomogeneities. In practice, these have to be carefully considered too. As a practical example by how much would we expect the observed wavelength of Lyman α for a galaxy at redshift 8 to drift within a decade. [10 pts]

4. The Future of the Accelerating Universe

a) Show that at late times (i.e., $\Omega_\Lambda \gg \Omega_m$) in a ΛCDM cosmology, the scale factor increases exponentially with time. [10 pts]

b) Consider a source which emits radiation at a cosmic time $t_{\rm em}$, which is observed by us today at cosmic time t_o, where $t_o \approx 14$Gyr is the current age of the universe. If the source continues to emit at a later time $t'_{\rm em}$, it will be observed at some later time t'_o. However, since the comoving coordinate remains constant,

$$r = \int_{t_{\rm em}}^{t_o} \frac{c\,dt}{a(t)} = \int_{t'_{\rm em}}^{t'_o} \frac{c\,dt}{a(t)} \tag{1}$$

Use this to solve for $t'_{\rm em}$ as a function of t'_o, for objects currently observed to be at redshift z_o. You should find that $t'_{\rm em}$ asymptotes to a constant value, which is progressively smaller for objects with higher z_o. This implies that for any currently measured redshift z_o of a source, there is a maximum intrinsic age up to which we can see that source, even if we continue to monitor it indefinitely. [20 pts]

c) Consider the luminosity distance d_L, the angular diameter distance d_A, and the future redshift z as a function of time for a source currently observed to be at some redshift z_o. Argue that the luminosity distance and redshift diverge exponentially in the future, but the angular diameter distance approaches a constant value. Thus, as the source image freezes in the future, its flux continues to decline and its redshift increases, but it will occupy a fixed angular diameter in the sky. [10 pts]

d) Describe the implications for future cosmologists in hundreds of billions of years. [5 pts]