Problem Set 2

Due Mon Jan 28

As always, assume a canonical cosmological model of $(\Omega_m, \Omega_\Lambda, h) = (0.3, 0.7, 0.7)$. Be prepared to solve numerical integrals. In Problem 1, assume $(\Omega_m, \Omega_X, h) = (0.3, 0.7, 0.7)$, where Ω_X is the fraction of critical density contributed by dark energy.

1. **Measuring the Dark Energy Equation of State** Suppose dark energy has an equation of state $P = w\rho$, where we now allow $w(z)$ to be a function of redshift (for a cosmological constant, $w = -1$). Show that the Hubble expansion rate is now given by:

$$\frac{H^2(z)}{H_o^2} = \Omega_m(1+z)^3 + \Omega_X \exp\left[3\int_0^z [1+w(x)]d\ln(1+x)\right] \tag{1}$$

Hence plot the (a) age of the universe (b) luminosity distance (c) comoving volume (d) $\Delta z/\Delta\theta$ (for an object of size 1 Mpc; the Alcock-Pacynski test from the last problem set) as a function of redshift for 4 different models, $w = -1, w = -1/3, w = -0.5 + 0.1z, w = -0.5 - 0.05z$, for $0 < z < 5$. These differences are intended to be exploited in the next generation of experiments to determine $w(z)$. One problem is that they involve integrals of $w(z)$ over redshift, and thus there may be degeneracies between different models of $w(z)$. Show if we had a very precise clock which could measure age differences between objects at different redshifts (passively evolving stellar populations?), $w(z)$ in principle could be directly determined from differential ages:

$$H_o^{-2}\frac{d^2z}{dt^2} = \frac{[H_o^{-1}(dz/dt)]^2}{(1+z)}[5/2 + 3/2w(z)] - \frac{3}{2}\Omega_m(1+z)^4 w(z) \tag{2}$$

2. **Cosmography by counting quasars.**

So-called volume-tests are based on the concept that if we know the density of a certain class of objects we can count their number for a given solid angle as a function of redshift and use the results to infer the cosmic volume and therefore cosmological parameters. Consider as an example the following case.

1) Use the black hole mass function shown in Figure 4 of the paper by Kelly & Shen 2012 (arXiv.1209.0477) to estimate the density of active black holes with mass above 10^9 $M_\odot$ at $z = 0.40$.

2) Assuming no evolution in the active black hole mass function compute the expected number of black holes with mass above 10^9 $M_\odot$ per square degree of sky in three redshift windows $z \in [0.3 - 0.5]$, $z \in [0.9 - 1.1]$, $z \in [3.5 - 4]$. Use the standard concordance cosmology, the Einstein de Sitter Cosmology with $h = 0.7$, and a cosmology with $\Omega = 0.3$, $\Omega_\kappa = 0.7$ $h = 0.7$. What is the least amount of square degrees of sky that would you need to observe to distinguish the three scenarios? *hint: use poisson statistics to get a rough idea of the counting uncertainties.*

3) Repeat the calculations above relaxing the no-evolution assumption and using instead the actual measured values by Kelly & Shen (neglect the fact that they are obtained for the concordance cosmology and take them at face value).

4) Compare the results in 2 and 3 and discuss the degenaracy between cosmological parameters and black hole evolution in this volume test.