Problem Set 4 = Homework 5

Due Feb 11 2013

As always, assume a canonical cosmological model of $(\Omega_m, \Omega_\Lambda, h) = (0.3, 0.7, 0.7)$.

1. Recombination of H and He

A) Assuming a pure hydrogen plasma use Saha's equation (as derived in class) to calculate the redshift at which n_p/n_H=0.1. [5 pts]

B) Consider now a more realistic model where the plasma is made of H and He (resp. $\sim 75\%$ and $\sim 25\%$ in mass). Defining Y as the mass fraction of baryons in He modify Saha's equation describing H recombination and write down the equations describing He recombination. [10 pts]

C) Using the equations derived in B, determine the redshifts at which $n_p/n_H = 0.1$, $n_{\rm He+}/n_{\rm He} = 0.1$ and $n_{\rm He++}/n_{\rm He+} = 0.1$. [10 pts]

2. Comoving entropy

Show using standard thermodynamics that the entropy density is given by

$$s = \frac{u + p}{T}. \tag{1}$$

Hence, using the using the First and Second laws of Thermodynamics, show that in thermal equilibrium, the entropy per comoving volume

$$S = \frac{a^3(u + p)}{T} \tag{2}$$

is conserved. You may assume a zero chemical potential. [20pts]