HW6

Due Monday Feb 18

As always, assume a canonical cosmological model of $(\Omega_m, \Omega_\Lambda, h) = (0.3, 0.7, 0.7)$.

1. **Cosmological Neutrino Background** Consider the cosmological relic population of neutrinos. In class, we showed that they freeze out at $k_B T \sim 1\,\mathrm{MeV}$, and that they have a temperature $T_\nu = (4/11)^{1/3} T_\gamma$, where $T_\gamma = 2.73\mathrm{K}$ is the CMB temperature today. [5 pts]
(a) Given that $T_\gamma = 2.73\mathrm{K}$, compute the number density of neutrinos and anti-neutrinos per species today. Recall that a thermal distribution of fermions has 3/4 as many particles as a thermal distribution of bosons at the same temperature. Assume a zero chemical potential throughout. [5 pts]
(b) Suppose that each neutrino (and its anti-particle) has a small mass $m_\nu \ll 1\mathrm{MeV}$, so it is still relativistic at decoupling. This means the number density computed in (a) still holds. Hence show that

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{91.5\mathrm{eV}} \tag{1}$$

where the sum is over all neutrino species. Given present-day measured values of Ω, h, this allows us to place an upper bound on neutrino masses! [5pts]
(c) Assume that one of the neturinos is massive and has mass 0.1eV. Given that relativistic neutrinos are in a thermal distribution for a massless fermion and that momenta (careful, not energy or velocity) scale as $(1+z)^{-1}$, compute the velocity distribution of the massive neutrinos today. [10pts]
(d) Compare this distribution with a thermal distribution of a massive non-relativistic particle at temperature T_ν. Is it the same? Compute the mean velocity $\langle v \rangle$ for both distributions, and show they differ quite a bit. It is therefore not really meaningful to speak of a "temperature" for the relic neutrino population [10pts].

2. Cosmological Limits on Particle Cross-Sections

Consider that we change particle physics so as to include a yet-undiscovered stable massive particle. For simplicity, we will make it spin-0 (meaning that $g = 1$) and call it X. X and its antiparticle $\bar{X}$ interact quickly enough in the early universe that their number densities reach thermal equilibrium. We will imagine that the mass m_X is large, of order the proton mass or larger.

a) If X interacts rarely enough, then it will decouple (interaction rate less than Hubble parameter) when the universe is still hotter than $m_X c^2$. Show that this is a cosmological catastrophe by computing $\Omega_m h^2$ in terms of m_X (where the latter is measured in GeV). You may assume that $\chi = 100$; χ is the total number of relativistic spin-states, including a penalty of 7/8 for fermions; the extra degrees of freedom correspond to particles (quarks, gluons, etc) that haven't frozen-out yet. In this notation the energy density of the universe will be $\epsilon = \chi a T^4$ [10pts].

b) If X interacts more quickly, then it remains able to follow the thermal equilibrium prediction as the temperature drops below the rest mass. In other words, as the temperature drops, the X

and $\bar{X}$ can annihilate. However, as the number density drops, the annihilation reaction slows and freezes out. This leaves a relic population of X and $\bar{X}$ particles that might be the dark matter today.

Compute the relic abundance of X and $\bar{X}$ particles as a function of the annihilation cross-section σ_a and the mass m_X. You may assume that the reaction ends when the reaction rate is equal to the Hubble parameter, and that the cross section is independent of velocity. You may assume that the Hubble parameter is to be computed at a time when the temperature of the universe is $0.05 m_X c^2/k$. Assume a zero chemical potential, and $\chi = 100$ [10 pts].

Hence, show that *if* the X particle is to be the dark matter ($\Omega_X h^2 \approx 0.11$), then it must have a cross-section that is predicted (at least to a factor of $\sim$3) by its mass!

This is an example of how cosmology can put limits on particle physics: we've just placed an lower limit on the cross-section of a particle as a function of its mass (barring heroic attempts to violate our assumptions, of course). This is exciting because it tells us that stable, interacting particles (e.g. WIMPs–weakly interacting massive particles) won't have undetectably low cross-sections.