HW7

Due Monday March 4

Mixed (Cold + Hot) Dark Matter Neutrinos are often referred to as "Hot" Dark Matter since they have large streaming velocities compared with (much more massive) Cold Dark Matter particles.

(a) The comoving distance which a free-streaming particle can travel by epoch t is:

$$r_{\rm FS} = \int_0^t \frac{v(t')}{a(t)} dt'. \tag{1}$$

Assume that a particle is relativistic with $v \approx c$ for $t < t_{\rm nr}$ and that it is non-relativistic with $v \propto a^{-1}$ for $t > t_{\rm nr}$. Show that by the epoch of matter-radiation equality, the dark matter particle could have travelling a maximum comoving distance:

$$r_{\rm FS} = \left(\frac{ct_{\rm NR}}{a_{\rm NR}}\right)\left[2 + \ln\left(\frac{t_{\rm eq}}{t_{\rm NR}}\right)\right]. \tag{2}$$

Consider neutrinos of mass $m_\nu \sim 30$eV. Plug in numbers to show that the free-streaming length-scale is $\lambda_{\rm FS} \approx 30(m_\nu/30\,eV)^{-1}$Mpc, and hence that the free-streaming mass is $M_{\rm FS} \approx 4 \times 10^{15}\left(\frac{m_\nu}{30\,eV}\right)^{-2} M_\odot$. This is very big!!—which is why Hot Dark Matter models are called "Top Down" scenarios, where the big things form first.

(b) (This is independent of part (a)). Consider a critical density $\Omega_{\rm tot} = 1$ universe in which massive neutrinos contribute Ω_ν to the density parameter. Show that on scales smaller than the free-streaming lengthscale (on which the neutrino component is smooth), cold dark matter perturbations will grow at a rate $\delta \propto t^\alpha$, where $\alpha = [(25 - 24\Omega_\nu)^{1/2} - 1]/6$.