Cosmology W13

Lecture 5: January 30 2013

Strong Lensing

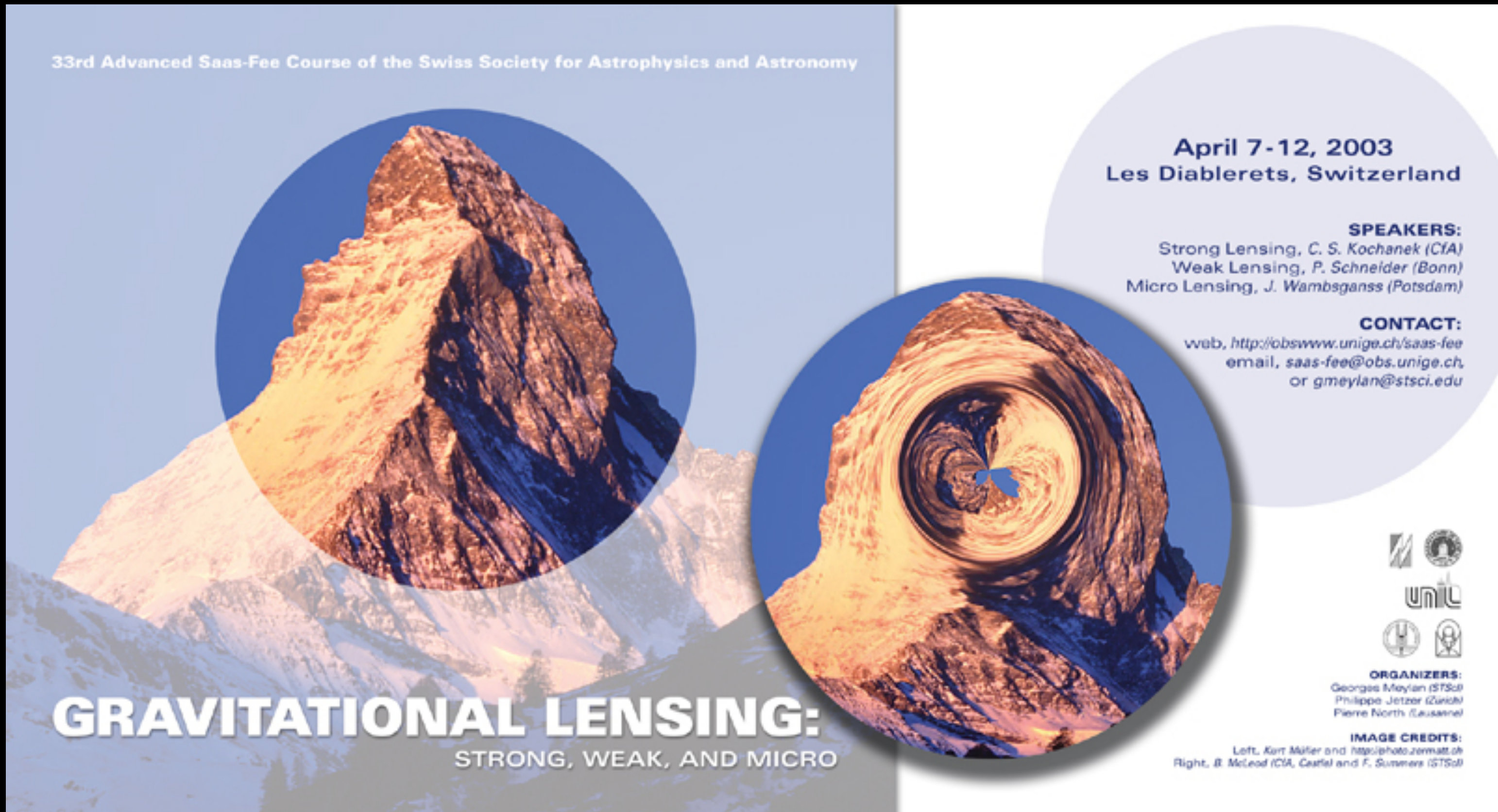
- Introduction to strong gravitational lensing
- strong lensing as a tool for cosmology: cosmography from gravitational time delays

References

- General
  - Schneider, Kochanek and Wambsganss 2006
  - Bartelmann 2010, CQGra, 27, 23
- Strong lensing by galaxies
  - Treu ARA&A, 2010, 87, 48
- Lensing by clusters
  - Kneib & Natarajan 2011, AARev, 19, 47
- Cosmology from gravitational time delays
  - Suyu et al. 2013, ApJ, arXiv 1208.6010

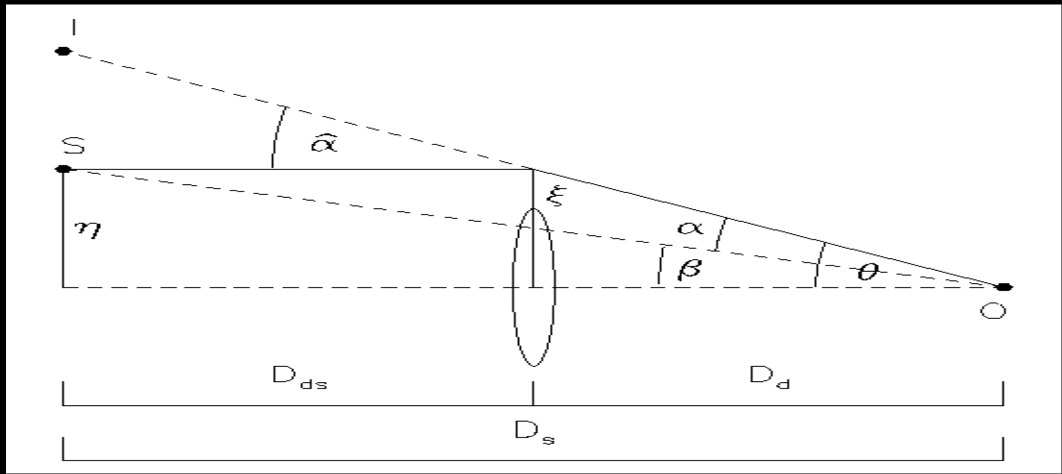
A very useful reference

- http://www.astro.uni-bonn.de/~peter/SaasFee.html



You are expected to study and know the content of the introductory chapter by Schneider

How does it work? Fermat's principle



Fermat distance

Shapiro delay

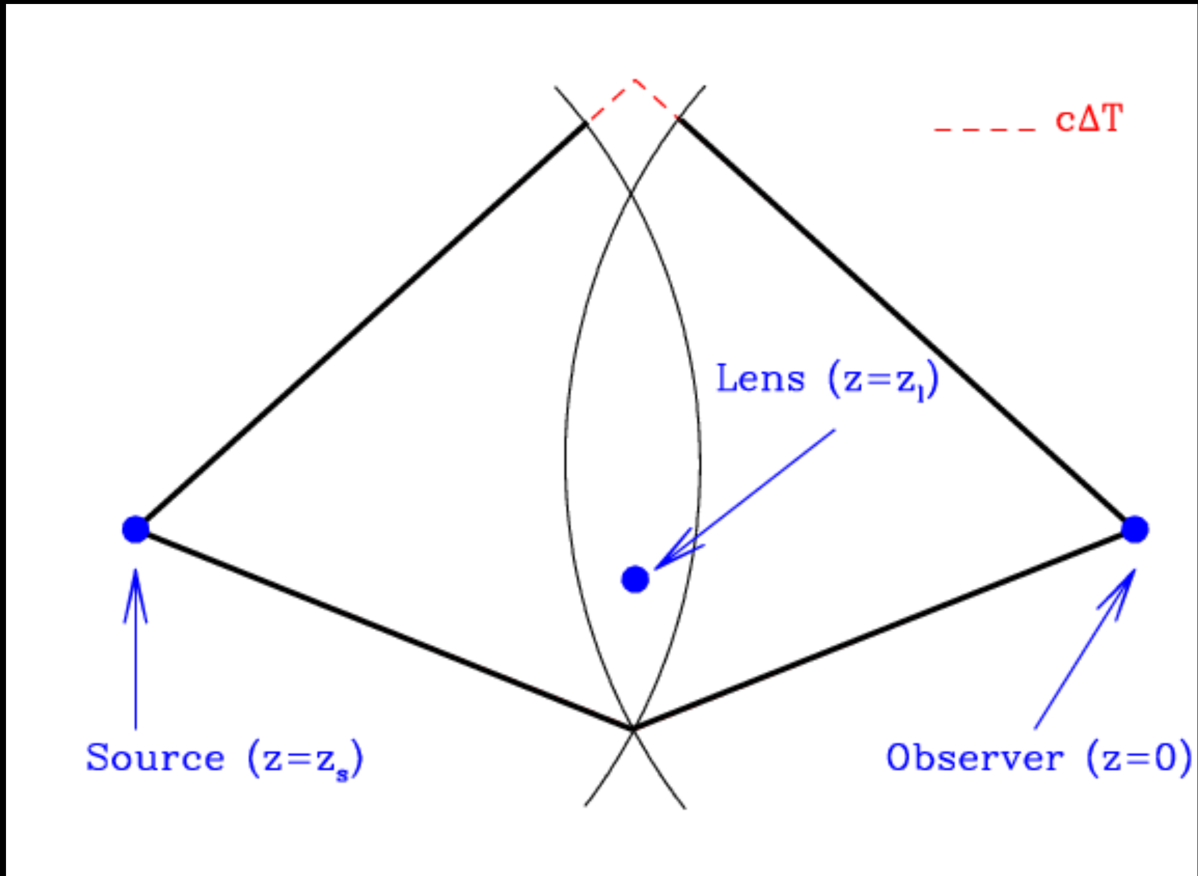
$$t(\vec{\theta}) = \frac{(1+z_\mathrm{d})}{c}\frac{D_\mathrm{d}D_\mathrm{s}}{D_\mathrm{ds}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right]$$

Excess time delay

geometric time delay

How does it work?

$$t(\vec{\theta}) = \frac{(1+z_{\mathrm{d}})}{c}\frac{D_{\mathrm{d}}D_{\mathrm{s}}}{D_{\mathrm{ds}}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right]$$



Recap of useful formulae - notation

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\, dz$$

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\, d^2\xi'$$

$$\hat{\alpha}(\xi) = \frac{4GM(\xi)}{c^2\xi}$$

$$M(\xi) = 2\pi \int_0^{\xi} \Sigma(\xi')\xi'\, d\xi'$$

$$\vec{\alpha} = \frac{D_{\rm ds}}{D_{\rm s}}\, \vec{\hat{\alpha}}$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\Sigma_{\rm cr} = \frac{c^2}{4\pi G}\, \frac{D_{\rm s}}{D_{\rm d} D_{\rm ds}} = 0.35\,{\rm g\,cm^{-2}} \left(\frac{D}{1\,{\rm Gpc}}\right)^{-1}$$

Recap of useful formulae - notation

$$\psi(\vec{\theta}) = \frac{D_{\rm ds}}{D_{\rm d} D_{\rm s}} \frac{2}{c^2} \int \Phi(D_{\rm d}\vec{\theta}, z)\, dz$$

$$\vec{\nabla}_\theta \psi = D_{\rm d} \vec{\nabla}_\xi \psi = \frac{2}{c^2} \frac{D_{\rm ds}}{D_{\rm s}} \int \vec{\nabla}_\perp \Phi\, dz = \vec{\alpha}$$

$$\nabla_\theta^2 \psi = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \int \nabla_\xi^2 \Phi\, dz = \frac{2}{c^2} \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}} \cdot 4\pi G\, \Sigma = 2 \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}} \equiv 2\kappa(\vec{\theta})$$

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla} \psi = \frac{1}{\pi} \int \kappa(\vec{\theta}')\, \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}\, d^2\theta'$$

$$\mathcal{A} \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right) = \mathcal{M}^{-1}$$

$$\mu = \det M = \frac{1}{\det \mathcal{A}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

What can strong lensing do for us?

- Strong lensing measures very accurately:
    - Mass enclosed within Einstein Radius
    - Derivative of enclosed mass (sometimes)
    - Projected orientation of the mass
    - Projected ellipticity of the mass
- Strengths:
    - Mass independent of presence of tracers
    - Mass independent of dynamic state
    - Measures total mass
- Weaknesses:
    - Projection effects
    - Measures total mass

Example: SIS as a simple model for galaxy scale lenses

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

$$\Sigma(\xi) = \int_{-\infty}^{\infty} \mathrm{d}r_3\, \rho\left(\sqrt{\xi^2 + r_3^2}\right) = \frac{\sigma_v^2}{2G}\,\xi^{-1}$$

$$\theta_{\mathrm{E}} = 4\pi\left(\frac{\sigma_v}{c}\right)^2 \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}}$$

$$\kappa(\theta) = \frac{\theta_{\mathrm{E}}}{2|\theta|}\,; \quad \bar{\kappa}(\theta) = \frac{\theta_{\mathrm{E}}}{|\theta|}\,; \quad |\gamma|(\theta) = \frac{\theta_{\mathrm{E}}}{2|\theta|}\,; \quad \alpha(\theta) = \theta_{\mathrm{E}}\,\frac{\theta}{|\theta|}$$

$$x = \theta/\theta_{\mathrm{E}},\; y = \beta/\theta_{\mathrm{E}}$$

$$\beta = \theta - \theta_{\mathrm{E}}\,\frac{\theta}{|\theta|}\,, \quad \text{or} \quad y = x - \frac{x}{|x|}$$

In practice you need elliptical models at least and those cannot be treated analytically

For simple elliptical mass distiributions

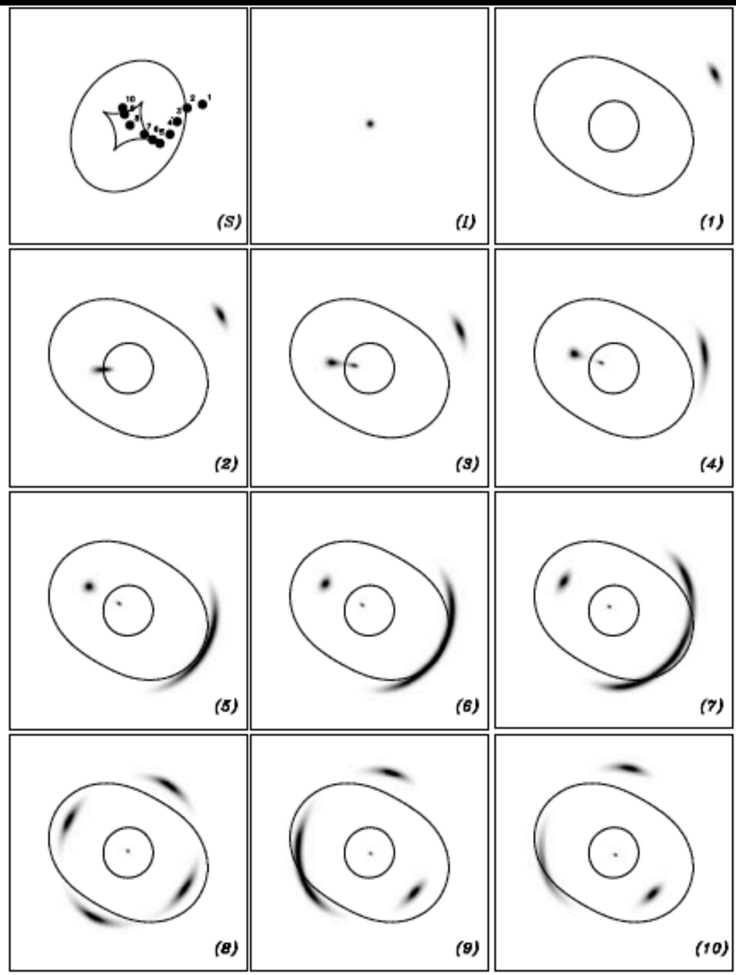
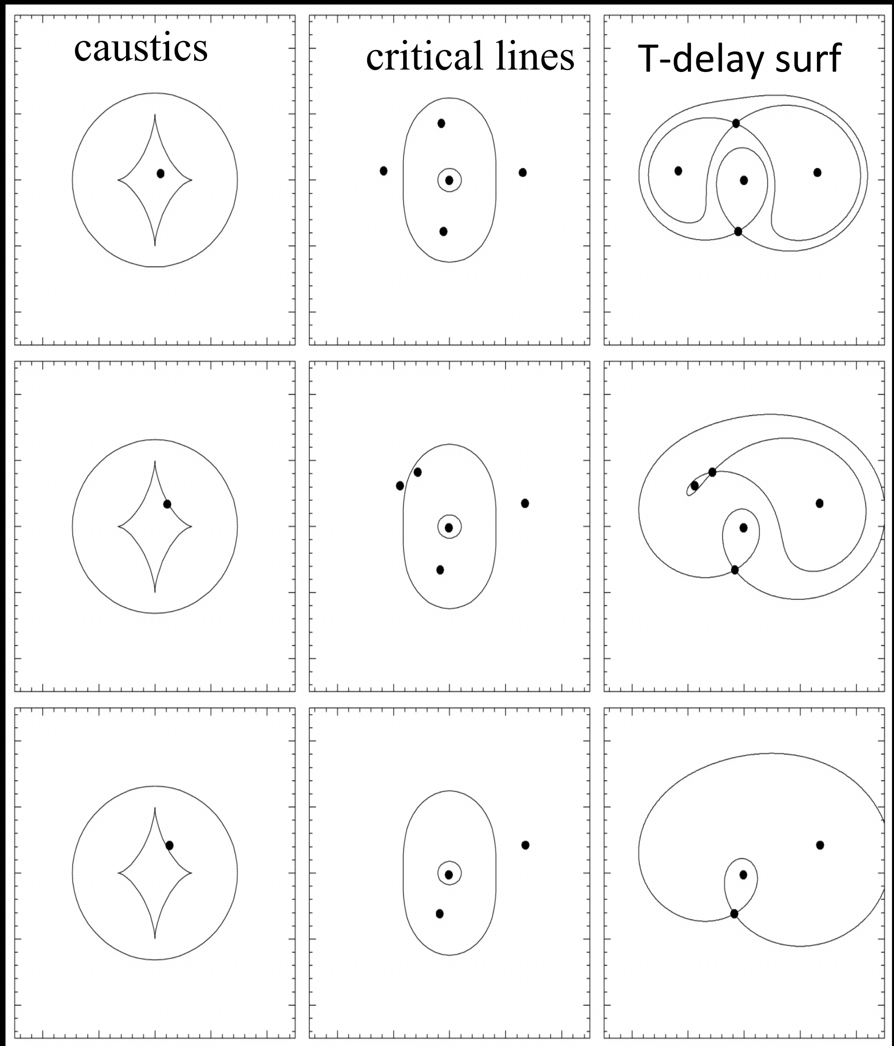


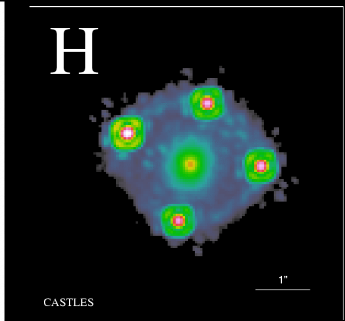
Fig. 12 Multiple-image configurations produced by a simple elliptical mass distribution. The panel (**S**) shows the caustic lines in the source plane and the positions numbered *1* to *10* denote the source position relative to the caustic lines. The panel (**I**) shows the image of the source without lensing. The panels (**1**) to (**10**) show the resulting lensed images for the various source positions. Certain configurations are very typical and are named as follows: (**3**) radial arc, (**6**) cusp arc, (**8**) Einstein cross, (**10**) fold arc

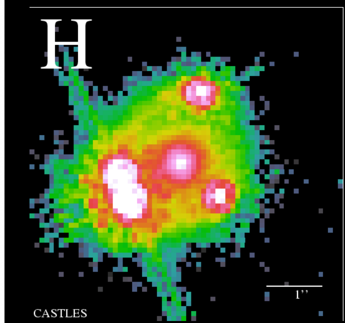
Kneib & Natarajan 2011

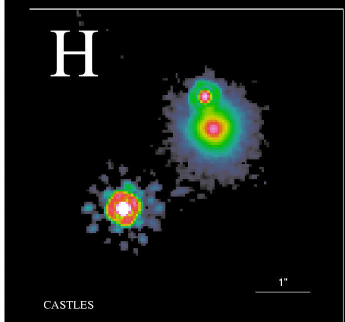
Examples of galaxy-scale lenses



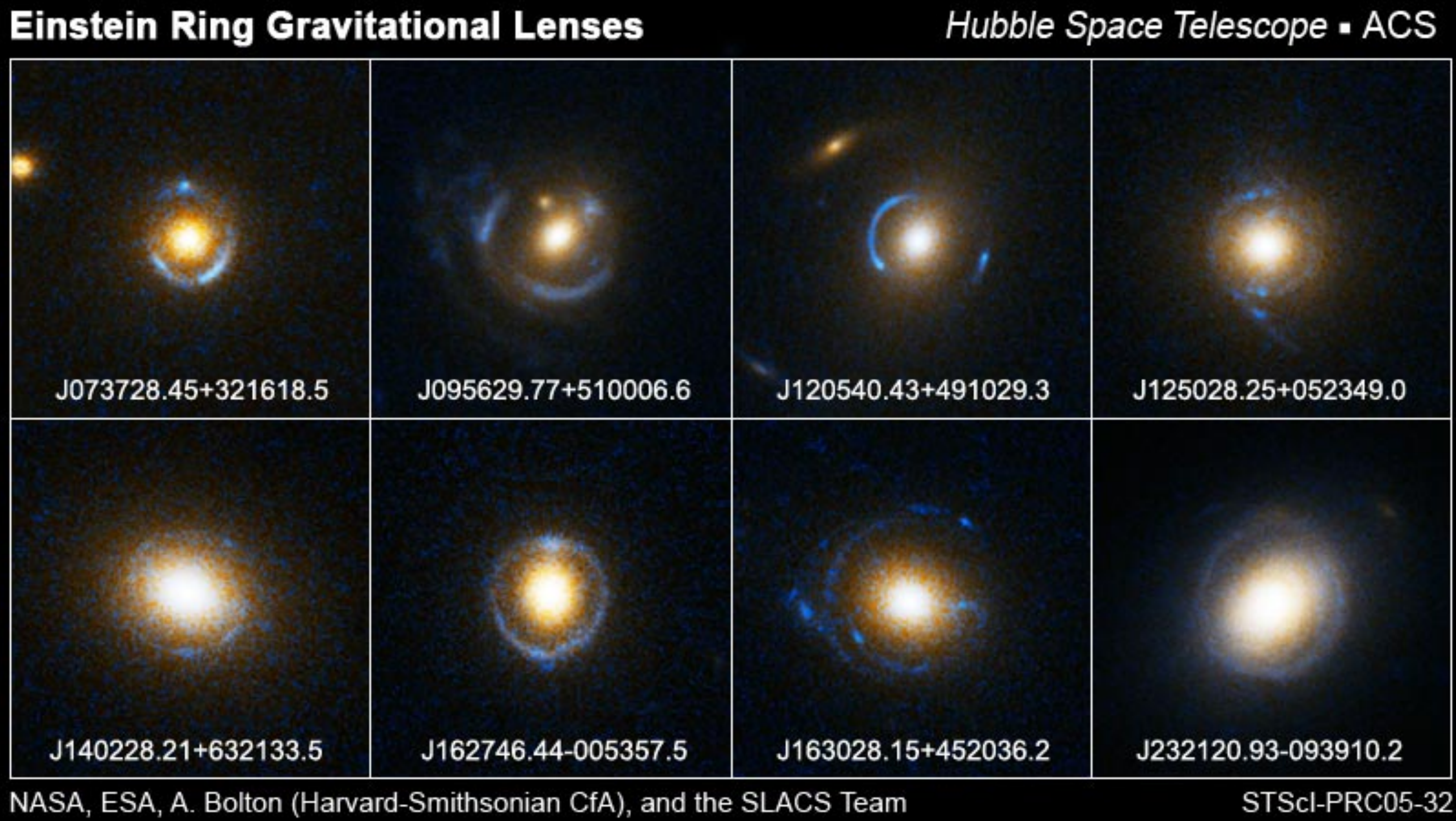
Saha & Williams 2003







Examples of galaxy-scale lenses
Classify the configurations



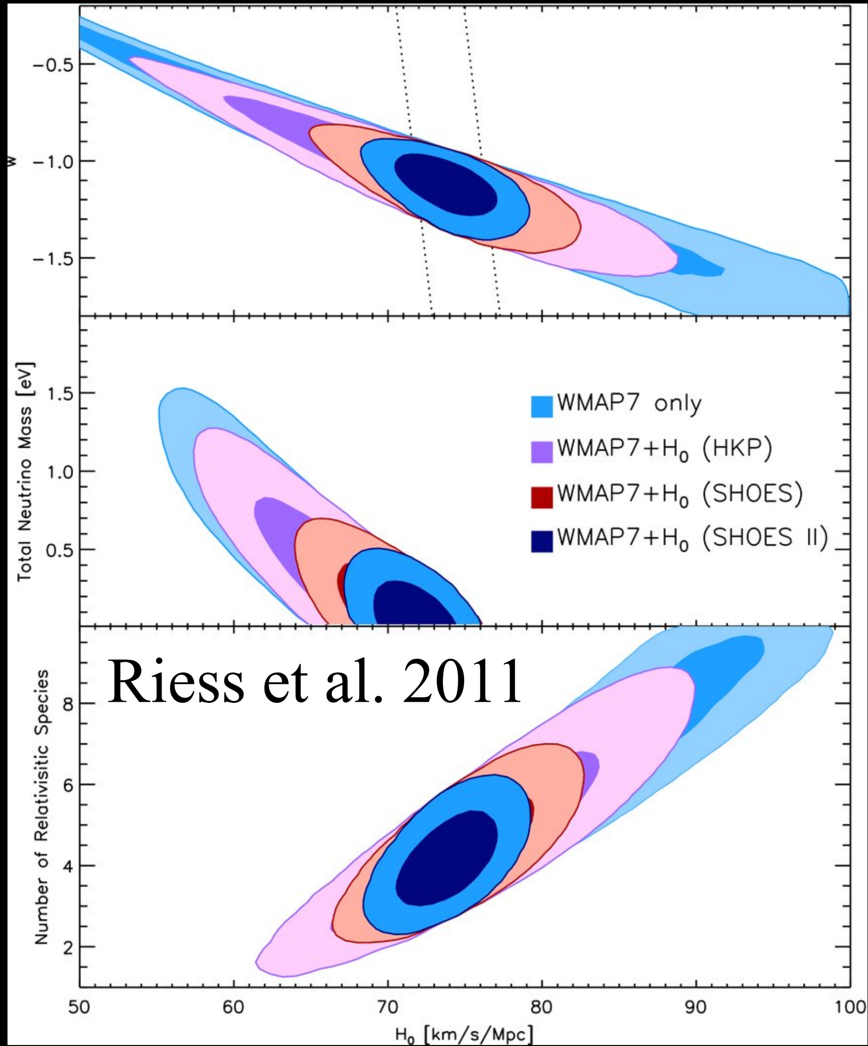
Cosmography with time-delays

Time delay distance in practice

$$\Delta t \propto D_{\Delta t}(z_s, z_d) \propto H_0^{-1} f(\Omega_m, w, ...)$$

Steps:

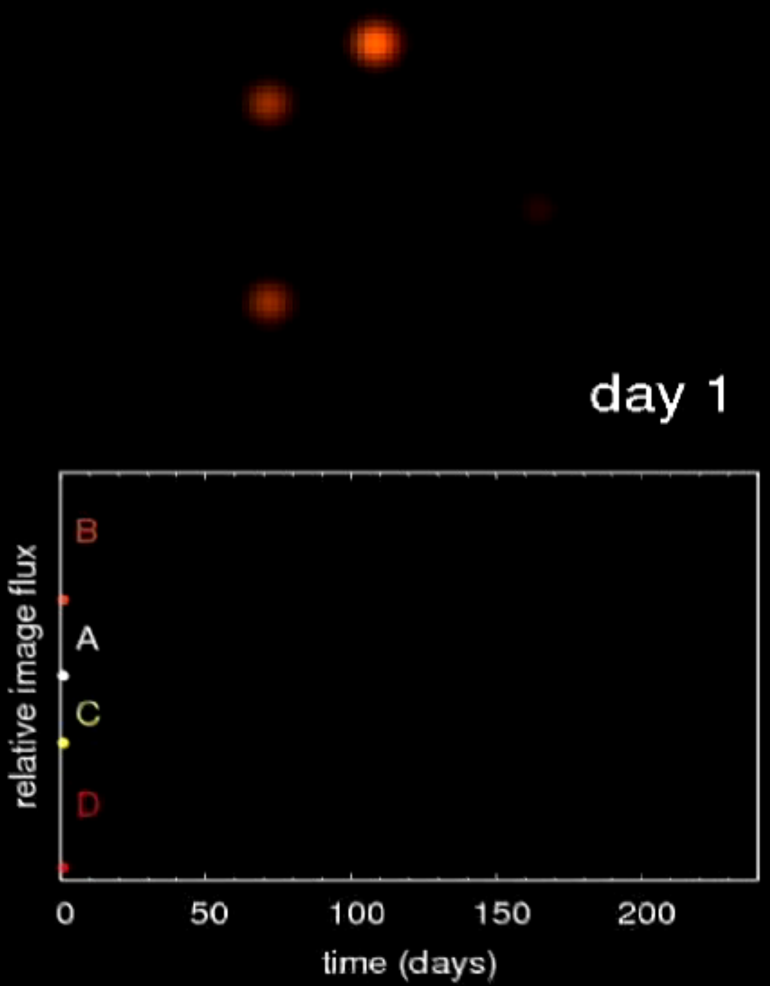
- Measure the time-delay between two images
- Measure and model the potential
- Infer the time-delay distance
- Convert it into cosmlogical parameters



Cosmography with strong lenses: the 4 problems solved

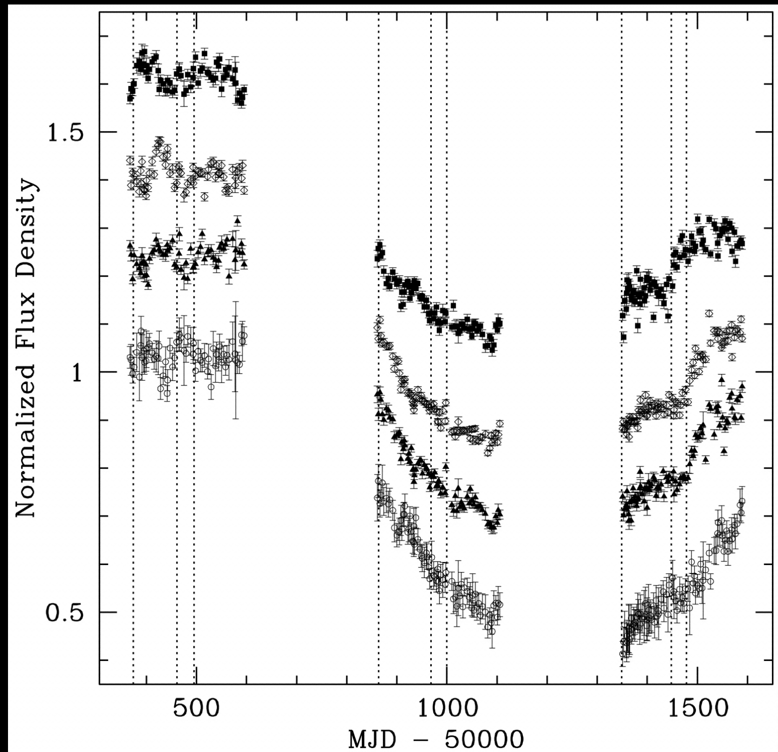
- Time delay – 2-3 %
  - Tenacious monitoring (e.g. Fassnacht et al. 2002); COSMOGRAIL (Meylan/Courbin)
- Astrometry – 10-20 mas
  - Hubble/VLA/(Adaptive Optics?)
- Lens potential (2-3%)
  - Stellar kinematics/Extended sources (Treu & Koopmans 2002; Suyu et al. 2009)
- Structure along the line of sight (2-3%) and the mass-sheet degeneracy,
  - Galaxy counts and numerical simulations (Suyu et al. 2009)
  - Stellar kinematics (Koopmans et al. 2003)

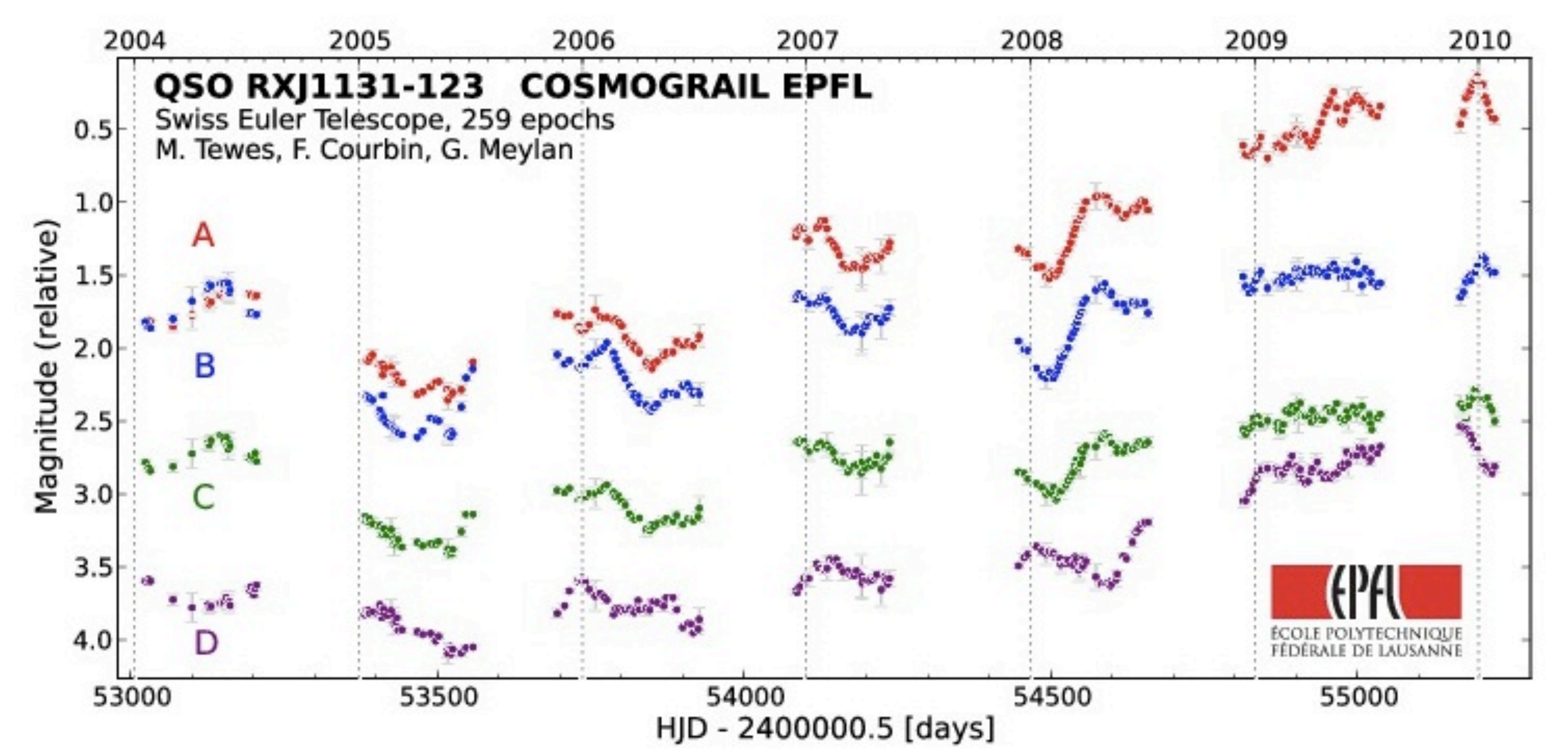
Gravitational Lens Time Delays



B1608+656 Variability in Radio Observations

Credits: S. H. Suyu, C. D. Fassnacht, NRAO/AUI/NSF



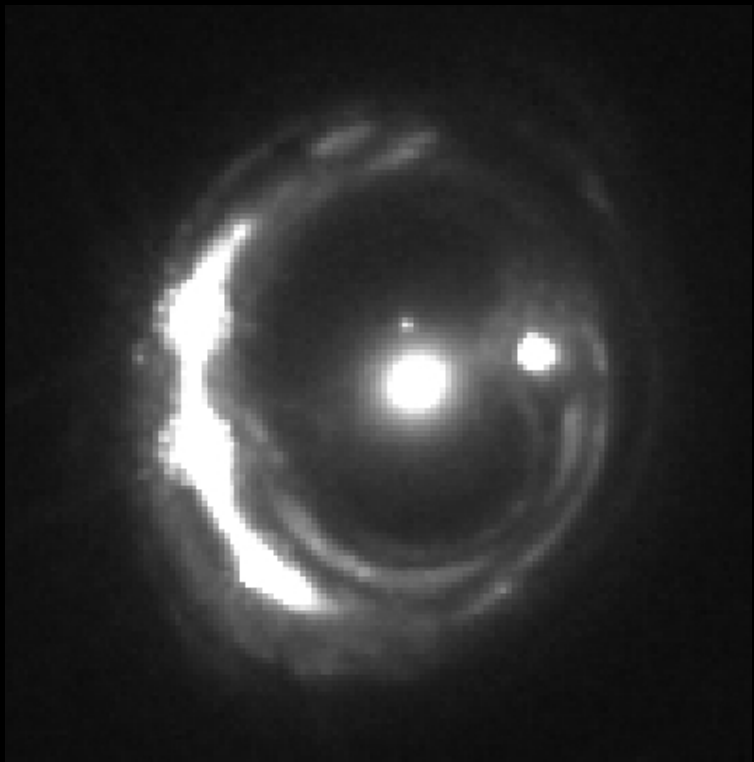

QSO RXJ1131-123 COSMOGRAIL EPFL
Swiss Euler Telescope, 259 epochs
M. Tewes, F. Courbin, G. Meylan
2004
2005
2006
2007
2008
2009
2010
A
B
C
D
Magnitude (relative)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
53000
53500
54000
54500
55000
HJD - 2400000.5 [days]
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

The slope-H_0 degeneracy

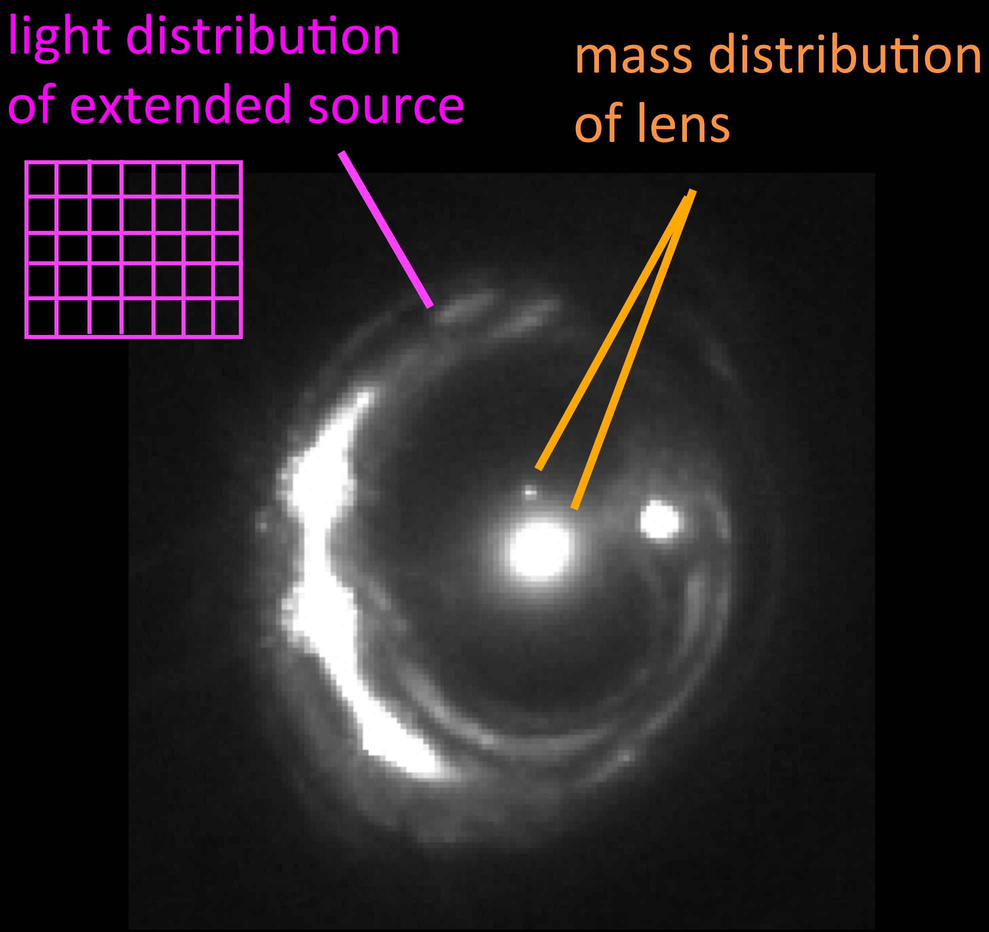
- Uncertainty of the mass density profile of the main lens is the single largest source of uncertainty.
- If mass density profile is a power law $\rho \sim r^{-\gamma'}$ then $H_0(\gamma') \sim H_0(2)(\gamma'-1)$ (Wucknitz 2002)
- Can be broken using lensing (Suyu et al. 2010) and stellar kinematic data (Treu & Koopmans 2002)

Lens Model

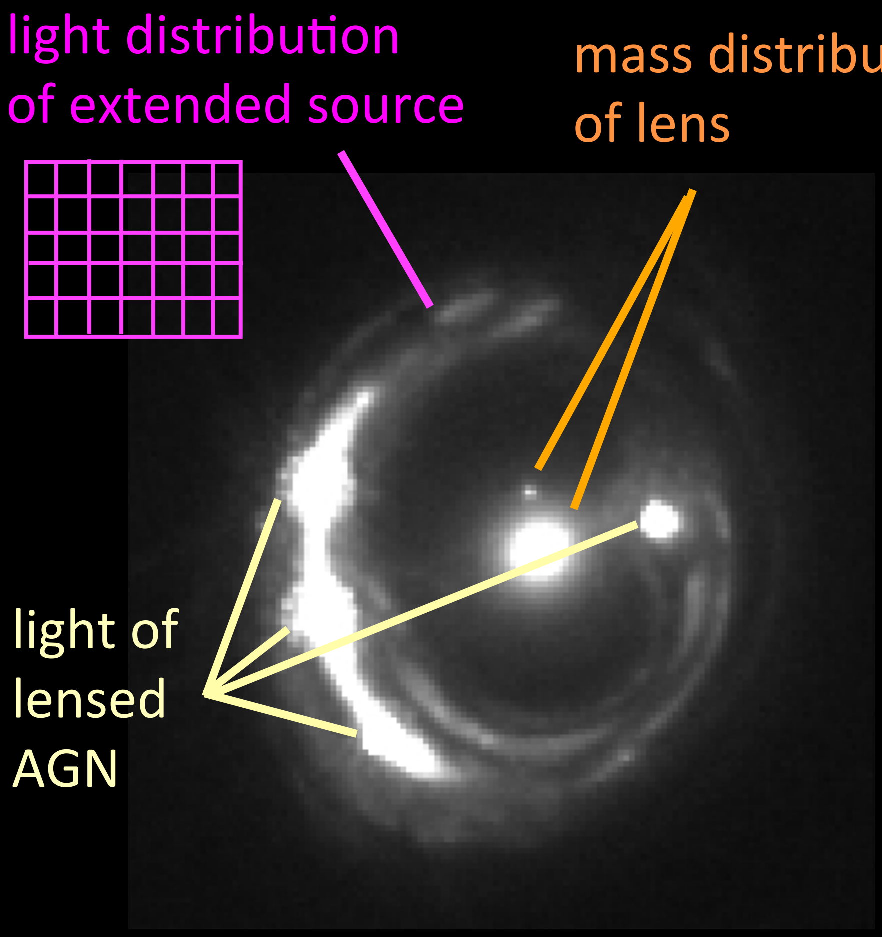
Observed Image



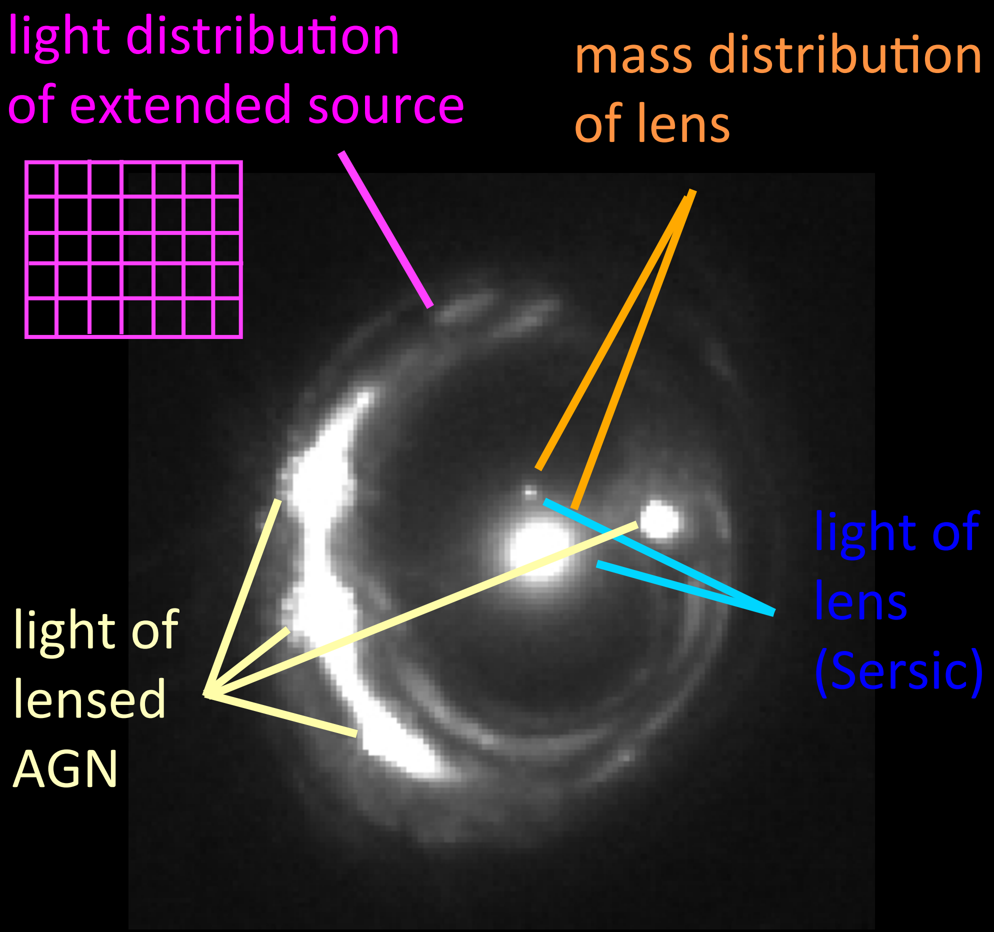
Lens Model


light distribution
of extended source
mass distribution
of lens

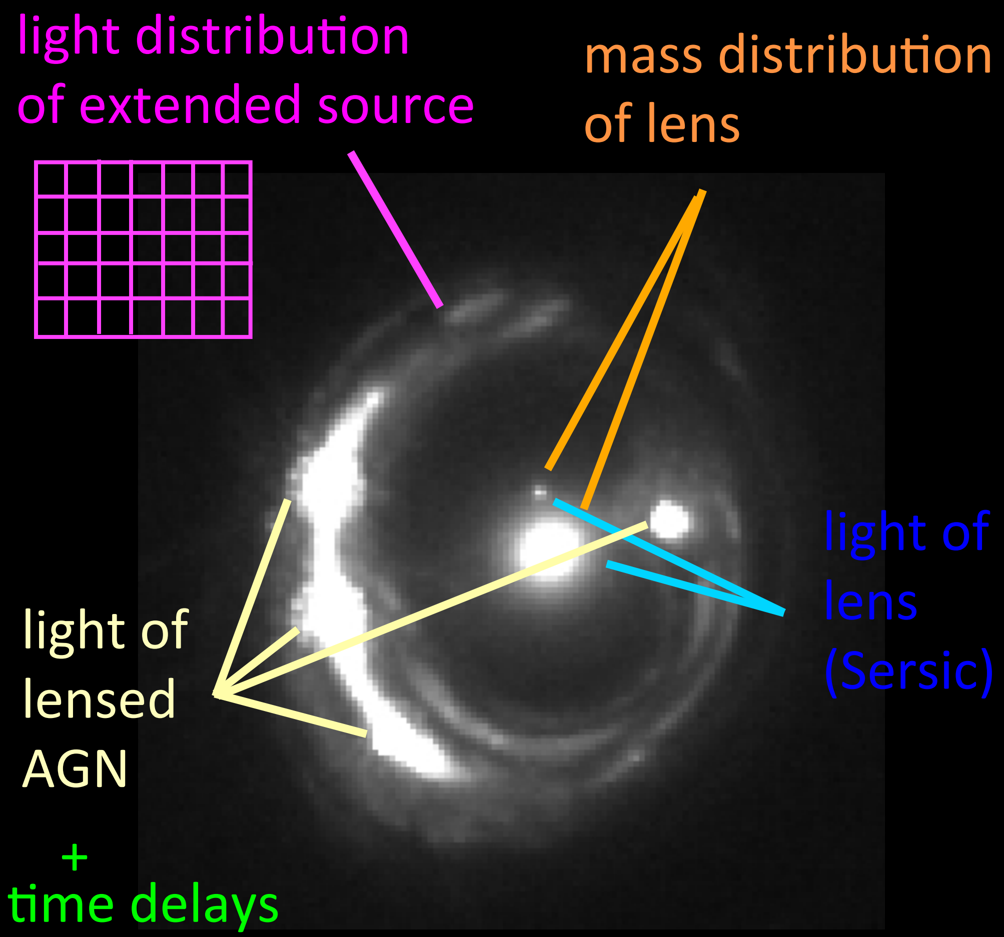
Lens Model


light distribution
of extended source
mass distribution
of lens
light of
lensed
AGN

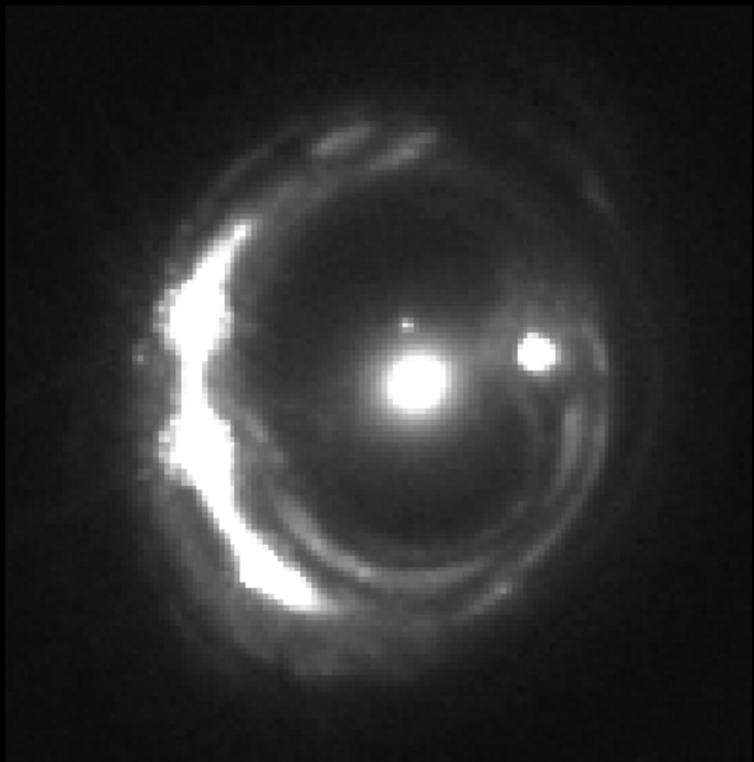
Lens Model

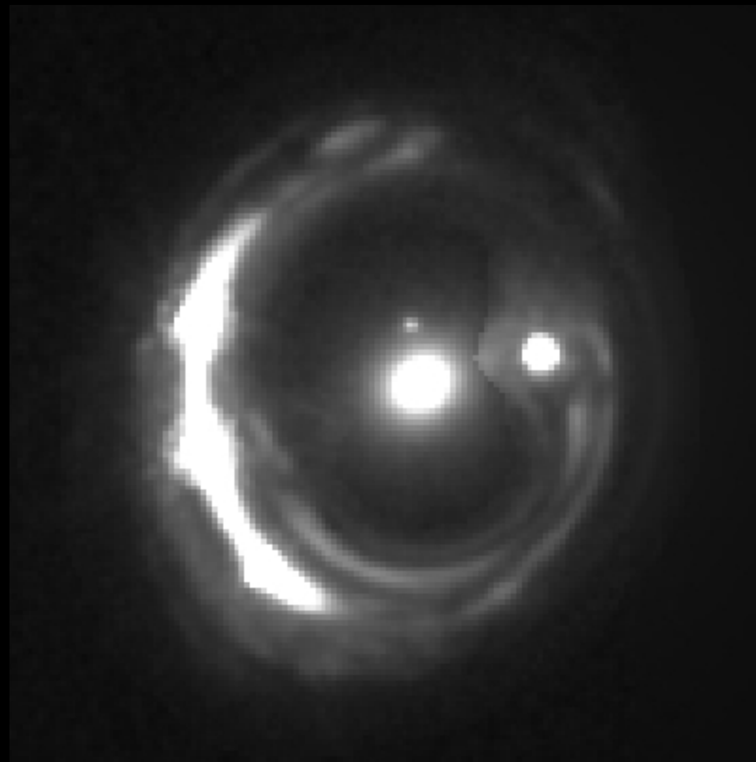

light distribution
of extended source
mass distribution
of lens
light of
lens
(Sersic)
light of
lensed
AGN

Lens Model

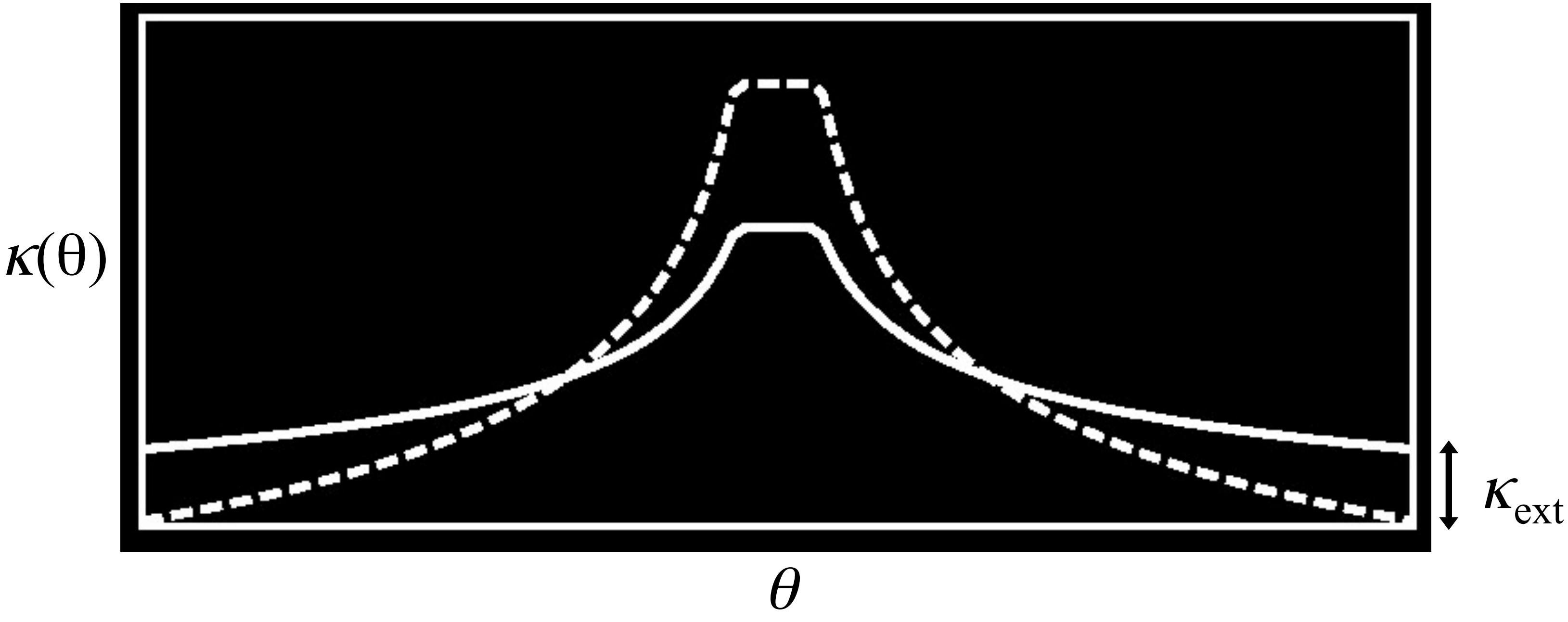

light distribution
of extended source
mass distribution
of lens
light of
lens
(Sersic)
light of
lensed
AGN
+
time delays

Lens Model





Mass-sheet degeneracy



lensing observables
do not change, but

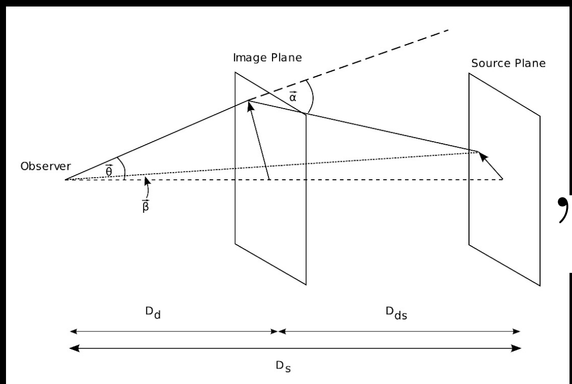
$$D_{\Delta t}^{\mathrm{true}} = \frac{D_{\Delta t}^{\mathrm{model}}}{1 - \kappa_{\mathrm{ext}}}$$

Mass-sheet degeneracy: why?

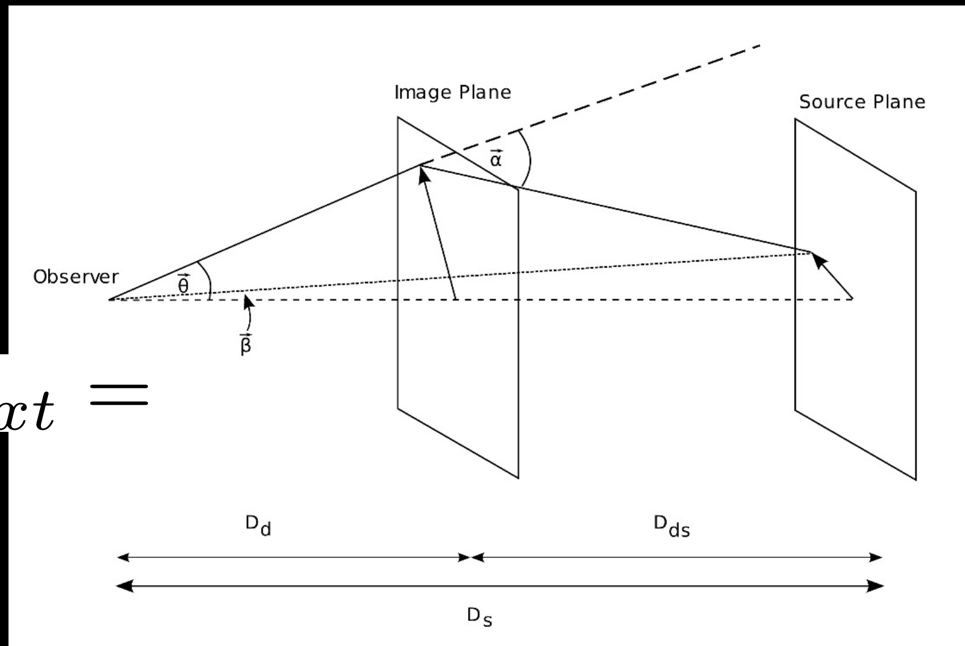
$$\kappa_\lambda = (1-\lambda) + \lambda\kappa; \quad \beta_\lambda = \beta/\lambda$$

$$\Delta t_\lambda = \lambda \Delta t; \quad \mu_\lambda = \mu_\lambda^2$$

$$\lambda = 1 - \kappa_{ext}$$

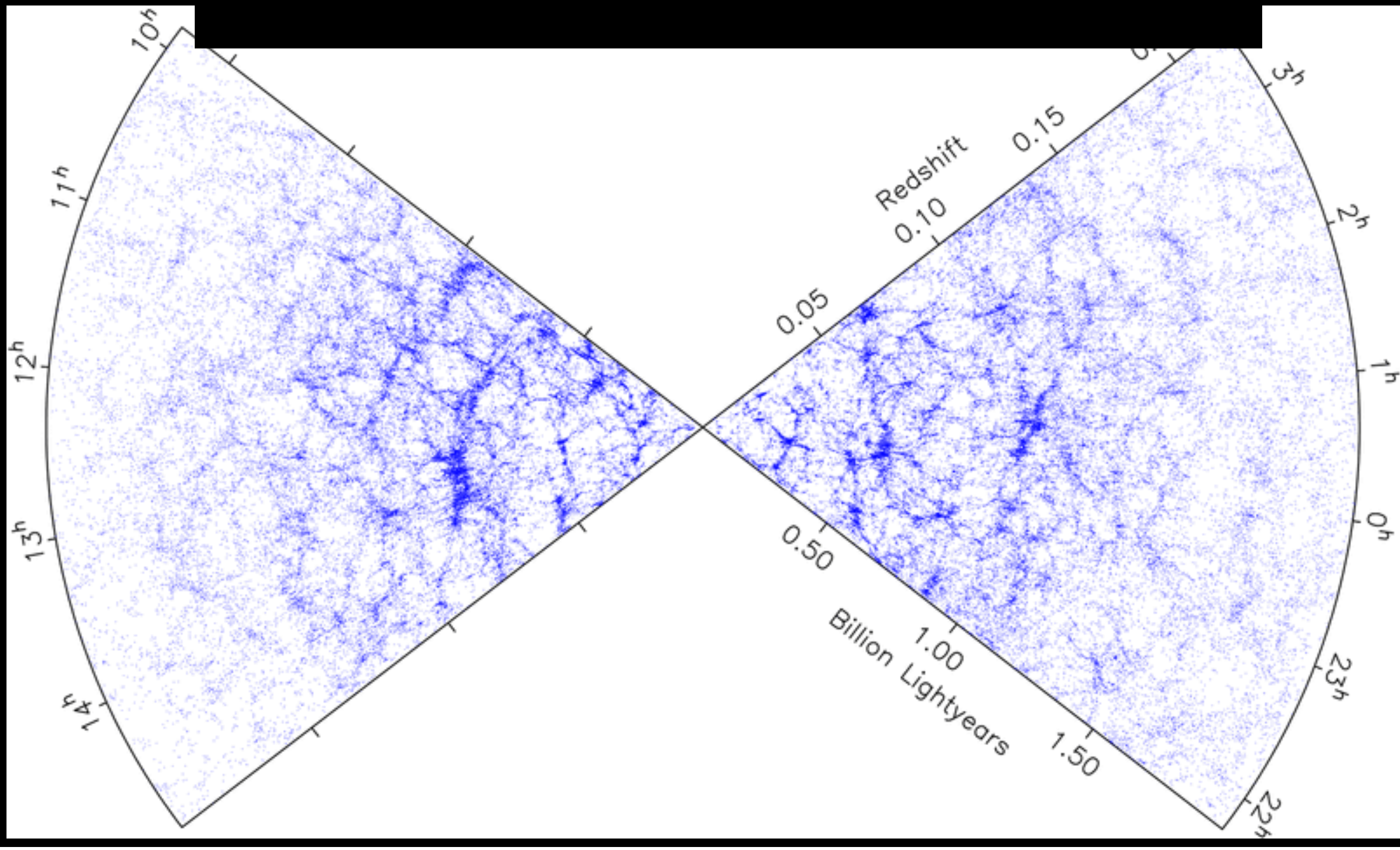


$$" \times " \kappa_{ext} =$$



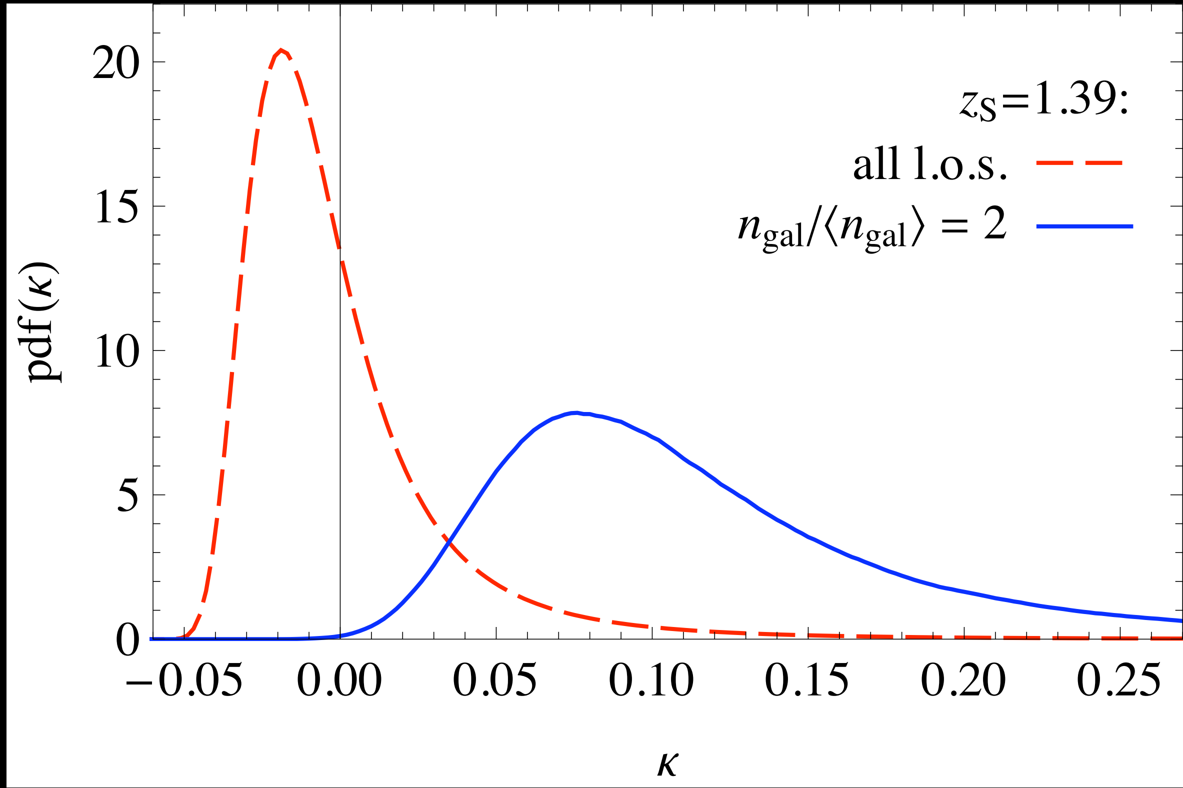
Line of sight vs mass sheet

$$t(\vec{\theta}) = \frac{(1+z_{\rm d})}{c} \frac{D_{\rm d} D_{\rm s}}{D_{\rm ds}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right]$$



Structure along the line of sight

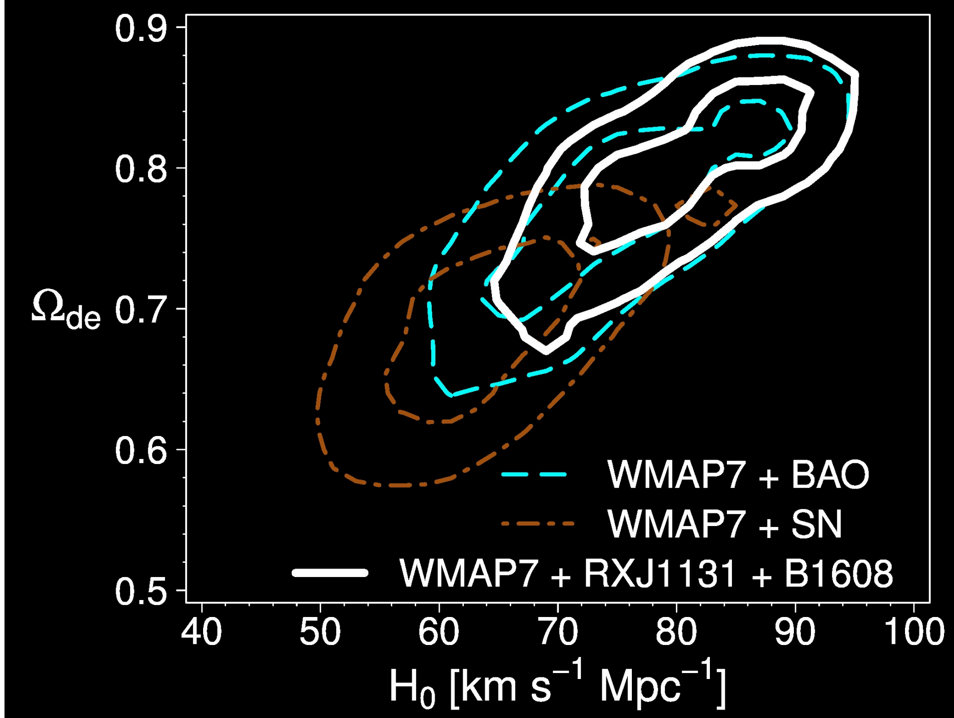
- $H_0(\kappa_{ext})=H_0(0)*(1-\kappa_{ext})$
- Ray-trace through simulations
- Choose light cones comparable to those of the lens
- Observables:
  - Number counts
  - Shear
- Transform line-of-sight structure into kappa_ext!!

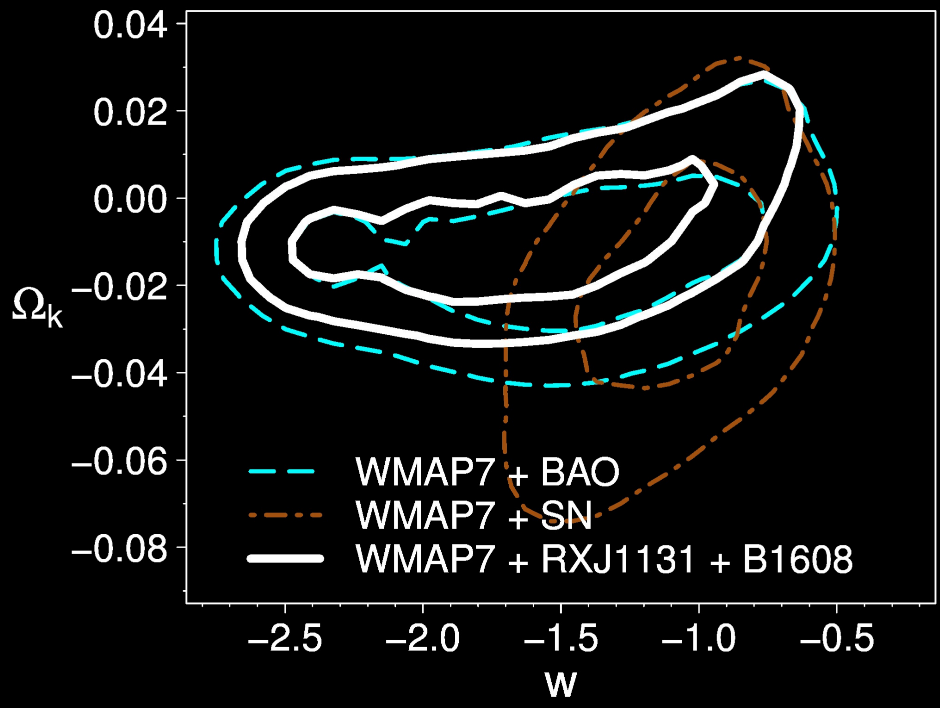


Time delays vs other probes

WMAP7owCDM prior

[Suyu et al. 2012]





- contour orientations are different: complementarity b/w probes
- contour sizes are similar: lensing is a competitive probe

The end