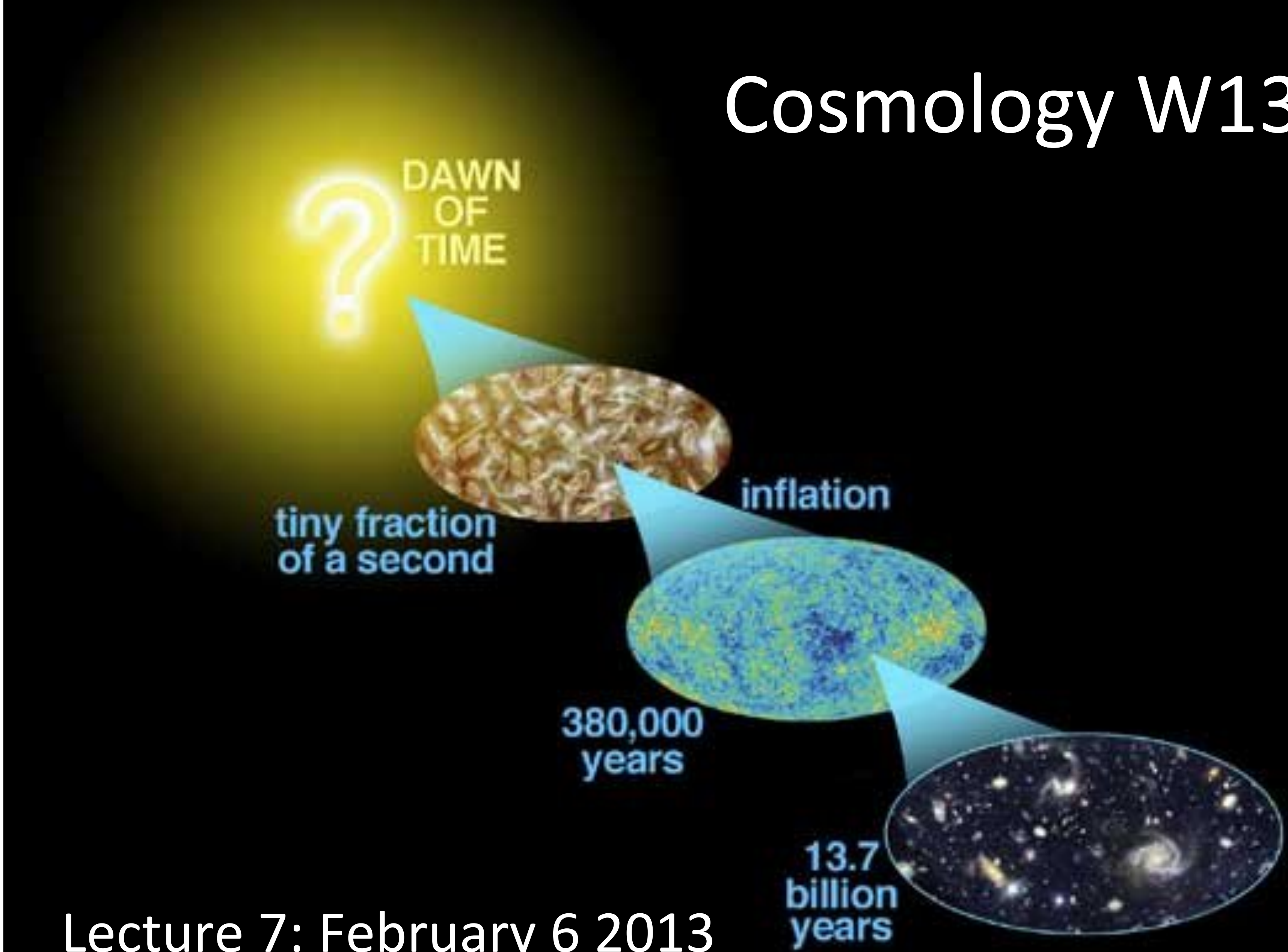
Cosmology W13



Lecture 7: February 6 2013

Thermal history of the universe

- Orders of magnitude
- The CMB
- Recombination
- Optical depth and the last scattering surface

Cosmological parameters. WMAP9

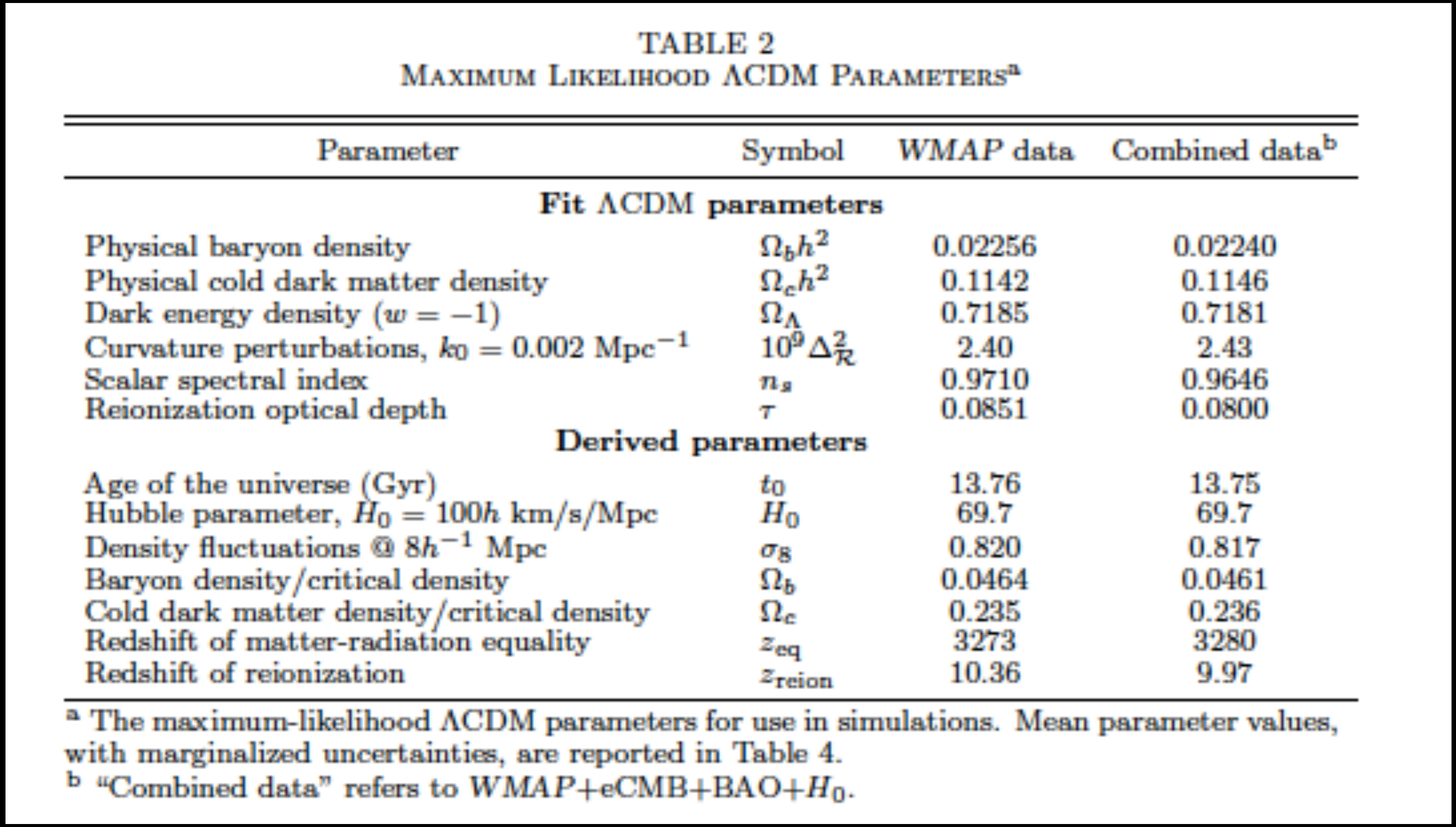


TABLE 2
MAXIMUM LIKELIHOOD ΛCDM PARAMETERS[a]

Parameter	Symbol	*WMAP* data	Combined data[b]
	Fit ΛCDM parameters		
Physical baryon density	$\Omega_b h^2$	0.02256	0.02240
Physical cold dark matter density	$\Omega_c h^2$	0.1142	0.1146
Dark energy density ($w = -1$)	Ω_Λ	0.7185	0.7181
Curvature perturbations, $k_0 = 0.002$ Mpc^{-1}	$10^9 \Delta^2_{\mathcal{R}}$	2.40	2.43
Scalar spectral index	n_s	0.9710	0.9646
Reionization optical depth	τ	0.0851	0.0800
	Derived parameters		
Age of the universe (Gyr)	t_0	13.76	13.75
Hubble parameter, $H_0 = 100h$ km/s/Mpc	H_0	69.7	69.7
Density fluctuations @ $8h^{-1}$ Mpc	σ_8	0.820	0.817
Baryon density/critical density	Ω_b	0.0464	0.0461
Cold dark matter density/critical density	Ω_c	0.235	0.236
Redshift of matter-radiation equality	$z_{\rm eq}$	3273	3280
Redshift of reionization	$z_{\rm reion}$	10.36	9.97

[a] The maximum-likelihood ΛCDM parameters for use in simulations. Mean parameter values, with marginalized uncertainties, are reported in Table 4.

[b] "Combined data" refers to *WMAP*+eCMB+BAO+H_0.

Hinshaw et al. 2012

Cosmological parameters. WMAP9

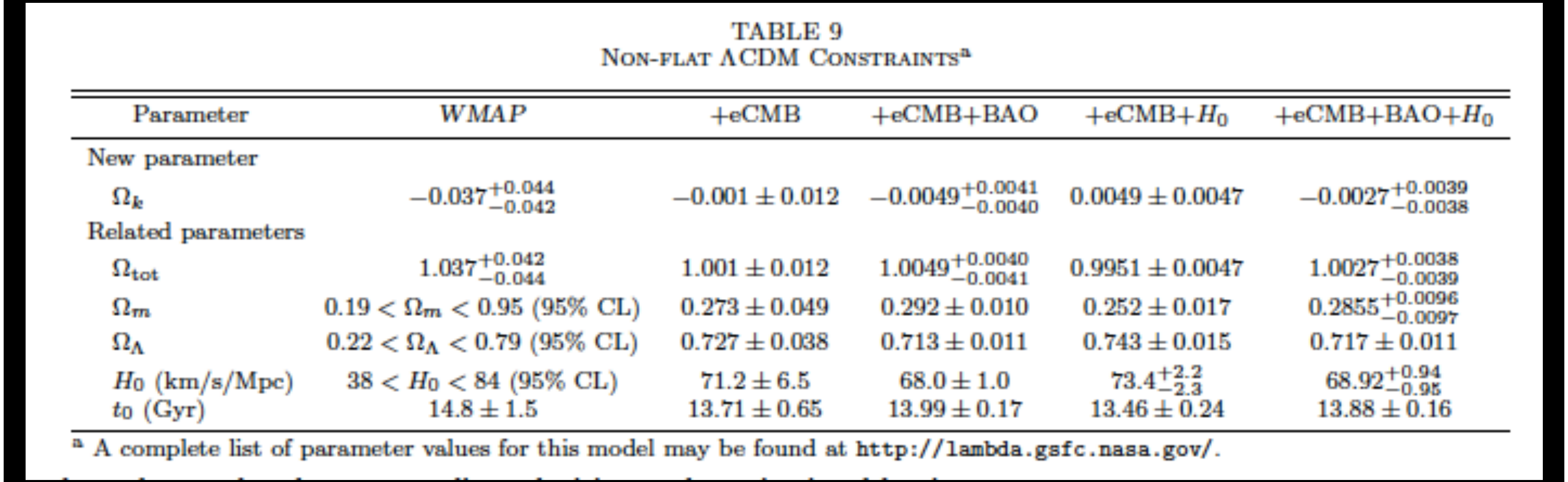


TABLE 9
NON-FLAT ΛCDM CONSTRAINTS[a]

Parameter	*WMAP*	+eCMB	+eCMB+BAO	+eCMB+H_0	+eCMB+BAO+H_0
New parameter					
Ω_k	$-0.037^{+0.044}_{-0.042}$	-0.001 ± 0.012	$-0.0049^{+0.0041}_{-0.0040}$	0.0049 ± 0.0047	$-0.0027^{+0.0039}_{-0.0038}$
Related parameters					
$\Omega_{\rm tot}$	$1.037^{+0.042}_{-0.044}$	1.001 ± 0.012	$1.0049^{+0.0040}_{-0.0041}$	0.9951 ± 0.0047	$1.0027^{+0.0038}_{-0.0039}$
Ω_m	$0.19 < \Omega_m < 0.95$ (95% CL)	0.273 ± 0.049	0.292 ± 0.010	0.252 ± 0.017	$0.2855^{+0.0096}_{-0.0097}$
Ω_Λ	$0.22 < \Omega_\Lambda < 0.79$ (95% CL)	0.727 ± 0.038	0.713 ± 0.011	0.743 ± 0.015	0.717 ± 0.011
H_0 (km/s/Mpc)	$38 < H_0 < 84$ (95% CL)	71.2 ± 6.5	68.0 ± 1.0	$73.4^{+2.2}_{-2.3}$	$68.92^{+0.94}_{-0.95}$
t_0 (Gyr)	14.8 ± 1.5	13.71 ± 0.65	13.99 ± 0.17	13.46 ± 0.24	13.88 ± 0.16

[a] A complete list of parameter values for this model may be found at `http://lambda.gsfc.nasa.gov/`.

Hinshaw et al. 2012

CMB global properties

- Blackbody with T_0=2.725+-0.001
- $T=T_0(1+z)$ preserving blackbody
- Energy density is $u=4\sigma T^4/c \sim (1+z)^4$
- In critical units $\Omega_{CMB} \approx 5 \cdot 10^{-5}$
- This is the majority of
  - the background density
  - energy content of the universe in relativistic particles (neutrinos add a litte)

Basic Thermodynamics Recap

- Dilute weakly intereacting quantum gas in thermal equilibrium has the following number and energy density

$$n = \frac{g}{h^3} \int_0^\infty f(\vec{p}) d^3p$$

$$\epsilon = \frac{g}{h^3} \int_0^\infty E(\vec{p}) f(\vec{p}) d^3p$$

$$f(\vec{p}) = [e^{\frac{E(\vec{p})-\mu}{kT}} \pm 1]^{-1}$$

Basic Thermodynamics Recap

- Non relativistic case (KT<<mc^2)

$$f(\vec{p}) = e^{-\frac{p^2/2m + mc^2 - \mu}{kT}}$$

$$n = \frac{g}{h^3}\int_0^\infty f(\vec{p})d^3p = g\left(\frac{mkT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{mc^2-\mu}{kT}}$$

$$\epsilon = mc^2 n$$

Basic Thermodynamics Recap

- Relativistic case (KT>>mc^2; kT>>μ)

$$n_b = \frac{g\zeta(3)}{\pi^2}\left(\frac{2\pi kT}{hc}\right)^3 \qquad [BOSE]$$

$$n_f = \frac{3}{4}\frac{g\zeta(3)}{\pi^2}\left(\frac{2\pi kT}{hc}\right)^3 \qquad [FERMI]$$

CMB global properties

- The CMB has lots of photons

$$n_{CMB} = \frac{g\zeta(3)}{\pi^2}\left(\frac{2\pi kT}{hc}\right)^3 = 4 \cdot 10^8 m^{-3}$$

- Approximately a billion photons/baryon!
- In terms of energy this is about 1Mev per baryon (c.f. converting H->He gives about 4Mev) and the number grows with (1+z)

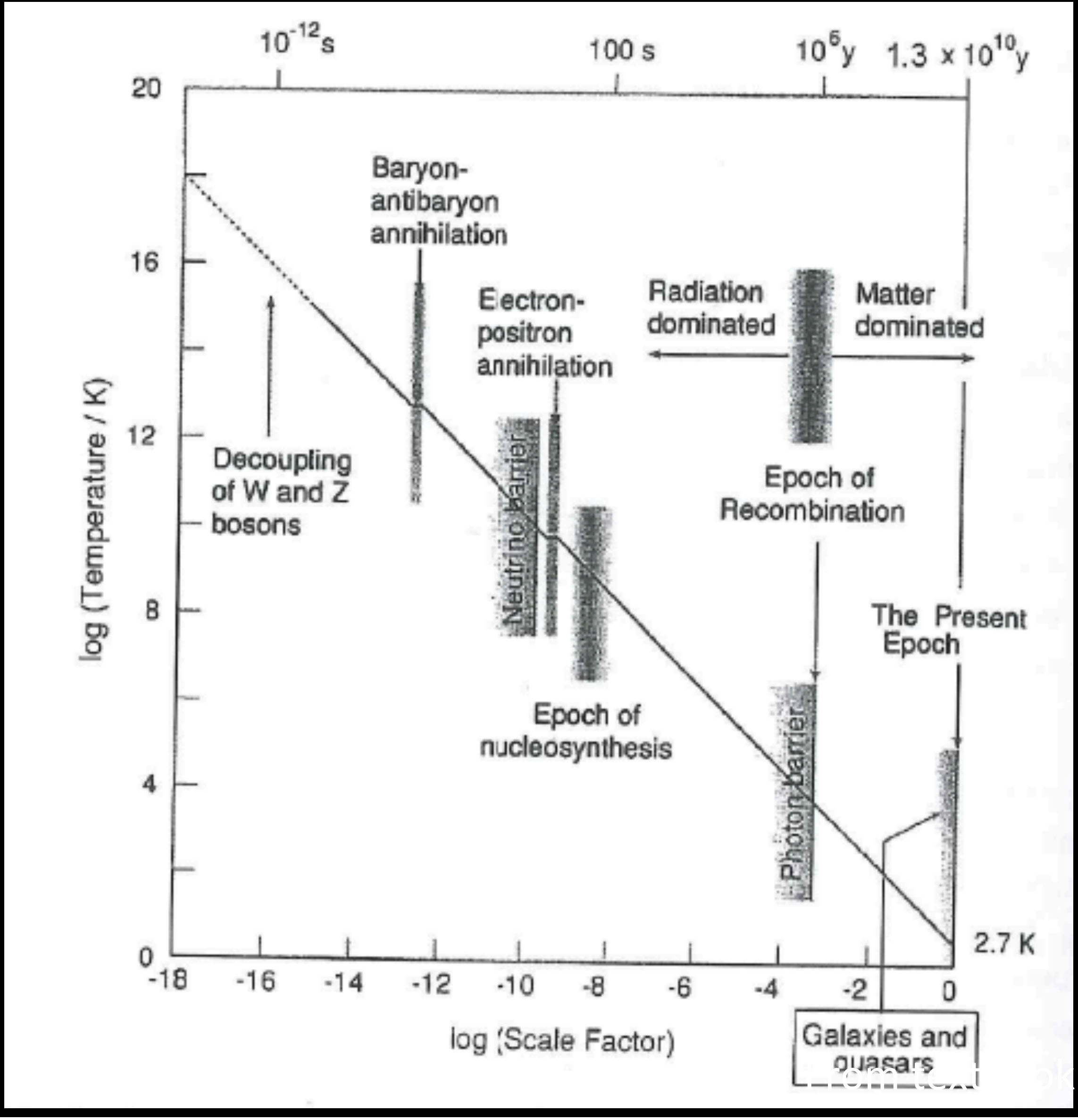
Basic thermal history

$$\Omega_\gamma >> \Omega_m(1+z) \rightarrow \mathrm{RAD\ DOMINATED} \rightarrow a \propto t^{1/2}$$

$$\Omega_\gamma << \Omega_m(1+z) << \Omega_\Lambda \rightarrow \mathrm{MAT\ DOMINATED} \rightarrow a \propto t^{2/3}$$

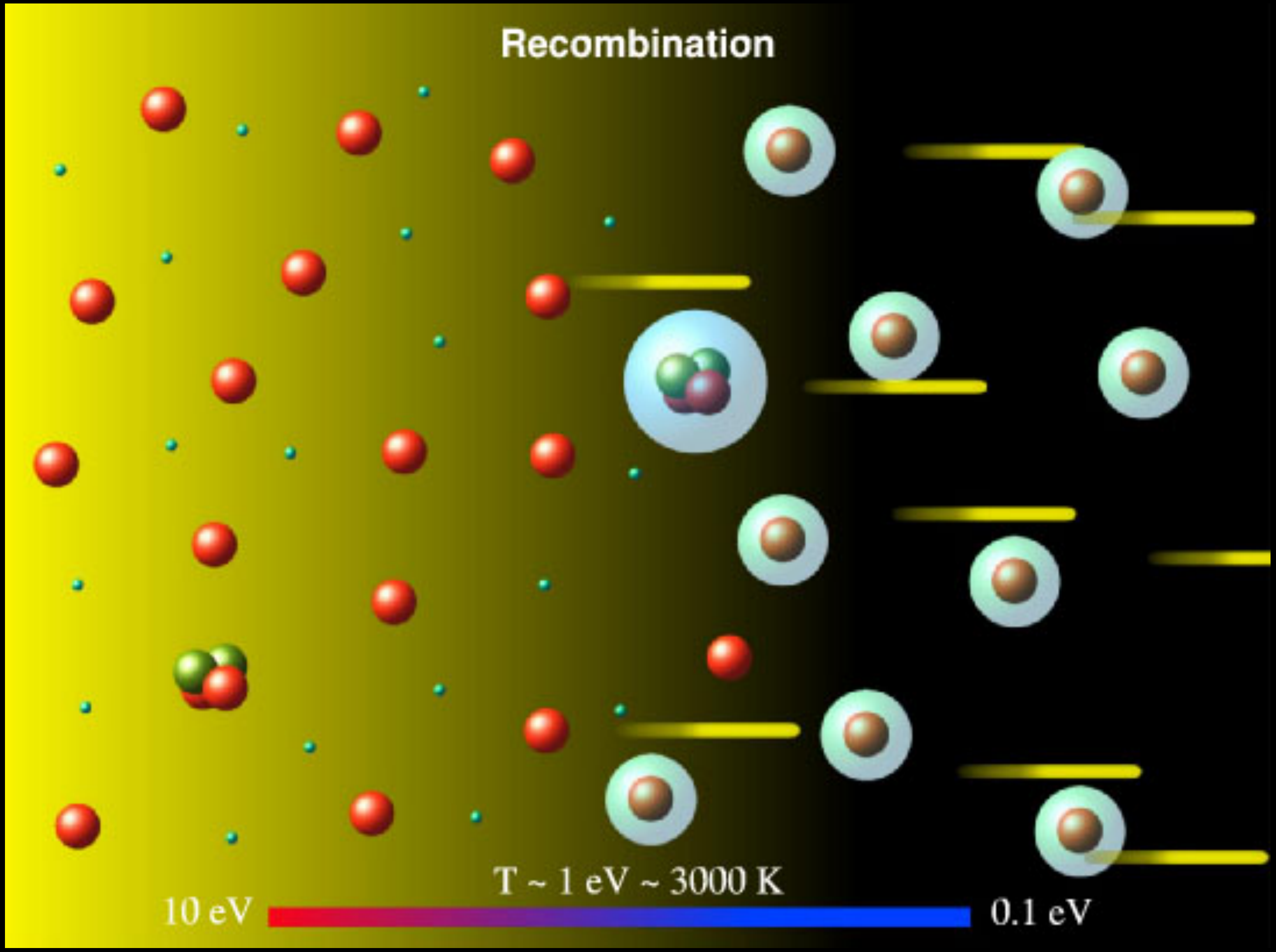
$$\Omega_\Lambda >> \Omega_m \frac{a_0}{a} \rightarrow \mathrm{2nd\ INFLATION} \rightarrow a \propto e^{\sqrt{\Omega_\Lambda} H_0 t}$$

Basic thermal history



From textbook

Recombination



Recombination

- Why does it happen at $kT \ll 12\,\mathrm{eV}$?
  - Qualitatively: a billion photon/baryon, it is sufficient to be in the tail of the distribution to keep H ionized [see Longair]
  - Quantitatively, Saha's equation [blackboard]

Optical depth for Thompson scattering of the CMB by free electrons

$$d\tau_T = \sigma_T N_e(z) dx = \sigma_T N_e(z) c \frac{dt}{dz} dz$$

$$\frac{dz}{dt} \approx -H_0 \Omega_m^{1/2} z^{5/2}$$

$$N_e = N_{H,0} x(z) (1+z)^3 \propto \Omega_b / m_p$$

$$\tau_T = \frac{9 \sigma_T H_0 c \Omega_b}{32 \pi G m_p \Omega_m^{1/2}} \int x(z') z'^{1/2} dz'$$

$$\tau_T = 0.035 \frac{\Omega_b}{\Omega_m^{1/2}} h z^{3/2} \quad [z >> 1; x = 1]$$

Optical depth for Thompson scattering of the CMB by free electrons

- In practice at z>>1000 the universe is opaque to CMB photons, last scattering surface
- There is some optical depth from lower redshift, after so-called cosmic reionization [see previous slides table by Hinshaw et al]

Radiation dominated era – thermal coupling of baryons and photons

- At first baryons and radiation are coupled
- After recombination they decouple and each expands adiabatically
  - Photons T ~ 1/a
  - Baryons T ~ $1/a^2$
    - Also recall that momentum scales as 1/a and therefore to conserve maxwell boltzmann

Radiation dominated era – thermal coupling of baryons and photons

- At first baryons and radiation are coupled

$$\frac{dT_e}{dt} = \frac{4}{3}\sigma_T \epsilon_r \left(\frac{T_r - T_e}{m_e c} \right)$$

- Heat capacity of CMB is much higher so changes in T electrons occur on timescales

$$\tau = \frac{T_e}{dT_e/dt} = 7.4 \cdot 10^{19} z^{-4} s$$

- Shorter than age of the universe

After recombination

- The fraction of free electrons is tiny ~1e-5
- Energy exchange between H and photons is suppressed by free electrons fraction

$$\tau = \frac{T_H}{dT_H/dt} = 1.47 \cdot 10^{24} z^{-4} s$$

- Compared to age of the universe

$$t = \frac{2.06 \cdot 10^{17}}{h\Omega_m^{1/2}} z^{-3/2} s$$

- Decoupling happens at z~150

The end