Cosmology W13

Lecture 9: February 13 2013

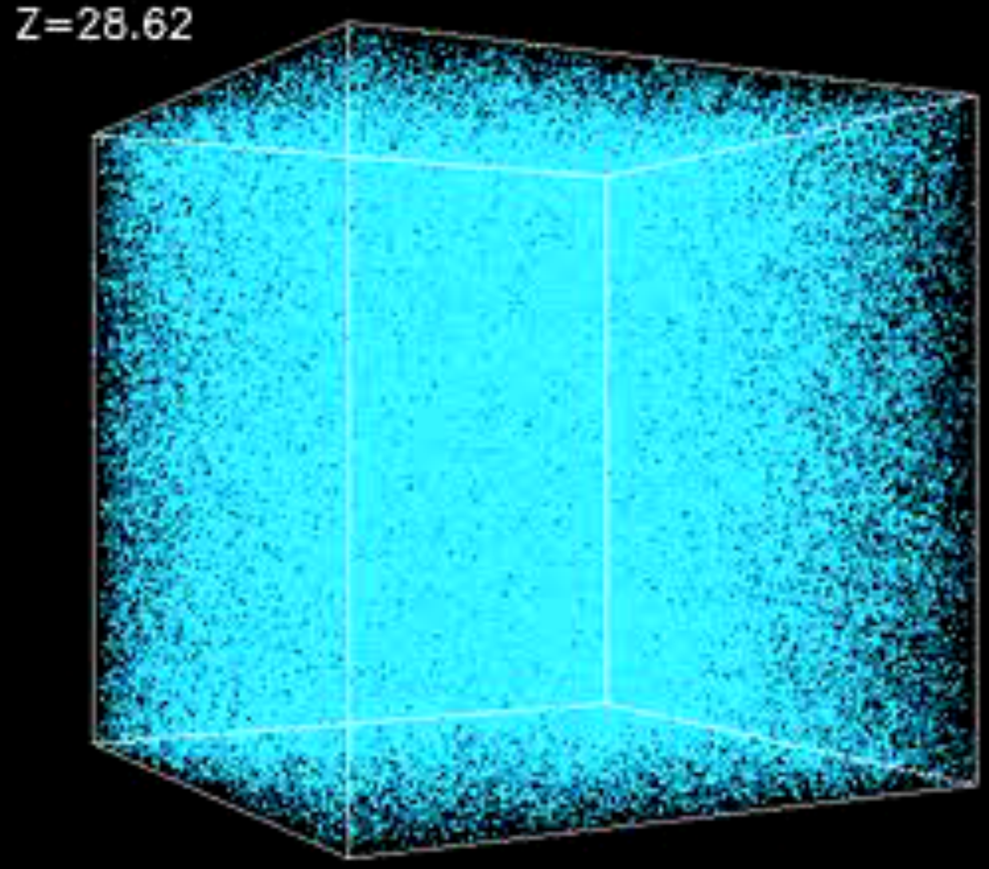
Growth of structure after recombination

- How do we get to where we are?
- Fluid equations in an expanding universe
- Jeans instability
- Jeans instability in an expanding universe
- Growth of small structures

Key concepts

- Current densities of galaxies and clusters show they must have collapsed at $z \ll 100$, well after recombination. Before then everything was linear
- Jeans instability is slow in an expanding universe (linear not exponential)

The growth of structure



kravtsov

Growth of small adiabatic perturbations in expanding universe

$$\frac{\partial p}{\partial \rho} = c_s^2 \qquad \Delta \equiv \frac{\delta \rho}{\rho}$$

$$\frac{d^2\Delta}{dt^2} + 2Hd\frac{d\Delta}{dt} = \frac{c_s^2}{\rho_0 a^2}\nabla_c^2 \delta\rho + 4\pi G\delta\rho$$

This is for perturbation smaller than horizon scale

In Fourier space

$$\Delta \propto e^{i(\mathbf{k_c}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{k_c} = a\mathbf{k}$$

$$\frac{d^2\Delta}{dt^2} + 2H\frac{d\Delta}{dt} = \Delta(4\pi G\rho_0 - k^2 c_s^2)$$

Now set H=0, i.e. non expanding

$$\Delta \propto e^{i(\mathbf{k_c}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{k_c} = a\mathbf{k}$$

$$\frac{d^2\Delta}{dt^2} + 2H\frac{d\Delta}{dt} = \Delta(4\pi G\rho_0 - k^2 c_s^2)$$

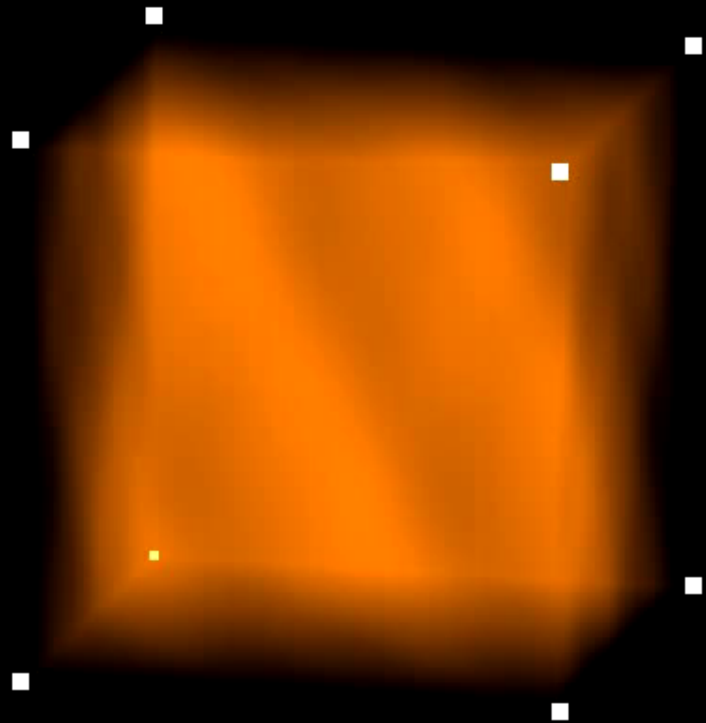
$$\omega^2 = k^2 c_s^2 - 4\pi G\rho$$

$$\lambda_j = c_s \left(\frac{\pi}{G\rho}\right)^{\frac{1}{2}}$$

Jeans instability – exponential for length greater than Jeans'

$$\lambda_j = c_s \left(\frac{\pi}{G\rho}\right)^{\frac{1}{2}}$$

$$\tau \sim (G\rho)^{-\frac{1}{2}}$$



Essential physical ingredient to form, e.g., stars from gas

But in expanding universe

$$\Delta \propto e^{i(\mathbf{k_c}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{k_c} = a\mathbf{k}$$

$$\frac{d^2\Delta}{dt^2} + 2H\frac{d\Delta}{dt} = \Delta(4\pi G\rho_0 - k^2 c_s^2)$$

The expansion provides a damping term

For long wavelengths and EdS

$$4\pi G\rho = \frac{2}{3t^2} \qquad H = \frac{2}{3t}$$

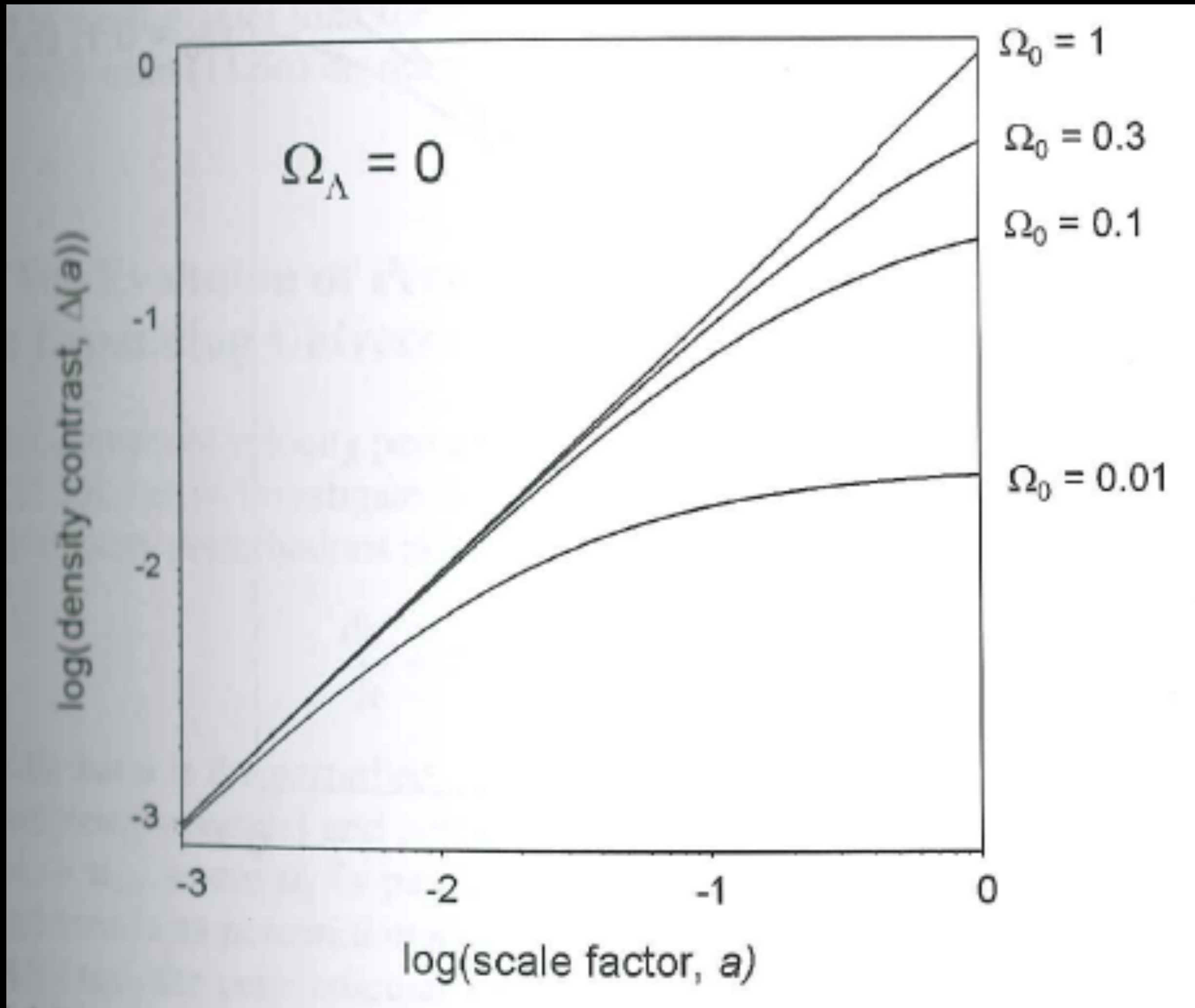
$$\frac{d^2\Delta}{dt^2} + \frac{4}{3t}\frac{d\Delta}{dt} - \frac{2}{3t^2}\Delta = 0$$

$$\Delta \propto t^{\frac{2}{3}} \rightarrow \Delta \propto a \propto (1+z)^{-1}$$

INSTABILITIES ONLY GROW LINEARLY!!

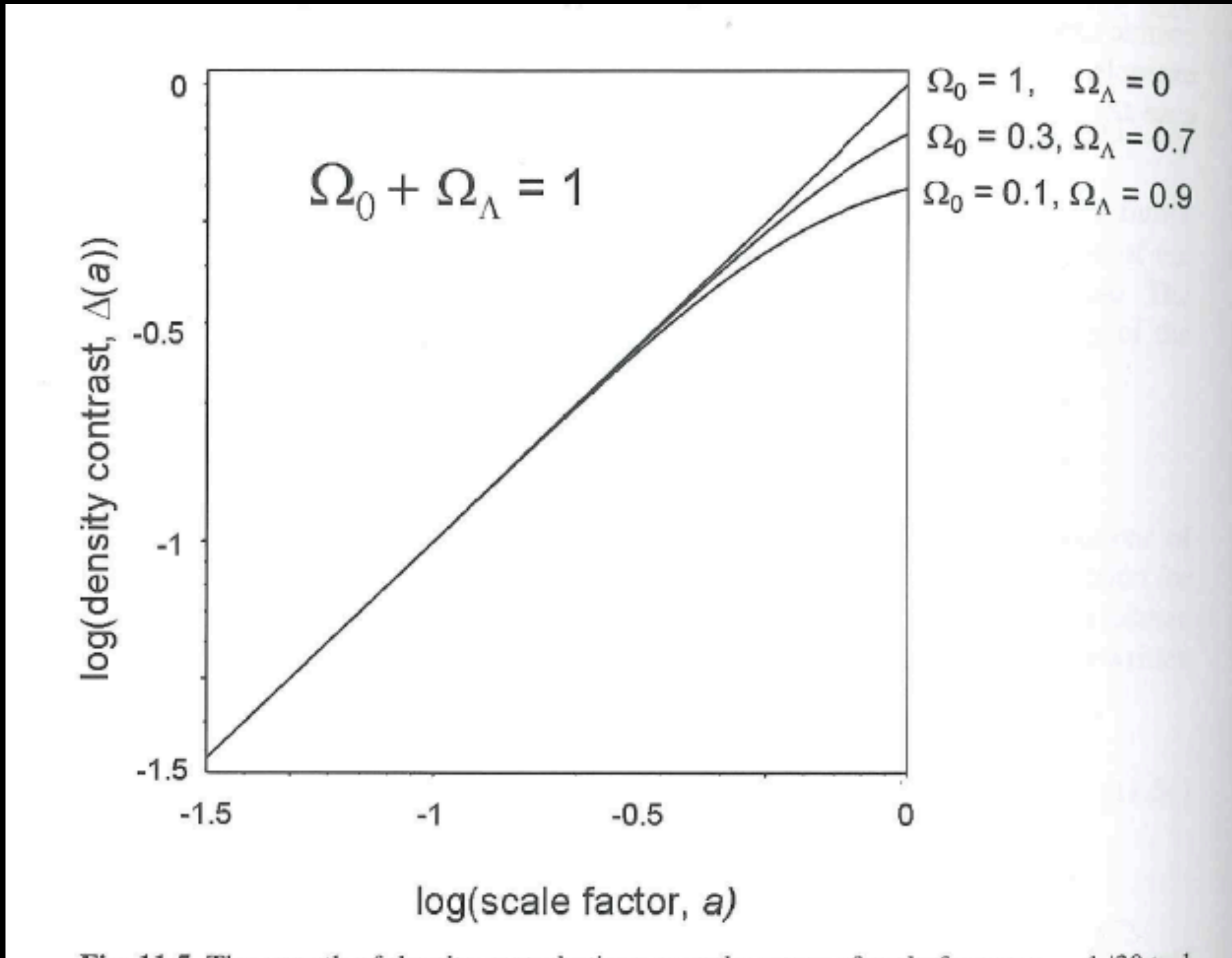
P.S. in empty universe they don't grow at all!!

In general for no lambda universe



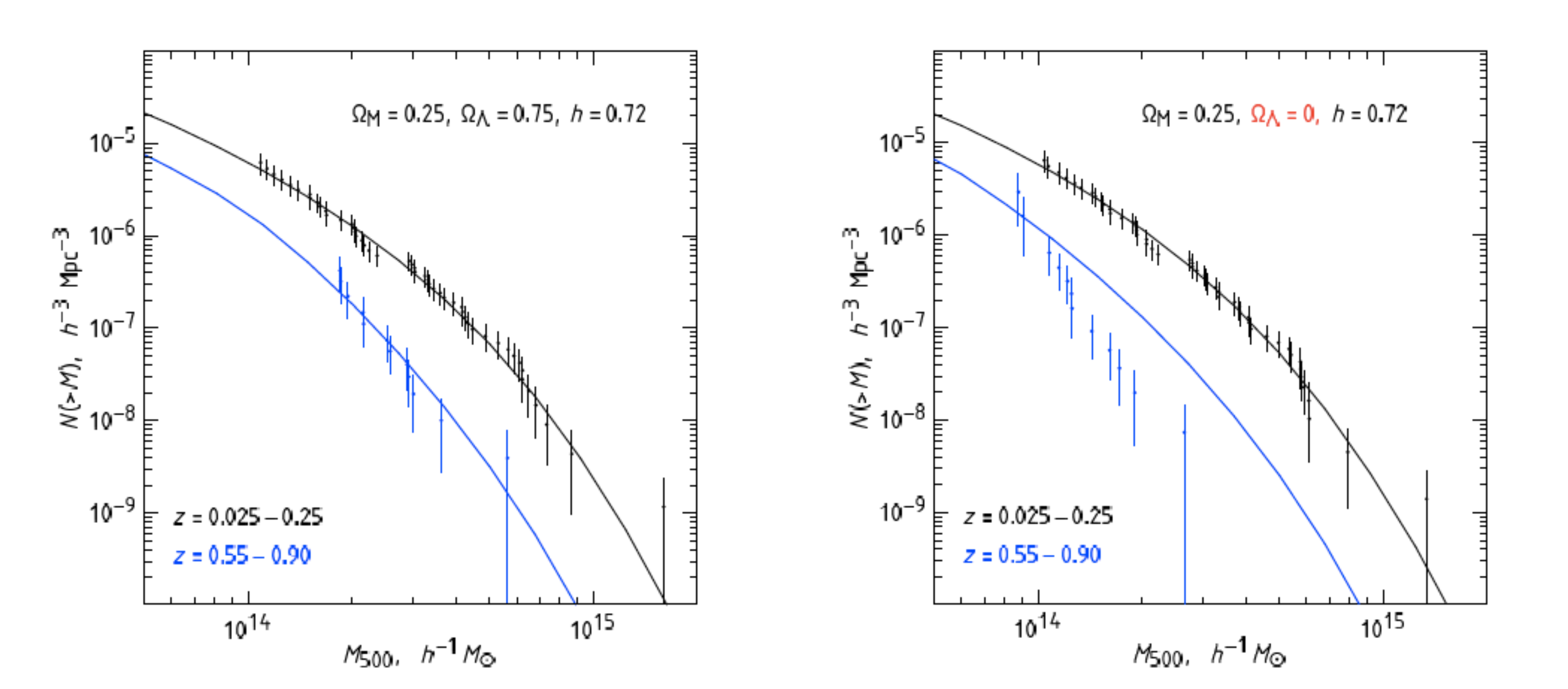
From textbook

In general for flat cosmology



From textbook

Growth of structure as a test of cosmology



What are the challenges?

Allen, Evrard & Mantz 2011

Peculiar velocities

$$\frac{d\mathbf{u}}{dt} + 2H\mathbf{u} = -\frac{1}{a^2}\nabla_c \delta\phi$$

- U is perturbed comoving velocity
- Component perpendicular to potential gradients is damped by expansion
- Component parallel to potential gradients depend on scale (k)

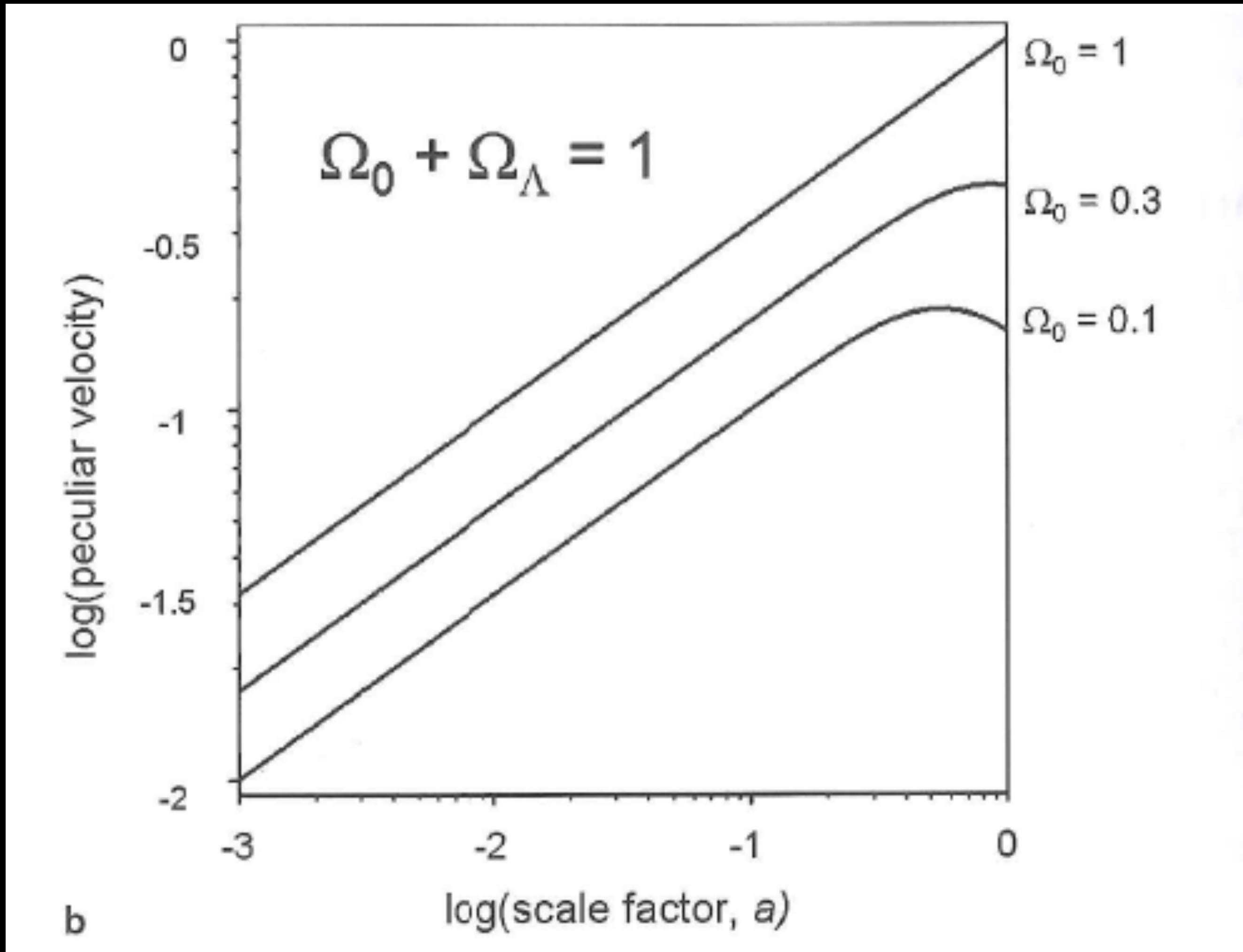
$$|\delta v_{||}| = \frac{a}{k_c}\frac{d\Delta}{dt}$$

For EdS

$$|\delta v_{||}| = \frac{H_0}{k}\left(\frac{\delta\rho}{\rho}\right)_0 (1+z)^{-\frac{1}{2}}$$

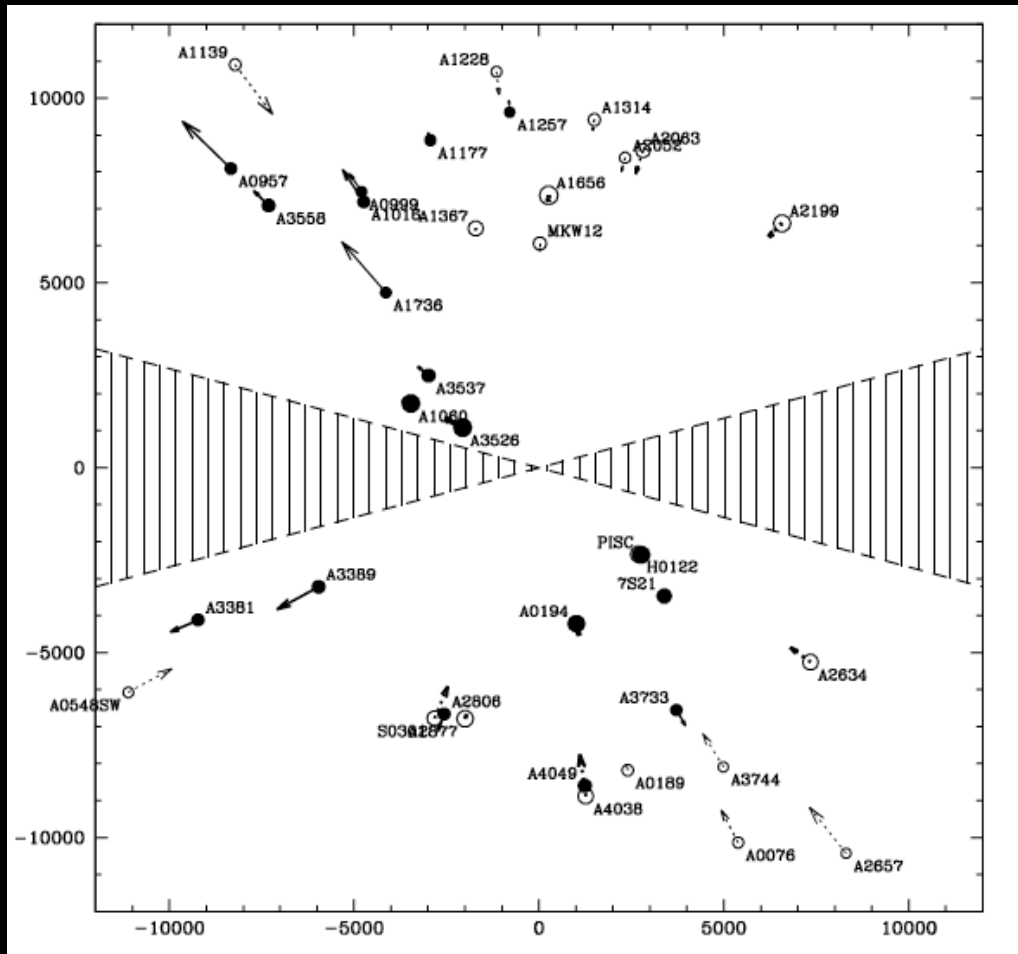
Peculiar velocities grow with time and are driven by largest scales (small k)

In general



Peculiar velocities grow with time and are driven by largest scales (small k)

Peculiar velocities as a test for cosmology



Hudson et al. 2004; What are the Challenges?

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