Homework 5 Solutions

1. (a) To estimate the total amount of hydrogen consumed over the past 4.56 billion years, we first need to obtain the total energy lost over the past 4.56 billion years, E. Since the Sun's luminosity $L_\odot$, which is the rate of energy radiated, is assumed to be constant, we simply multiply $L_\odot$ by the length of time $t = 4.56$ billion years to get E:

$$\begin{aligned} E &= L_\odot \times t \\ &= (3.90 \times 10^{26}\ \mathrm{W}) \times (4.56 \times 10^9\ \mathrm{yr}) \times \left(\frac{3.15 \times 10^7\ \mathrm{s}}{1\ \mathrm{yr}}\right) \\ &= 5.602 \times 10^{43}\ \mathrm{J} \end{aligned}$$

This energy radiated must be equal to the energy released from hydrogen burning. Now, in hydrogen burning, 4 hydrogen atoms produces a helium atom, and it is the mass difference between the helium atom and the four hydrogen atoms that is converted to energy. The total mass difference over the past 4.56 billion years, M_{diff}, is related to the total energy released in this period via the mass-energy relation, $E = M_{\mathrm{diff}}c^2$.

We can relate this to the total mass of hydrogen consumed since in hydrogen burning, for each 1 kg of hydrogen being consumed, only 0.007 kg is converted to energy. In other words, the total mass difference is 0.7% of the total mass of hydrogen consumed, $M_{\mathrm{diff}} = 0.007M_{\mathrm{H}}$. Thus, the total amount of hydrogen consumed is

$$\begin{aligned} E &= M_{\mathrm{diff}}c^2 = 0.007M_{\mathrm{H}}c^2 \\ M_{\mathrm{H}} &= \frac{E}{0.007c^2} \\ &= \frac{5.602 \times 10^{43}\ \mathrm{J}}{0.007(3 \times 10^8\ \mathrm{m/s})^2} \\ &= \boxed{8.89 \times 10^{28}\ \mathrm{kg}} \end{aligned}$$

Alternatively, one can read from Box 16-1, and multiply the rate of hydrogen consumption 6×10^{11} kg/s by 4.56 billion years to get the total amount of hydrogen consumed:

$$M_{\mathrm{H}} = (6 \times 10^{11}\ \mathrm{kg/s}) \times (4.56 \times 10^9\ \mathrm{yr}) \times \left(\frac{3.15 \times 10^7\ \mathrm{s}}{1\ \mathrm{yr}}\right) = \boxed{8.62 \times 10^{28}\ \mathrm{kg}}$$

where we note that the numerical value of M_{H} obtained here is slightly different from that obtained in the first method, because the rate of hydrogen consumption 6×10^{11} kg/s is a rounded number.

The total mass lost over the past 4.56 billion years is what we called M_{diff} previously, which is related to the total mass of hydrogen consumed M_{H} via $M_{\mathrm{diff}} = 0.007M_{\mathrm{H}}$. Thus,

$$M_{\mathrm{diff}} = 0.007M_{\mathrm{H}} = 0.007(8.89 \times 10^{28}\ \mathrm{kg}) = \boxed{6.22 \times 10^{26}\ \mathrm{kg}}$$

(b) Our answers to part (a) are an $\boxed{\text{overestimate}}$. Since the Sun's luminosity was lower in the past than in the present, the rate of hydrogen burning required to power the Sun's luminosity was also lower in the past. Therefore, the total amount of hydrogen lost and the total amount of mass lost are both overestimated.

2. We use the Doppler shift equation, which says that the ratio of the wavelength shift $\Delta\lambda = \lambda - \lambda_0$ to the unshifted wavelength λ_0, is proportional to the ratio of the source velocity along the line of sight v to the speed of light c:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

We only need the magnitude of the maximum wavelength shift, so we take the absolute value of the wavelength shift and of the velocity:

$$|\Delta\lambda| = \frac{|v|}{c}\lambda_0 = \frac{0.1 \text{ m/s}}{2.9979 \times 10^8 \text{ m/s}}(557.6099 \text{ nm}) = \boxed{1.86 \times 10^{-7} \text{ nm}}$$

3. We know that the blackbody temperature of the umbra of a sunspot is typically $T_{\text{umbra}} = 4300$ K, that of the penumbra of a sunspot is typically $T_{\text{penumbra}} = 5000$ K and that of the Sun's photosphere is about $T_{\text{photosphere}} = 5800$ K. We can relate the blackbody temperatures T to the energy flux F via the Stefan-Boltzmann law, which says the energy flux is proportional to the fourth power of the blackbody temperature, $F \propto T^4$.

(a) The ratio of energy flux from a sunspot's penumbra F_{penumbra} to the energy flux from an equally large patch of photosphere $F_{\text{photosphere}}$ is therefore

$$\frac{F_{\text{penumbra}}}{F_{\text{photosphere}}} = \left(\frac{T_{\text{penumbra}}}{T_{\text{photosphere}}}\right)^4 = \left(\frac{5000 \text{ K}}{5800 \text{ K}}\right)^4 = \boxed{0.552}$$

Since $F_{\text{penumbra}}/F_{\text{photosphere}} < 1$, the photosphere is brighter.

(b) The ratio of energy flux from a sunspot's penumbra F_{penumbra} to the energy flux from an equally large patch of umbra F_{umbra} is

$$\frac{F_{\text{penumbra}}}{F_{\text{umbra}}} = \left(\frac{T_{\text{penumbra}}}{T_{\text{umbra}}}\right)^4 = \left(\frac{5000 \text{ K}}{4300 \text{ K}}\right)^4 = \boxed{1.83}$$

Since $F_{\text{penumbra}}/F_{\text{umbra}} > 1$, the penumbra of a sunspot is brighter.

4. (a) From Box 17-1, the tangential velocity of a star v_{t} (in km/s) is related to its proper motion μ (arcsec/yr) and distance d (in parsec) via $v_{\text{t}} = 4.74\mu d$. We first want to obtain d in parsec, from the parallax (in arcsec), via

$$d = \frac{1}{p} = \frac{1}{0.255} = 3.922 \text{ pc}$$

and thus the tangential velocity is

$$v_{\text{t}} = 4.74\mu d = 4.74(8.67)(3.922) = \boxed{161 \text{ km/s}}$$

(b) The actual speed v can be obtained from the tangential speed v_{t} and radial speed v_{r} by the Pythagorean theorem,

$$v = \sqrt{v_{\mathrm{t}}^2 + v_{\mathrm{r}}^2} = \sqrt{(161\ \mathrm{km/s})^2 + (246\ \mathrm{km/s})^2} = \boxed{294\ \mathrm{km/s}}$$

(c) The radial velocity is the component of velocity along the observer's line-of-sight and is related to the Doppler shift via $(\lambda - \lambda_0)/\lambda_0 = v_{\mathrm{r}}/c$. Since $v_{\mathrm{r}} > 0$ for Kapteyn's star, then $\lambda > \lambda_0$ which means there is a redshift in spectral lines. The star is thus $\boxed{\text{moving away}}$ from the Sun.

5. (a) The brightest star in Figure 17-6b has an apparent magnitude of $m_{\mathrm{bright}} = +2.84$, and the dimmest has an apparent magnitude of $m_{\mathrm{dim}} = +9.53$ (recall that a star with a higher apparent brightness has a lower apparent magnitude). We can relate the apparent magnitude m to the absolute magnitude M if we know the distance d in parsec, via $m - M = 5 \log d - 5$. Using $d = 110$ pc, the absolute magnitude of the brightest star is

$$M_{\mathrm{bright}} = m_{\mathrm{bright}} - 5 \log d + 5 = 2.84 - 5 \log(110) + 5 = \boxed{-2.37}$$

and that of the dimmest star is

$$M_{\mathrm{dim}} = m_{\mathrm{dim}} - 5 \log d + 5 = 9.53 - 5 \log(110) + 5 = \boxed{+4.32}$$

(b) The limiting apparent magnitude for the naked eye is $m = +6$. Using again $d = 110$ pc, the absolute magnitude for such a star is

$$M = m - 5 \log d + 5 = 6 - 5 \log(110) + 5 = \boxed{+0.793}$$

The Sun has an absolute magnitude of $M_{\odot} = +4.8$. Since a more luminous star has a lower absolute magnitude, the Pleaides star at the naked eye limit is more luminous.

6. (c) The spectra of very hot stars peak in the blue visible or the ultraviolet and radiate progressively less flux in longer wavelengths, as shown in the figure for the 12,000 K star. There is more flux in the U band than in the B band, and more in the B band than in the V band, and so $b_{\mathrm{V}}/b_{\mathrm{B}}$ and $b_{\mathrm{B}}/b_{\mathrm{U}}$ are less than 1.

Similarly, very cool stars peak in the red visible and even the infrared, and radiate progressively less flux in shorter wavelengths, as shown in the figure for the 3000 K star. There is less flux in the U band than in the B band, and less in the B band than in the V band, and so $b_{\mathrm{V}}/b_{\mathrm{B}}$ and $b_{\mathrm{B}}/b_{\mathrm{U}}$ are greater than 1.

7. (a) Singly ionized helium lines start to form for temperatures hotter than 30,000 K, so we would expect to find them in the spectrum of a star with a surface temperature of 35,000 K.

(b) We expect to find prominent molecular lines such as titanium oxide (TiO) in a star with surface temperature of 2800 K.

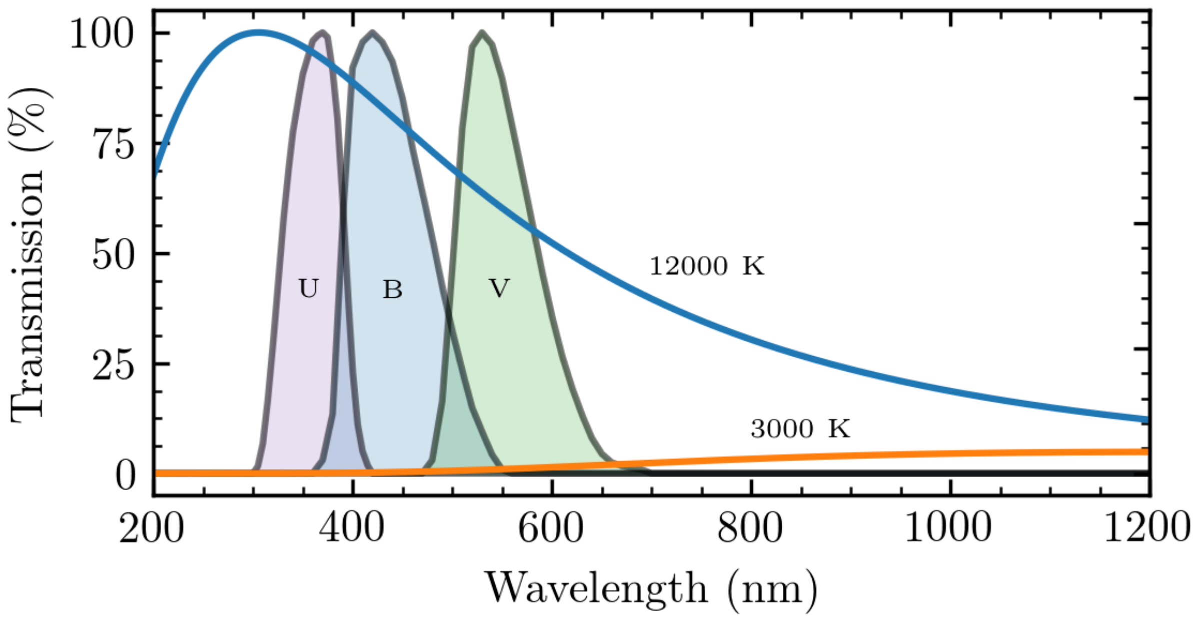

Figure 1: Problem 6a and 6b.

(c) We expect to find prominent ionized and neutral metal lines in a star with surface temperature of 5800 K.

These stars have such different spectra because their surface temperatures excite different atomic/molecular species. A 35,000 K star is hot enough to singly ionize helium, and excite singly helium lines. A 5800 K star is not hot enough to excite helium atoms so as to produce prominent helium lines, but it is hot enough to excite ionized and neutral metal lines. A 2800 K star is not hot enough to excite helium or metal lines, but it is cool enough to allow formation of molecules and produce prominent molecular lines.