Homework 6, Astro 1

Due November 15th, 2019

1. (U11-18.12)

For every 1 kpc, only 15% of the light could pass through the interstellar medium. To travel a distance of 3 kpc, the light remains is $(0.15)^3 = 3.375 \times 10^{-3} = \boxed{0.3375\%}$ of the original amount.

2. (U11-18.28)

(a) The length of the scale bar is around 5.8 cm. On the image, the diameter of the disk is around 3 cm, i.e. the radius is 1.5 cm. Therefore, for the accretion disk, radius r is

$$r = \frac{1.5 \text{ cm}}{5.8 \text{ cm}}(1000 \text{ AU}) \approx \boxed{260 \text{ AU}}$$
$$\approx 260 \text{ AU}\left(\frac{1.496 \times 10^8 \text{ km}}{1 \text{ AU}}\right) = \boxed{3.87 \times 10^{10} \text{ km}}$$

(b) We use the following form of Kepler's Thrid Law with the masses in solar units (i.e. in terms of $M_\odot$), semi-major axis a in AU and period P in years. Let M_1 be the mass of the star, and M_2 be the mass of an accreting particle, therefore $M_2 \ll M_1$, i.e. $M_1 + M_2 \approx M_1$.

$$M_1 + M_2 = \frac{a^3}{P^2} \quad \Rightarrow \quad M_1 = \frac{a^3}{P^2}$$

Rearranging the equation, substitute $a = 260$ AU and $M_1 = 1M_\odot$,

$$P = \sqrt{\frac{a^3}{M_1}} = \sqrt{\frac{260^3}{1}} = \boxed{4190 \text{ years}}$$

(c) On the image, the jet is around 5.0 cm from the mid-plane. Distance travelled along the jet is

$$d = \frac{5.0 \text{ cm}}{5.8 \text{ cm}}(1000 \text{ AU})\left(\frac{1.496 \times 10^8 \text{ km}}{1 \text{ AU}}\right) = 1.29 \times 10^{11} \text{ km}$$

With the given speed v as 200 km s^{-1}, the time required t is

$$d = vt \quad \Rightarrow \quad t = \frac{d}{v} = \frac{1.29 \times 10^{11}}{200} = 6.45 \times 10^8 \text{ s} = \boxed{20.4 \text{ years}}$$

3. (U11-18.42)

The shock wave has expanded 3 pc outward in 300 years. Converting 3 pc into kilometers and 300 years into seconds, and using 1 pc $= 3.09 \times 10^{13}$ km and 1 year $= 3.16 \times 10^7$ s,

$$d = 3\ \text{pc} = 3(3.09 \times 10^{13}\ \text{km}) = 9.27 \times 10^{13}\ \text{km}$$
$$t = 300\ \text{years} = 300(3.16 \times 10^7\ \text{s}) = 9.48 \times 10^9\ \text{s}$$

Note that speed of light $c = 3.00 \times 10^5\ \text{km s}^{-1}$, the average speed of the shock wave is

$$\begin{aligned} d = vt \quad \Rightarrow \quad v = \frac{d}{t} = \frac{9.27 \times 10^{13}}{9.48 \times 10^9} &= \boxed{9800\ \text{km s}^{-1}} \\ &= 9800\ \text{km s}^{-1}\left(\frac{c}{3.00 \times 10^5\ \text{km s}^{-1}}\right) \\ &= \boxed{0.0327c} \end{aligned}$$

4. (U11-18.46)

Given the protosun had a luminosity L of $1000L_\odot$ and surface temperature T of 1000 K, to find radius R, we use the equation $L = 4\pi R^2 \sigma T^4$. For convenience, we relate this to our present day Sun, $L_\odot = 4\pi R_\odot^2 \sigma T_\odot^4$. Using the two equations, we have

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4 \quad \Rightarrow \quad \frac{R}{R_\odot} = \sqrt{\left(\frac{L}{L_\odot}\right)\left(\frac{T_\odot}{T}\right)^4} = \left(\frac{T_\odot}{T}\right)^2 \sqrt{\frac{L}{L_\odot}}$$

Since we know the surface temperature of our present Sun $T_\odot$ is 5800 K,

$$\frac{R}{R_\odot} = \left(\frac{5800}{1000}\right)^2 \sqrt{\frac{1000L_\odot}{L_\odot}} = 1060 \quad \Rightarrow \quad \boxed{R = 1060R_\odot}$$

To express the radius R in kilometers and astronomical units, note that $R_\odot = 6.96 \times 10^5$ km and 1 AU $=$ 1.496×10^8 km,

$$\begin{aligned} R = 1060R_\odot = 1060(6.96 \times 10^5\ \text{km}) &= \boxed{7.38 \times 10^8\ \text{km}} \\ &= (7.38 \times 10^8\ \text{km})\left(\frac{1\ \text{AU}}{1.496 \times 10^8\ \text{km}}\right) \\ &= \boxed{4.93\ \text{AU}} \end{aligned}$$

5. (U11-19.8)

The star should still be in the main sequence after 1 billion (10^9) year. From Table 19-1 in *Universe*, along the main sequence, stars of $3M_\odot$ and $1.5M_\odot$ have main sequence lifetime of 800×10^6 years and 4500×10^6 years respectively. Therefore, this main sequence lifetime of 10^9 year corresponds to a mass of star of $\boxed{\text{slightly under } 3M_\odot}$.

Alternatively, we could actually calculate the mass using $t \propto 1/M^{2.5}$ for stars in the main sequence. Since a $3M_\odot$ main sequence star corresponds to a main-sequence lifetime of 800×10^6 years from Table 19-1, set $t_1 = 800 \times 10^6$ years and $M_1 = 3M_\odot$. Consider $t_2 = 10^9$ years, we find the corresponding mass M_2 using

$$\frac{t_2}{t_1} = \left(\frac{M_1}{M_2}\right)^{2.5} \quad \Rightarrow \quad M_2 = M_1 \left(\frac{t_1}{t_2}\right)^{\frac{1}{2.5}} = 3M_\odot \left(\frac{800 \times 10^6}{10^9}\right)^{0.4} = 2.7M_\odot$$

6. (U11-19.12)

Hydrogen contains a nucleus with only a single proton while Helium contains two protons and two neutrons. Therefore, the repulsive force between any two Helium nuclei is four times that of two Hydrogen nuclei, given that the separation distance between atoms remains constant. Because of this greater repulsion, Helium fusion requires greater force and higher speeds in order to collide more violently, and that is only attainable via higher temperatures.

7. (U11-19.35)

Given the apparent brightness of δ Cephei has a period of 5.4 days, using the period-luminosity relation for Cepheids in Figure 19-20 of *Universe*, this corresponds to a luminosity of around $3 \times 10^3 L_\odot$.
We use the equation of apparent brightness $b = L/(4\pi d^2)$, where d is the distance from the source. Set b_1 as the apparent brightness of the Sun, d_1 as the Sun-Earth distance 1 AU $= 1.496 \times 10^8$ km, and luminosity $L_1 = L_\odot$. Given δ Cephei has an apparent brightness 5.1×10^{-13} that of the Sun, i.e. $b_2 = 5.1 \times 10^{-13} b_1$, and we found $L_2 = 3 \times 10^3 L_\odot$ from the graph,

$$\begin{aligned} \frac{b_2}{b_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{d_1}{d_2}\right)^2 \quad \Rightarrow \quad d_2 &= d_1 \sqrt{\left(\frac{b_1}{b_2}\right)\left(\frac{L_2}{L_1}\right)} \\ &= (1\ \text{AU}) \sqrt{\left(\frac{1}{5.1 \times 10^{-13}}\right)\left(\frac{3 \times 10^3 L_\odot}{L_\odot}\right)} \\ &= 7.67 \times 10^7\ \text{AU} = \boxed{1.15 \times 10^{16}\ \text{km}} \end{aligned}$$

8. (U11-19.46)

Escape velocity $v_{\text{esc}} = \sqrt{2GM/r}$

(a) For the present-day Sun, $M = 1M_\odot = 1.99 \times 10^{30}$ kg, and $r = R_\odot = 6.96 \times 10^5$ km $= 6.96 \times 10^8$ m,

$$v_{\text{esc,present}} = \sqrt{\frac{2GM_\odot}{R_\odot}} = \sqrt{\frac{2(6.67 \times 10^{-11})(1.99 \times 10^{30})}{6.96 \times 10^8}} = \boxed{6.18 \times 10^5 \text{ m s}^{-1}}$$

(b) When the Sun becomes a red giant, given $M = 1M_\odot$ and $r = 100R_\odot$,

$$v_{\text{esc,rg}} = \sqrt{\frac{2GM_\odot}{100R_\odot}} = \frac{1}{10}\sqrt{\frac{2GM_\odot}{R_\odot}} = \frac{1}{10}v_{\text{esc,present}} = \boxed{6.18 \times 10^4 \text{ m s}^{-1}}$$

(c) The escape velocity of the Sun as a red giant is lower than that in the present day (the main sequence). Therefore the particle speed required is lower in order to escape from the red giant, i.e. mass can be lost more easily from a red giant star.