Homework 4 Solutions

Astronomy 1

Due October 25, 2019

1. (7.13)
   We are asked for the escape speed, which is given by:

   $$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

   Where G is the universal gravitational constant, M is the mass of the objects whose gravitational field we are trying to overcome, and R is the distance from to the objects center of mass. For part (a), we are trying to escape Europa's gravitational field. So the mass we want to use is the mass of Europa($M = 4.80 \times 10^{22}$ kg), and the radius we want to use is the radius of Europa ($R = 1,560,000$ m). Thus:

   $$v_{\text{escape}} = \sqrt{\frac{2(6.67 \times 10^{-11})(4.80 \times 10^{22})}{1,560,000}} = 2,030\frac{\text{m}}{\text{s}}$$

   For part (b) we use the same formula. However now we are trying to escape Jupiter's gravitational field. So the mass we need to consider is the mass of Jupiter ($M = 1.90 \times 10^27$ kg and the radius is the distance from Jupiter to Europa (given in the problem to be $R = 670,900,000$ m). Thus:

   $$v_{\text{escape}} = \sqrt{\frac{2(6.67 \times 10^{-11})(1.90 \times 10^{27})}{670,900,000}} = 19,400\frac{\text{m}}{\text{s}}$$

   In part (a) we found the velocity necessary to overcome the gravitational field of Europa. In part (b) we found the velocity necessary to overcome the gravitational field of Jupiter. In order to escape the entire system and return home, the spacecraft would need to overcome both the gravitational field of the Europa and Jupiter simultaneously. And in order to do this we would need a velocity greater than either (a) or (b).

2. (7.24)
   Newton's form of Kepler's third law relates the orbital period for two objects as:

   $$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}$$

   Where P is the period, M_1 and M_2 are the masses of the two objects and a is the semimajor axis. In this problem, we will assume that the mass of the sun is much larger than the trans-Neptunian object and therefore we can ignore the mass of the trans-Neptunian object. The mass of the sun is $1.99 \times 10^{30} kg$ and $1au = 1.5 \times 10^{11} m$

$$P = \sqrt{\frac{4\pi^2(100 \times 1.5 \times 10^{11})^3}{6.67 \times 10^{-11}(2 \times 10^{30})}} = 3.16 \times 10^{10} s = 1000 \text{ years}$$

The object moves 360 degrees in 1000 years, since the period is defined to be the time it takes the object to complete one full circle. Therefore to find how long it would take to move one arcminute we apply dimensional analysis

$$\frac{1000 \text{ years}}{360 \text{ degrees}} \times \frac{360 \text{ degrees}}{60 \times 360 \text{ arcminutes}} = \frac{1000 \text{ years}}{60 \times 360 \text{ arcminutes}} = .0463 \frac{\text{years}}{\text{arcminute}}$$

Recall that your pinky measures about one degree of the night sky. We just found that it takes .0463 years $\approx$ 17 days for the object to move one sixtieth of your pinky across the night sky. To notice any appreciable change we are going to have wait quite a long time to see the object move across the celestial sphere.
We are looking for very small changes in movement across the celestial sphere. Therefore we need to have large telescopes with very high angular resolution in order to measure the small changes.

3. (8.46)
We need a formula that relates the temperature to the wavelength. Wein's law for a blackbody states:

$$\lambda_{\max} = \frac{0.0029\text{Km}}{T}$$

Plugging in the temperatures

$$\lambda_{\max} = \frac{0.0029\text{Km}}{1130} = 2.57 \times 10^{-6}\text{m}$$

We use the same equation for part (b), only now the temerature is different

$$\lambda_{\max} = \frac{0.0029\text{Km}}{6030} = 4.81 \times 10^{-7}\text{m}$$

The planets maximum emission wavelength is in the infrared, so it would be better for the telescope to use infrared light.

4. (8.47)
The distance from the star to earth is 170 lightyears. Figure 8-16 tells us the distance from the star to the planet is 55au=8.70×10^{-4} lightyears. Since the distance from the star to earth is much greater than the distance from the star to the planet, we can use the small angle approximation.

$$D = \frac{\alpha \times d}{206265}$$

Thus

$$\alpha = \frac{206265 \times D}{d} = \frac{206265 \times 8.7 \times 10^{-4}}{170} = 1.06 \text{ arcseconds}$$

Using Newton's form of Kepler's third law and neglecting the mass of the planet (similar calculation/reasoning as problem 2)

$$P = \sqrt{\frac{4\pi^2(55 \times 1.5 \times 10^{11})^3}{6.67 \times 10^{-11}(.025 \times 2 \times 10^{30})}} = 8.15 \times 10^{10} s = 2588 \text{ years}$$

An astronomer would most certainly not observe a complete orbit in one lifetime!

5. (10.22)
(**Box10 − 1**) We are told that the tidal force is given by

$$F_{\text{tidal}} = \frac{2GM_{\text{Earth}}md}{r^3}$$

Where G is the universal gravitational constants, M_{Earth} is the mass of the earth, m is the mass of the object we are calculating the tidal force for, d is the separation distance of the two points we are comparing, and r is the distance from the Earth to the Moon. We take d to be the diameter of the Moon, 3.48×10^6 m and $r_{\text{perigree}} = 3.63 \times 10^8$ m

$$F_{\text{tidal,perigree}} = \frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times (1) \times (3.48 \times 10^6)}{(3.63 \times 10^8)^3} = 5.78 \times 10^{-5} \text{ N}$$

For part b, the only thing that changes is $r_{\text{apogee}} = 4.06 \times 10^8$ m

$$F_{\text{tidal,apogee}} = \frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times (1) \times (3.48 \times 10^6)}{(4.06 \times 10^8)^3} = 4.15 \times 10^{-5} \text{ N}$$

The ratio between the two forces is:

$$\frac{F_{\text{tidal,perigree}}}{F_{\text{tidal,apogee}}} = \frac{5.78 \times 10^{-5}}{4.15 \times 10^{-5}} = 1.39$$

6. (10.23)
(**Box10 − 1**) When the Moon originally coalesced, it may have been only one tenth as far from Earth as it is now.
Again, we are going to use:

$$F_{\text{tidal}} = \frac{2GM_{\text{Earth}}md}{r^3}$$

The current average distance from the Earth to the Moon is 3.84×10^8 m. Taking one tenth of this gives $r = 3.84 \times 10^7$ m.

$$F_{\text{tidal}} = \frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times (1) \times (3.476 \times 10^6)}{(3.84 \times 10^7)^3} = 4.89 \times 10^{-2} \text{ N}$$

In order to lift rocks off the lunar surface, this force would need to exceed the gravitational force exerted on the rock by the moon. This force is given by newtons formula for the gravitational force:

$$F_g = \frac{GM_{\text{Moon}}M_{\text{rock}}}{r^2}$$

Note that this r is different than the r above used to calculate the tidal force. The r in the gravitational force equation is the distance from the rock to the Moon's center of Mass, which should be the radius of the Moon.

$$F_g = \frac{(6.67 \times 10^{-11}) \times (7.35 \times 10^{22}) \times (1)}{(1.74 \times 10^6)^2} = 1.62 \text{ N}$$

Since the gravitational force is larger than the tidal force, the Earth's tidal force was not strong enough to lift rocks off the lunar surface.
The current tidal force was calculated in box 10-1 to be 4.88×10^{-5} N. Our answer found in part (a) is 1000 times greater than the current value.