Astronomy 1 fall 2009
Prof: R Antonucci Office: Broida 2015K

Solutions to Assignment 7

by Elijah L. Quetin
email questions to: elquetin@physics.ucsb.edu

1

(a) Hydrostatic equilibrium means that all of the forces on a parcel of matter inside the sun feel are precisely balanced, so that the star isn't expanding or contracting or twisting. See Figure 16-2 in the book for a diagram. The main idea is that the pressure from all the photos streaming out of the star's core has to balance the star's tendency to contract gravitationally. Since a pressure is a force over an area, an equation would look like $P=F_gA$, where P is the pressure, F_g is the force due to gravity, and A is the area of the parcel.
(b) The equation of state describes how this pressure relates density and temperature. The ideal gas law would suffice here: PV=NRT (P is pressure, V is volume, N is number of particles, R is another constant like G, σ, h, etc, and T is temperature. A more compact form or this equation was given to you in lecture $P{\sim}\rho T$, where ρ is the density, and ~ means "proportional to".
(c) The nuclear energy generation is what is supplying the radiation pressure, which keeps it from collapsing under its own gravity. Similarly, a higher density, temperature, or both mean a faster nuclear energy generation rate. Matter is converted to energy in the core via $E=mc^2$.

2

Since this star is much more massive than the Sun, we should expect to find a lifetime much shorter than that of the Sun (see following problems). Mass is converted to energy via $E=mc^2$. Since luminosity is the energy emitted per second, we can write

$$L = \frac{E}{sec} = \frac{mc^2}{sec} = (\frac{m}{sec})c^2$$

and solve for m/sec, which is the rate we're looking for

$$\frac{m}{sec} = \frac{L}{c^2}$$

Plugging in our luminosity gives us a rate of 4.33 x 10^{15}kg/sec. Similarly, we can find the rate at which H is converted to He by including the efficiency factor of 0.7% or 0.007.

$$L = \frac{0.007mc^2}{sec} = (\frac{m}{sec})0.007c^2$$

which leads to

$$\frac{m}{sec} = \frac{L}{0.007c^2}$$

and gives us a rate of 6.19 x 10^{17}kg/sec. Finally, we're given that 10% or 0.1 of the star's total mass of 60 x 2 x 10^{30}kg, or 1.22 x 10^{31}kg, is available to convert to He. Once all of this H is turned into He, the star leaves the main sequence. Thus, if we divide the total amount of fuel available by the rate at which the fuel is transformed into He, we should get the lifetime of the star:

$$t = \frac{1.22x10^{31}kg}{6.19x10^{17}kg/sec} = 1.93x10^{13}sec \approx 614000yrs$$

As expected, this is a much shorter lifetime than that of the Sun.

3 Ch. 19 #4

High-mass main sequence stars are much more luminous than their low-mass counterparts. A higher luminosity means that there is much more energy produced in the star's core. Energy is produced by thermonuclear reactions, converting matter into energy in the process. Even though a high-mass star has more matter (fuel), it uses it up much more quickly than low-mass stars. Notice how short is the lifetime of the 60 solar-mass star you investigated in problem 3. Think of the analogy of the Hummer and the Prius. The Hummer has a larger gas tank than the Prius, but it consumes fuel so quickly that a Prius would travel farther than a Hummer even though the Prius has a smaller gas tank.

Ch. 19 #9

Temperature can be viewed as a quantity representing the average motion of the particles in a substance. If the particles are zipping around very quickly, the temperature is high, and vice versa. The high temperatures required for fusion are due to the fact that nuclei have positive charge and repel each other. In order to smash nuclei together and fuse into heavier elements, the particles must be traveling very quickly to overcome the electric repulsion, hence the necessity of high temperature. Now consider the fact that a Helium nucleus contains two protons, whereas a Hydrogen nucleus contains only one proton. Two positively charged protons would repel two other positively charged protons more so than the one proton will repel another proton. Thus, a Helium gas must have a higher temperature to smash particles together than a Hydrogen gas, in order to overcome a stronger repulsion between nuclei.

Ch. 19 #10

Unlike an ordinary gas, the pressure of a degenerate gas does not depend on temperature.

4

Ch. 19 #13

To say that a star moves from one place to another on the H-R diagram has nothing to do with the actual physical motion of the star through space. Recall that an H-R diagram is a plot of Luminosity against Temperature. Thus as a star evolves it may become more or less luminous and thus move up or down on the H-R diagram. Similarly, if the temperature of the star changes, it's position on the H-R diagram will shift to the right or left.

Ch. 19 #16

Since the stars with higher mass have higher luminosities and shorter lifetimes, they will move off the main sequence faster than the lower-mass stars. This lets one read off the approximate age of the cluster as the age of the highest mass (and hence highest luminosity) stars left on the main sequence. See Fig. 19-10 in the text for a graphical representation of this.

Ch. 19 #17

Stars in spectral class M have the lowest masses of all the spectral classes, and thus the longest lifetimes—hundreds of billions of years! So the universe hasn't been around nearly long enough for them to move off the main sequence.

Ch. 19 #18

Astronomers can gauge the age of a cluster by looking at the turn-off point mentioned in the previous question. It has been found that there are no high-mass stars left on the main sequence in globular clusters. Thus the clusters must be very old.

5 Answer will vary