Quantum Lifshitz point and a multipolar cascade for frustrated ferromagnets

Leon Balents, KITP, UCSB

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Collaborators

Oleg Starykh
U. Utah

Teddy Parker
UCSB
What this talk is not

• (almost) nothing topological
• No gauge fields
• Nothing fractional
• No anyons. Not even fermions
• No CFT, no bootstrap
What this talk is not

• (almost) nothing topological
• No gauge fields
• Nothing fractional
• No anyons. Not even fermions
• No CFT, no bootstrap
• Not even a complete solution 😞
What it is about

• I will discuss the simplest example of a “frustrated ferromagnet”, and argue that there is a simple QFT description of such systems, with surprisingly rich phenomenology

• It is clear that this description extends to higher dimensions and perhaps the phenomenology does as well
Outline

• Introduction and phenomena:
  • a QCP, and multipolar phases
  • QFT: what we need
  • Lifshitz NLsM
  • Limits and analysis
Frustrated ferromagnet

1d $S=1/2$ chain

$J_1 < 0$ FM

$J_2 > 0$ AF

$H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z$

<table>
<thead>
<tr>
<th>Compound</th>
<th>$J_1$, $J_2$ (K)</th>
<th>$\angle$ Cu-O-Cu (deg)</th>
<th>$T_N$ (K)</th>
<th>$H_s$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li$_2$ZrCuO$_4$[12, 13]</td>
<td>$-151$, $35$</td>
<td>94.1</td>
<td>6.4</td>
<td>-</td>
</tr>
<tr>
<td>Rb$_2$Cu$_3$Mo$<em>3$O$</em>{12}$[14, 15]</td>
<td>$-138$, $51$</td>
<td>89.9, 101.8</td>
<td>&lt; 2</td>
<td>14</td>
</tr>
<tr>
<td>PbCuSO$_4$(OH)$_2$[16–18]</td>
<td>$-100$, $36$</td>
<td>91.2, 94.3</td>
<td>2.8</td>
<td>5.4</td>
</tr>
<tr>
<td>LiCuSbO$_4$[19]</td>
<td>$-75$, $34$</td>
<td>89.8, 95.0</td>
<td>&lt; 0.1</td>
<td>12</td>
</tr>
<tr>
<td>LiCu$_2$O$_2$[20–22]</td>
<td>$-69$, $43$</td>
<td>92.2, 92.5</td>
<td>22.3</td>
<td>110</td>
</tr>
<tr>
<td>LiCuVO$_4$[23–31]</td>
<td>$-19$, $44$</td>
<td>95.0</td>
<td>2.1</td>
<td>44.4</td>
</tr>
<tr>
<td>NaCuMoO$_4$(OH)</td>
<td>$-51$, $36$</td>
<td>92.0, 103.6</td>
<td>0.59</td>
<td>26</td>
</tr>
</tbody>
</table>

K. Nawa et al, arXiv:1409.1310
Frustrated ferromagnet

1d $S=1/2$ chain

$J_1<0$ FM

$H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z$

$J_2/(|J_1|+J_2)$

M

FM

PM

gap

$\text{gap}$
Frustrated ferromagnet

1d $S=1/2$ chain

$J_1<0$ FM

$J_2>0$ AF

$H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z$

$\frac{J_2}{(|J_1|+J_2)}$

$M$

0 1/5 1

FM PM gap
Frustrated ferromagnet

1d $S=1/2$ chain

$J_2>0$ AF

$J_1<0$ FM

\[ H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z \]

$J_2/(|J_1|+J_2)$

FM

PM

gap

very weakly dimerized
Multipolar phases

\[ \frac{H}{|J_1| + J_2} \]

\[ \frac{1}{5} \]

FM

VC

Hikihara et al., 2008

Sudan et al., 2009
Multipolar phases

\[ J_2/(|J_1|+J_2) \]

0 \[\rightarrow\] 1

\[ H/(|J_1|+J_2) \]

Hikihara et al, 2008
Sudan et al, 2009

FM

VC

1/5

2

4

3
Magnon BEC

1-magnon

$E - E_{FM} = \varepsilon_1 + h$

E

$S^z = -1$

Magnon (quasi-)BEC

$\langle S_i^- \rangle \sim \Psi e^{i q x_i}$
1-magnon

\[
E - E_{FM} = \varepsilon_1 + h
\]

T. Radu et al, 2007
Magnon BEC

$E - E_{FM} = \varepsilon_1 + h$

T. Giamarchi et al, 2008
Magnon binding

For $d>1$ at $T=0$ this is a molecular BEC = true spin nematic
Hidden order

No dipolar order

\[ \langle S_i^z \rangle - M = 0 \]
\[ \langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi} \]
\[ \langle S_i^{\pm} \rangle = 0 \]
\[ \langle S_i^+ S_{i+a}^+ \rangle \neq 0 \]

Nematic order

\[ S^z = 1 \text{ gap} \]

Magnetic quadrupole moment

Symmetry breaking \( U(1) \rightarrow Z_2 \)

can think of a fluctuating fan state
Multipolar phases

H/(|J_1|+J_2)

0 1/5

1 J_2/(|J_1|+J_2)

A progression of higher and higher multipolar phases on approaching the QCP!
Multipolar phases

Is there a QFT that describes this region?

Hikihara et al., 2008
Sudan et al., 2009
A QFT?

- Is this behavior generic?
- Is the cascade infinite, or does it terminate?
- Can a single QFT describe an infinite number of order parameters?
- Is this specific to one dimension?

$$\Psi_n \sim \langle (S^-)^n \rangle$$
A QFT?

- A strong constraint:
  - Entire green area including the QCP itself has exact trivial FM ground state
- Not a CFT

\[ \Psi_n \sim \langle (S^-)^n \rangle \]
Lifshitz Point

- Effective action - NLσM

\[ S = \int dx d\tau \left\{ i \sigma A_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

WZW/Berry phase term

\[ A_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}. \]

tunes QCP

two symmetry allowed interactions at \( O(q^4) \)

All properties near Lifshitz point obey “one parameter universality” dependent upon \( u/K \) ratio
Lifshitz Point

\[ S = \int dxd\tau \{ isA_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h\hat{m}_z \} \]

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

\[ x \rightarrow \sqrt{\frac{K}{|\delta|}} x \quad \tau \rightarrow \frac{K}{\delta^2} \tau \]

\[ S = \sqrt{\frac{K}{\delta}} \int dxd\tau \{ isA_B[\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \} \]

Large parameter: saddle point!

\[ v = \frac{u}{K} \quad \overline{h} = \frac{hK}{\delta^2} \]
Lifshitz point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ i s A_B[\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

\( v \) derives from quantum fluctuations

Need it be positive?

\[ \hat{m} \cdot \hat{m} = 1 \quad \Rightarrow \quad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}| \]

Theory is stable for \( v > -1 \)

In fact, \( v < 0 \)

- Semiclassical large \( s \) limit: \( v \sim -3/2s \)
- \( s = 1/2 \) exact 2-magnon calculation \( v = -5/8 \)
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ i s A_B [\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

\[ \hat{m} = \begin{pmatrix} |\Psi| \cos(qx + \phi) \\ \pm |\Psi| \sin(qx + \phi) \\ \sqrt{1 - |\Psi|^2} \end{pmatrix} \]

-1 < v < -\frac{1}{4}

\[ h_c = \frac{\delta^2}{8K \sqrt{|v|(1 - \sqrt{|v|})}} > \frac{\delta^2}{2K} \]
Multipolar phases

H/(|J_1|+J_2)

J_2/(|J_1|+J_2)

FM

3

VC

Sudan et al., 2009

“metamagnetism”
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ isA_B[\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \bar{h}\hat{m}_z \right\} \]

N.B.: at saddle point level there is no scale for \( \delta \)
Beyond saddle point

• Issues:
  • Fate of ordered saddle points?
  • Endpoint of metamagnetic line?
  • Multipolar orders?
Zero field

• Saddle point is a spiral phase

\[ \hat{m}(x) = \hat{e}_1 \cos qx + \hat{e}_2 \sin qx \]

\((\hat{e}_1, \hat{e}_2, \hat{e}_3 = \hat{e}_1 \times \hat{e}_2)\) form an SO(3) matrix

• Fluctuations are described by an SO(3) NLsM

\[ S_{\text{eff}} = \frac{1}{g} \int d^2x \, \text{Tr} \left[ (\partial_\mu O)^2 \right] + iS_{\text{topo}} \]
Zero field

\[ S_{\text{eff}} = \frac{1}{g} \int d^2 x \ Tr \ [(\partial_\mu O)^2] + i S_{\text{ topo}} \]

- NLsM is asymptotically free

\[ \Pi_1(SO(3)) = Z_2 \quad "Z_2 \ vortex\" \ \text{instanton} \]

\[ S_{\text{ topo}} \quad \text{carries phase factor } (-1)^x \]

dimerization
Multipolar phases

\[
\frac{H}{|J_1| + J_2} = 0
\]

\[
\frac{|J_1| + J_2}{|J_2|/(|J_1| + J_2)}
\]

\[
dimerized
\]
Multipolar phases

\[ H/(|J_1|+J_2) \]

\[ J_2/(|J_1|+J_2) \]

dimerized

FM

2

3

VC??

\[ \hat{m} = \begin{pmatrix} |\Psi| \cos(qx + \phi) \\ \pm |\Psi| \sin(qx + \phi) \\ \sqrt{1 - |\Psi|^2} \end{pmatrix} \]

\[ S_{\text{eff}} = \frac{1}{g} \int d^2 x (\partial_\mu \phi)^2 \]

\[ c=1 \]

broken "TR" symmetry

\[ \hat{z} \cdot \langle S_i \times S_{i+1} \rangle \neq 0 \]
Metamagnetic endpoint?

\[ \frac{h}{K} \]

\[ \varepsilon_{FM} = \varepsilon_{cone} \]

\[ \epsilon_1 = 0 \]

\[ -1 < v < -\frac{1}{4} \]
Metamagnetic endpoint?

Quantum corrections penalize $E_{\text{cone}}$ but not $E_{\text{FM}}$
Metamagnetic endpoint?

Quantum corrections penalize \( E_{\text{cone}} \) but not \( E_{\text{FM}} \)

\[
\Delta E_{\text{cone}} = +f(v)\delta^{5/2}
\]
Metamagnetic endpoint?

\[ S = \int dx d\tau \{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \} \]

\[ \hat{m} = \sqrt{2 - n_1^2 - n_2^2 (n_1 \hat{e}_1(x) + n_2 \hat{e}_2(x)) + (1 - n_1^2 - n_2^2) \hat{e}_3(x)} \]

\[ \hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}_{sp}(x) \]

\[ \eta = n_1 + in_2 \quad \bar{\eta} = n_1 - in_2 \]

\[ S = S_{sp} + \int dx d\tau \{ \bar{\eta} \partial_{\tau} \eta + H(\bar{\eta}, \eta) \} + O(\eta^3) \]

Bogoliubov transformation gives correction to GS energy
Metamagnetic endpoint?

Corrected first order curve bends slightly downward to intersect second order line
Metamagnetic endpoint?

\[ \frac{h}{K} \]

\[ \mathcal{E}_{FM} = \mathcal{E}_{cone} \]

\[ \epsilon_1 = 0 \]

\[ \mathcal{E}_{FM} - \mathcal{E}_{cone} \sim a\delta^2 - f(v)\delta^{5/2} \]

Control?

\[ v = -1/4 - \varepsilon \]

\[ \mathcal{E}_{FM} - \mathcal{E}_{cone} \bigg|_{\epsilon_1=0} \sim \varepsilon^3 \delta^2 - \varepsilon^2 \delta^{5/2} \]

\[ \delta_c \sim \varepsilon^2 \ll 1 \]
Still need to understand multipolar phases!
Instabilities

- Choose $E_{FM} = 0$

What about multi-particle instabilities?
Low density limit

\[ \hat{m}^x + i\hat{m}^y = (2 - \overline{\psi}\psi)^{1/2} \psi \]

\[ \hat{m}^z = 1 - \overline{\psi}\psi \]

Low energy

\[ \psi \sim \psi_1 e^{iqx} + \psi_2 e^{-iqx} \]

\[ \mathcal{L} \sim \overline{\psi}_a (\partial_\tau + h - \frac{\delta^2}{2K} - 4\delta \partial_x^2)\psi_a \]

\[ + \gamma_1 [ (\overline{\psi}_1 \psi_1)^2 + (\overline{\psi}_2 \psi_2)^2 ] + \gamma_2 \overline{\psi}_1 \psi_1 \overline{\psi}_2 \psi_2 \]

\[ \gamma_1 = \frac{\delta^2}{4K} (1 + 4\nu) \]

\[ \sim -\varepsilon \delta^2 < 0 \]

\[ \gamma_2 = \frac{\delta^2}{K} (5 + 4\nu) \]

\[ \sim + \delta^2 \]
Low density limit

\[ H = -4\delta \sum_i \frac{\partial^2}{\partial x_i^2} + 2\gamma_1 \sum_{i<j} \delta(x_i - x_j) \]

\[ \gamma_1 \sim -\varepsilon \delta^2 < 0 \]

attractive delta-function gas!

\[ \epsilon_n = \epsilon_b \frac{n(n^2 - 1)}{6} \quad \epsilon_b = -\frac{\gamma_1^2}{8\delta} = -\frac{\varepsilon^2 \delta^3}{8} \]

collapse: bound states have size

\[ \ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon \delta} \]
At low density level it appears higher bound state instabilities dominate.
Instabilities

- Choose $E_{FM}=0$

But the bound states cannot get arbitrarily deep - low density approximation is violated.
A guess

• Scaling

\[ \epsilon_n \sim -\epsilon^2 \delta^3 n^3 \mathcal{F}(n\delta^{1/2}, \frac{\delta^{1/2}}{\epsilon}) \]

• Matching?

\[ n\delta^{1/2} \gg 1 \quad \mathcal{F}(X, Y) \sim 1/X^2 f(Y) \]

• Suggests maximum bound state

\[ n_{\text{max}} \sim \delta^{-1/2} \sim 1/\epsilon \]

(at this scale, 3-body interactions enter)
Instabilities

- Choose $E_{FM}=0$

Expect that $n$-boson bound states bend with increasing $n$ to approach continuum line
Instabilities

• Choose $E_{FM}=0$

Expect that $n$-boson bound states bend with increasing $n$ to approach continuum line.
Summary

Lifshitz point is a "parent" of many phases

\[ S = \int dx d\tau \left\{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]
Other frustrated ferromagnets

• In 1+1d, we could figure out (nearly) everything by numerically exact methods (DMRG)

• But in d>1, we have fewer tools but plenty of experiments
Eg. a frustrated ferrimagnet

volborthite

\[ S = \int dxd^{d-1}y d\tau \left\{ isA_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + c |\partial_y \hat{m}|^2 + K |\partial^2_x \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

same saddle point analysis applies...