Phase diagram of the S=1/2 checkerboard antiferromagnet

Oleg Starykh, University of Utah
Akira Furusaki, RIKEN
Leon Balents, UC Santa Barbara

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“Two-dimensional pyrochlore”: checkerboard antiferromagnet

Corner-sharing geometry
but
non-equivalent
$J$, $J_x$ bonds

Large-N: Bernier et al
Moessner et al;
Exact diagonalization:
Fouet et al. 2001;
Large-S: Tchernyshyov,
OS,Moessner,Abanov 2003

Singh,OS,Freitas 1998
Canals 2002

Chains
$J_2 \gg J_1$
(classically
decoupled)

Plaquette phase

$J_x/J$

$0$
$1$
$4/3$

$\uparrow$

$\downarrow$

$\bigcirc = 4$-spin singlet

$\rightarrow$
Plaquette VBS: $J_x \sim J$

- $p_1$ and $p_2$ labels the plaquettes.

$\bigcirc = 4$-spin singlet

$S=1$ triplet excitations

$$E(\bar{p}) = \sqrt{J_x [J_x + \frac{4}{3} (J - J_x) (\cos p_1 + \cos p_2)]}$$

- $J_x/J = 8/5$ triplet condensation at $P_{Neel}=0$: Neel state (but with bond modulation)

- $J_x/J = 8/11$ triplet condensation at $P_{Neel^*}=(\pi,\pi)$: Neel$^*$ state (ordered)

Lauchli et al. 2002; Tchernyshyov et al. 2003; Brenig et al. 2004
Key variables of a single Heisenberg chain ($J_x = 0$)

- Quantum “triad”: uniform magnetization $\mathbf{M} = \mathbf{M}_R + \mathbf{M}_L$, staggered magnetization $\mathbf{N}$ and staggered dimerization $\mathbf{\varepsilon}$ form closed OPE algebra

  \[ \tilde{S}_{n,m} / a = \tilde{M}_m(x) + (-1)^n \tilde{N}_m(x) \]

  \[ \tilde{S}_{n,m} \cdot \tilde{S}_{n+1,m} \rightarrow \mathbf{\varepsilon}_m(x) \]

- Operator product expansion $z = \nu \tau - i\chi$ (similar to commutation relations)

\[
\begin{align*}
&M_R^a(x, \tau)M_R^b(0, 0) = \frac{\delta^{ab}}{8\pi^2 z^2} + \frac{i\varepsilon^{abc}M_R^c(x, \tau)}{2\pi z} \\
&M_R^a(x, \tau)N^b(0, 0) = -i\delta^{ab}\varepsilon(x, \tau) + i\varepsilon^{abc}N^c(x, \tau) \\
&M_R^a(x, \tau)\varepsilon(0, 0) = \frac{iN^a(x, \tau)}{4\pi z}
\end{align*}
\]

- Scaling dimension 1/2 (relevant)  \[ \langle N^a(x, \tau)N^a(0, 0) \rangle \sim \frac{1}{\sqrt{\nu^2 \tau^2 + x^2}} \sim \langle \varepsilon(x, \tau)\varepsilon(0, 0) \rangle \]

- Scaling dimension 1 (marginal)  \[ \langle M_R^a(x, \tau)M_R^a(0, 0) \rangle \sim \frac{1}{(\nu \tau - i\chi)^2} \]
Inter-chain RPA \((J_x \ll J)\) - spin liquid? (OS, Singh, Levine 2002)

- Random phase approximation predicts 1d critical phase - sliding SU(2) Luttinger liquid

\[
\chi_{\text{RPA}}(\vec{K}) = \frac{\chi_1(k_x) + \chi_1(k_y) - 2J_1(\vec{K})\chi_1(k_x)\chi_1(k_y)}{1 - J_1^2(\vec{K})\chi_1(k_x)\chi_1(k_y)}
\]

\(T=0: \chi_1(k) \sim \frac{\sqrt{\ln(1/k)}}{k}\) but \(J_1(\vec{k}) = 2J_x \sin(k_x/2) \sin(k_y/2)|_{\vec{k} \to 0} = 0 \Rightarrow\) No instability!

\(K \text{ measured from } \pi\)
\(J_x = J, T = 0.01J, \omega = 0.1J\)

- Local current-current interaction is irrelevant

\(H_{\text{inter}} = J_x (S_1 + S_2) \cdot (S_3 + S_4) \Rightarrow J_x \mathbf{M}_H \cdot \mathbf{M}_V\)

Scaling dimension\(=2 > 1\)
Valence Bond Solid! (OS,Furusaki,Balents 2005)

• Keep irrelevant terms (as they help to generate symmetry-allowed relevant ones!)

\[ H_{\text{inter}} = (J_x a^2) \sum_{n,m} [2\tilde{M}_{h,m} + (-1)^n a \partial_x \tilde{N}_{h,m}] \cdot [2\tilde{M}_{v,n} + (-1)^m a \partial_y \tilde{N}_{v,n}] \]

• Expand in \( J_x \) and apply OPE

\[ [M_R^a(x, \tau) + M_L^a(x, \tau)] \partial_x N^b(x, 0) \sim \frac{-\epsilon(x, 0) \delta^{ab}}{\tau + \alpha \text{ sign}(\tau))^2} \]

• RG generates interaction of staggered dimerizations on crossing chains

\[
\delta H_{\text{inter}} = -g_\epsilon \sum_{x=na, y=ma} (-1)^{n+m} \epsilon_{h,m}(x) \epsilon_{v,n}(y), \quad g_\epsilon = \left( \frac{3J_x^2 a^2}{\pi^2 v^2} \right)(va)
\]

(respects all lattice symmetries)

• Chain mean-field: staggered dimer order on parallel chains

\[ \epsilon_h(x, m) = (-1)^m \langle \epsilon \rangle, \quad \epsilon_v(n, y) = (-1)^n \langle \epsilon \rangle \]

horizontal \quad vertical

m+1

m

n

n+1
Crossed-dimer VBS phase

- $J_x/J \ll 1$: $\langle \varepsilon \rangle = 0.08 J_x/J$  
  Spin gap $\Delta = 0.68 (J_x/J)^2$

Large-N (Bernier et al. PRB 69, 214427 (2004))

Exact diagonalization  
Sindzingre et al. 2002
Global phase diagram

- Scenario 1

- Scenario 2

$? = \text{First-order transition or co-existence phase}$
Extension to three dimensions


\[ J_x/J = 0.16 \]

but seems to order

\[ D = 3 \text{ crossed-dimer phase for } J_x \ll J \]
Conclusions

• Novel VBS phase: crossed-dimer phase in the quasi-1d limit of the checkerboard antiferromagnet ($J_x/J \ll 1$)
  
  Follows from consistent implementation of the renormalization group
  
  Clarifies previous large-N results on the “decoupled chains” phase(s)

• Global phase diagram: most of the quantum transitions are of 1st order

• Possible magnetically-ordered phase (Neel*) between the crossed-dimer and plaquette VBS phases

• Three-dimensional extension: crossed-dimer instability in GeCu$_2$O$_4$?
  
  Neel* in 3d materials - GeCu$_2$O$_4$ and ZnV$_2$O$_4$?