**Motivation**

- We are interested in resonators that allow for higher power readout. Higher readout powers may improve the signal to noise ratio by saturating two level systems.

- The base design is simple and avoids detailed lithography.

- Disks are well suited for detectors where the event energy spot size is large (e.g. phonon mediated membrane suspended devices).

- Disk resonators are analytically solvable as microwave circuit elements.

- Inductively coupled
  - Disk does not share feedline ground

- Capacitively coupled
  - Disk shares feedline ground

**Design Equations**

A disk resonator’s resonant frequency depends on its size, disk separation and its surface inductance. High surface inductance materials and small disk separations are needed to bring the resonant frequency into a usable range. Resonator footprints of 500 x 500 µm to 1000 x 1000 µm can be realized in the 1 – 10 GHz range.

Resonant frequencies are related to Bessel function zeros. \( j_{1,1} \) is the first zero of the first Bessel function.

\[
L_{s,1} = \frac{\mu h}{j_{1,1} + \mu h}
\]

Resonant Frequency

\[
f_r = \frac{c \sqrt{\mu h}}{2\pi \sqrt{1 + \frac{L_{s,1} + L_{s,2}}{\mu h}}}
\]

Lumped Element Approximation

\[
C = \frac{\varepsilon \pi r^2}{h}
\]

Coupling Q

\[
Q_c = \frac{2}{Z_0} \frac{L(C + C_c)}{\mu h}
\]

**Signal To TLS Noise**

\[
\text{SNR} \propto \frac{\epsilon}{\alpha_{k1} \left( \frac{1}{2} \sqrt{QQ_1} + \frac{1}{2} \right) + \delta E}{P_{\text{in}} V_{\text{ref}}}
\]

**Kinetic Inductance Fraction**

\[
\alpha_{k1} = \frac{1}{1 + \frac{\mu h}{L_{s,1} + L_{s,2}}}
\]

**Simulations**

- **Solid Disk**
  - Sonnet® E&M Simulation of TM02 resonance mode
  - \( f_r = 4.13 \text{ GHz} \)
  - \( r = 0.5 \text{ mm} \)

- **Meshed Disk**
  - Resonance still exists after meshing
  - Reduces resonance frequency by up to ~17%
  - Reduces metal volume
  - Avoids vortex losses from magnetic flux penetration

**Challenges and Ongoing Development**

- Meshing is being explored as a way to reduce the resonator volume and vortex penetration.

- ~10 nm thick dielectrics are needed to reduce the resonator size. We’ve currently reached internal quality factors of ~40,000.

- Nearby disk modes could interfere with reading out large numbers of resonators on the same feedline. Slits in the disk may suppress some of these modes.

- See Grégoire Coiffard’s talk (Thursday 12:45 O-65) for another example of a parallel plate resonator geometry that we are exploring.