Uncertainty, Cricket and the 2nd Law of Thermodynamics

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Deductive Reasoning

premises ----------> conclusions

rules of logic

Example: mathematical proofs.

- Conclusions are certain if you accept the premises
- Rare for this to be possible in science:

premises + data ----------> conclusions

rules of logic

Plausible Reasoning

- In science and many other fields of inquiry, certainty is rare.
- Think in terms of “degrees of plausibility”
- Can make generalised rules of logic to handle these
- It's just probability! (Cox 1946, Jaynes 2003)

How it works

(a probability for each possible hypothesis) + (observed data)

---------->

rules of probability

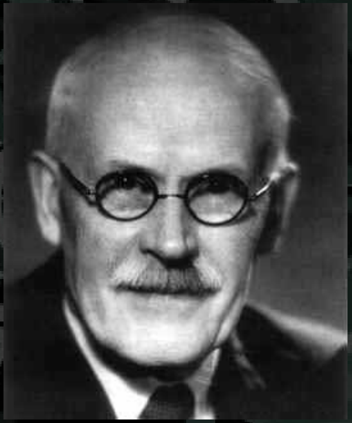
(an updated probability for each possible hypothesis)

“Bayesian”

- If you see this word, here's what it means.
- In my application, probability is to be interpreted as degree of plausibility.
- Probabilities describe **states of mind** not **states of the world**.
- Mix of subjective and objective elements. Reasonable people can disagree about the probabilities, but usually this disagreement decreases with more data.

Physicists > Statisticians

Laplace, looking pretty spunky



Harold Jeffreys. Well he was smart.

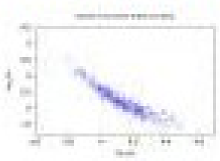
Motivation for Students Taking Stats

P-values
Power
Confidence Intervals

Probability can be applied to...

Facebook Quiz

Brendon Brewer I just made this quiz. Hope you like it. :)



Brendon created a new quiz - What's your statistical philosophy?!

Are you a frequentist, a Bayesian, or someone that mixes and matches or uses whatever someone taught you?

May 6 at 11:45pm · Comment · Like · Take this Quiz! · See more Quizzes

How it Works

- Rate the plausibility of each hypothesis you want to test. Assign *prior probabilities* accordingly. Prob(H) for each H
- For each hypothesis, describe your predictions for observed data. Prob(D|H) for all D and H
- Observe particular data D*

Bayes' Rule

- New plausibilities are the *posterior probabilities*: $P(H|D^*) \propto P(H)P(D^*|H)$

What were the odds of *that*?

- Observe particular data D*

Bayes' Rule

- New plausibilities are the *posterior probabilities*: $P(H|D^*) \propto P(H)P(D^*|H)$

- Probability for each hypothesis gets scaled according to how well each one predicted the data that we actually got.
- $P(D^*|H)$ as a function of H is called the *likelihood function*.

Early Days: the Case of Mike Hussey

- Great start to test career: 2188 runs, dismissed 28 times, average = 78.14
- Bradman averaged 99.94, next best over a long career is ~60
- Hussey looked like the 2nd best batsman ever, but it was "early days"
- How confident should our conclusions be?

A Model

- Consider a sequence of scores $\{x_1, x_2, ..., x_N, ...\}$
- Each x can be any non-negative integer
- If all we knew was that the mean of all of the x's was μ, we would assign independent geometric (discrete exponential) distributions for each x.
- $p(\{x_1, x_2, ..., x_N\} \mid \mu)$
- Can invert using Bayes's theorem. Get posterior probability distribution for μ given the scores.
- $p(\mu \mid \{x_1, x_2, ..., x_N\}) \propto p(\mu)\, p(\{x_1, x_2, ..., x_N\} \mid \mu)$

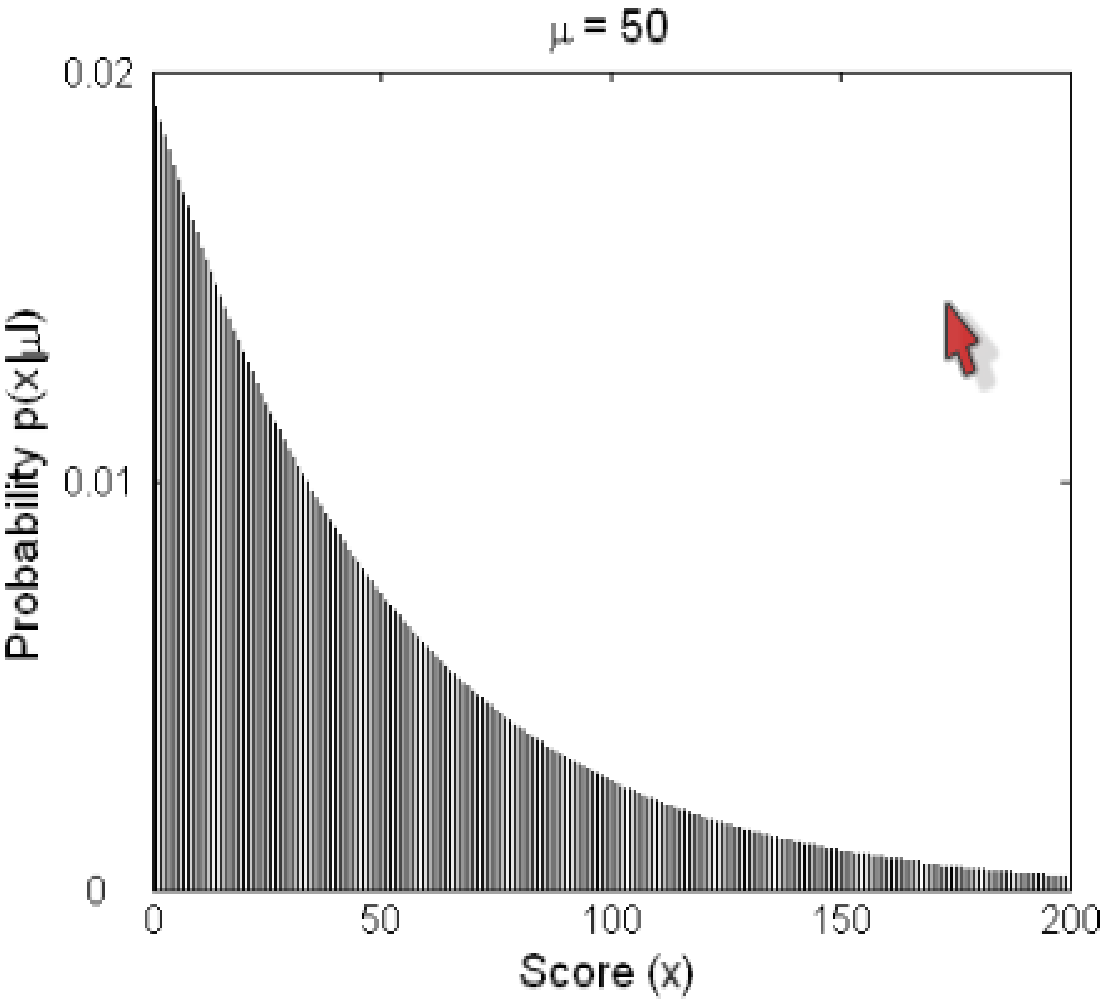


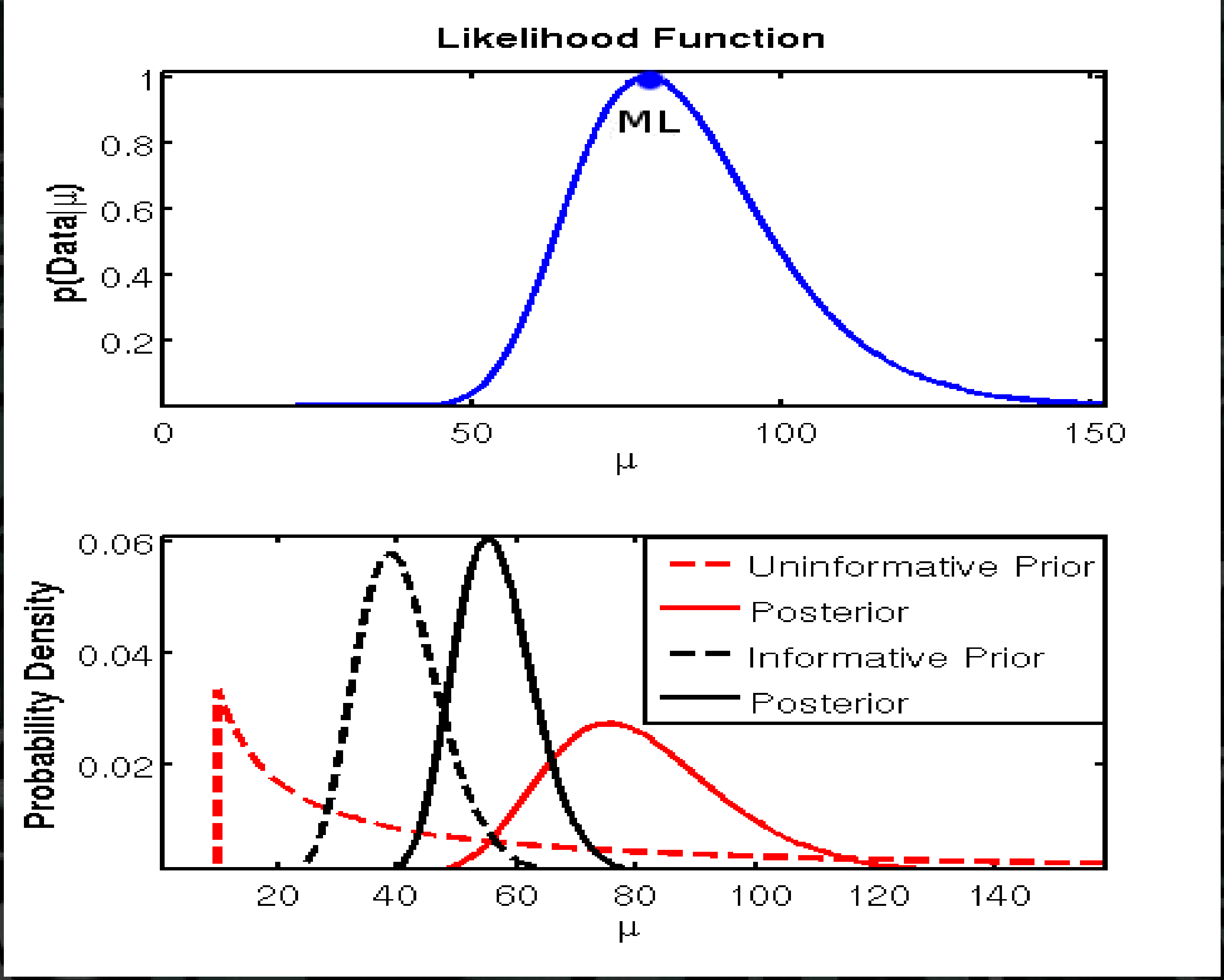
Figure 1 – Distribution of the score in a single innings, for an excellent batsman with a “true average” μ=50.

Two Prior Distributions

- ***Noninformative:*** How long is a piece of string? Twice the length from the middle to one end.
- ==> Jeffreys' Scale Invariant Prior (uniform distribution for log(μ)). I added cutoff at μ=10.

- ***Informative*:**
- log(μ) ~ Normal(Mean log(40), Sd 0.175)

78.14?
No, only 56.5 ± 6.8

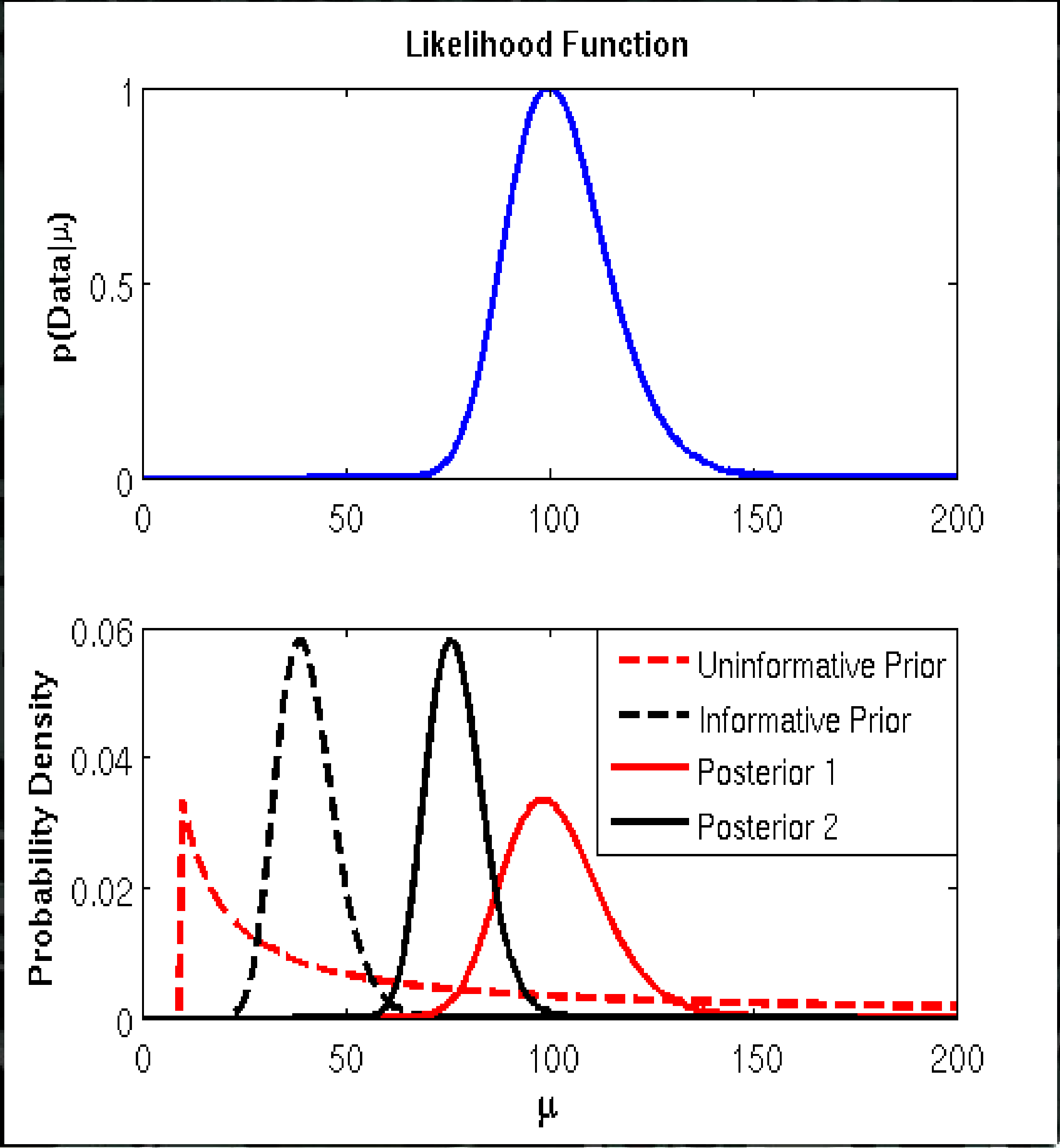


His actual test average is now 55.29

The Same for Bradman

77.0 ± 7.0

Heresy! But dependent on tails of prior. That's the way it is, get over it

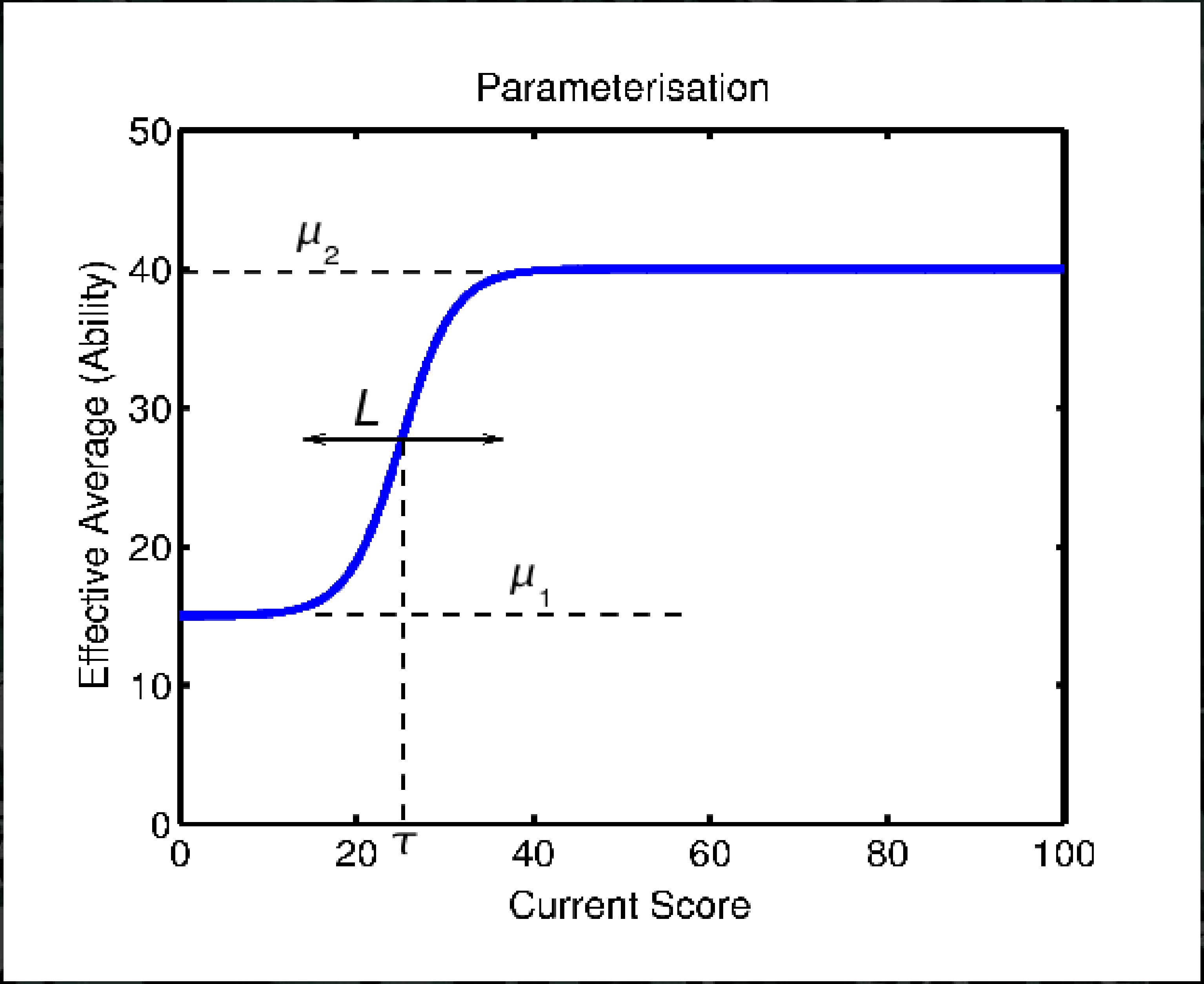


How to be the best

- Be really really good
- Be lucky as well
- The measured best player is almost certainly the one where these two effects coincided.
- This is why 99.94 is almost certainly an overestimate of Bradman's ability.
- His first-class average, excluding tests = 93.65 < 99.94

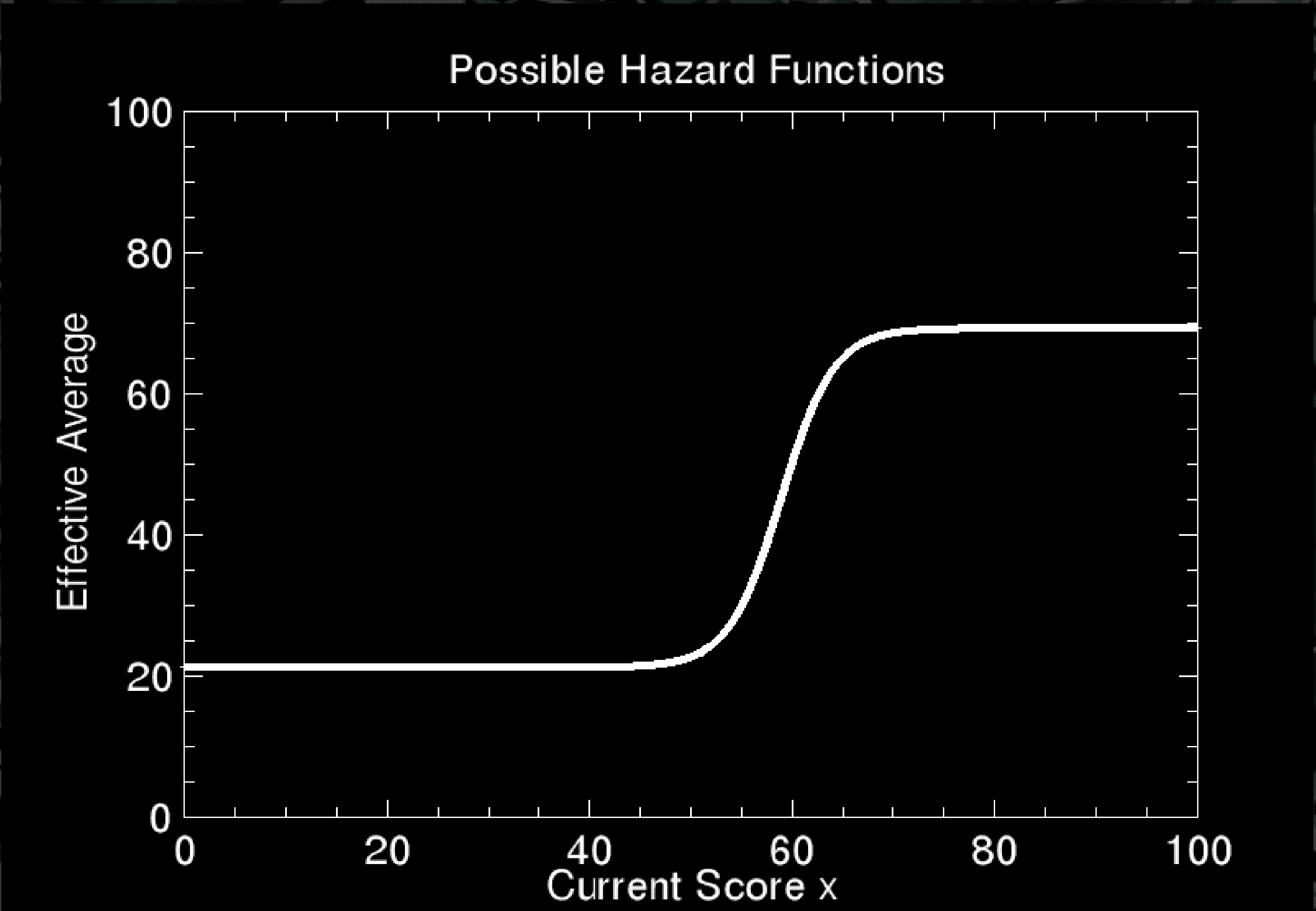
Getting Your Eye In

- Geometric distribution => constant probability per run of getting out
- We have more information than this
- Hazard function: $H(x) = P(X = x \mid X \geq x)$
- Infer "effective average" ≈ 1/H as a function of time.



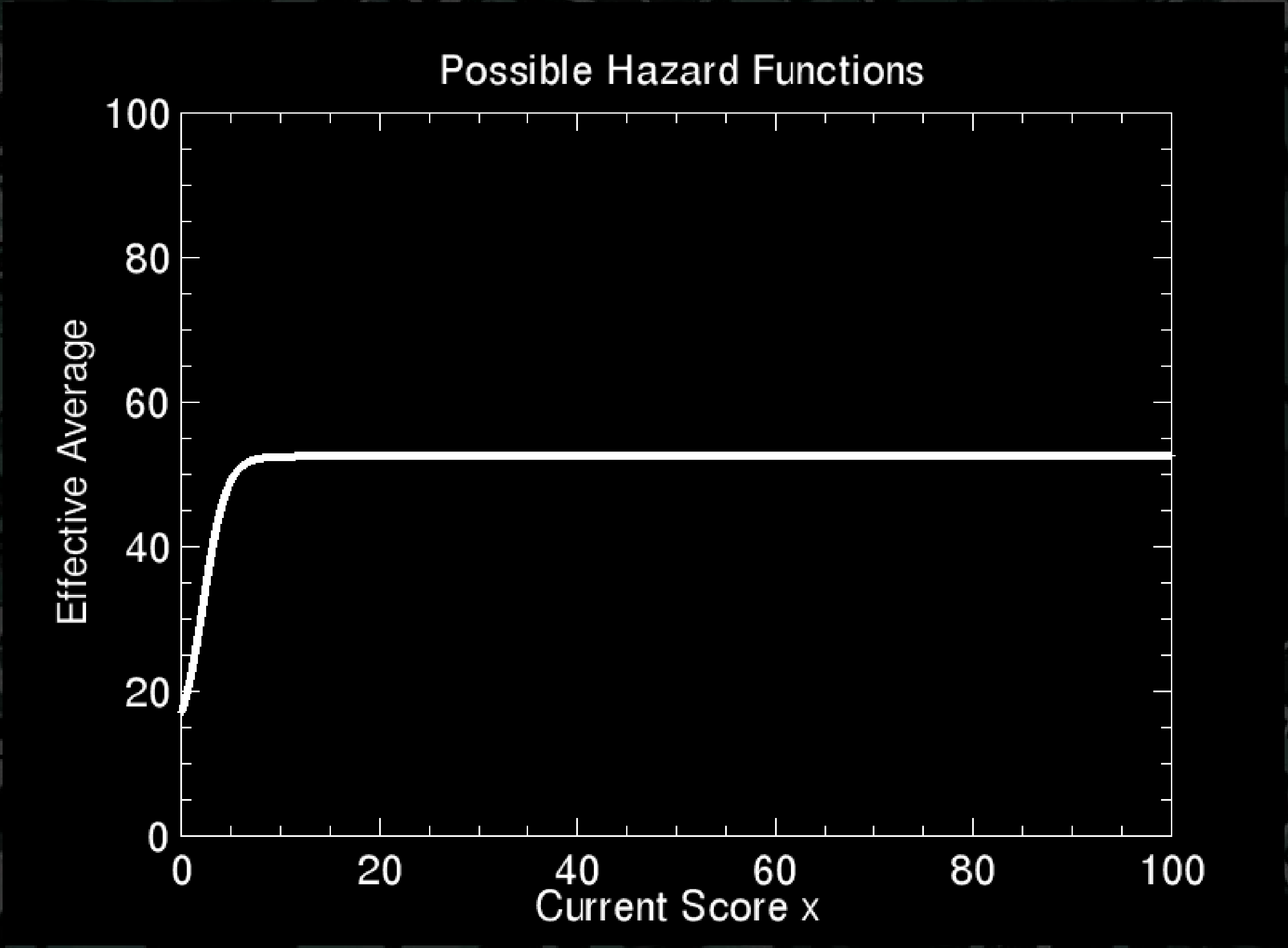
Parameterisation
Effective Average (Ability)
Current Score
μ2
μ1
L
τ
0
10
20
30
40
50
0
20
40
60
80
100

Prior



Possible Hazard Functions
Effective Average
Current Score x
0
20
40
60
80
100
0
20
40
60
80
100

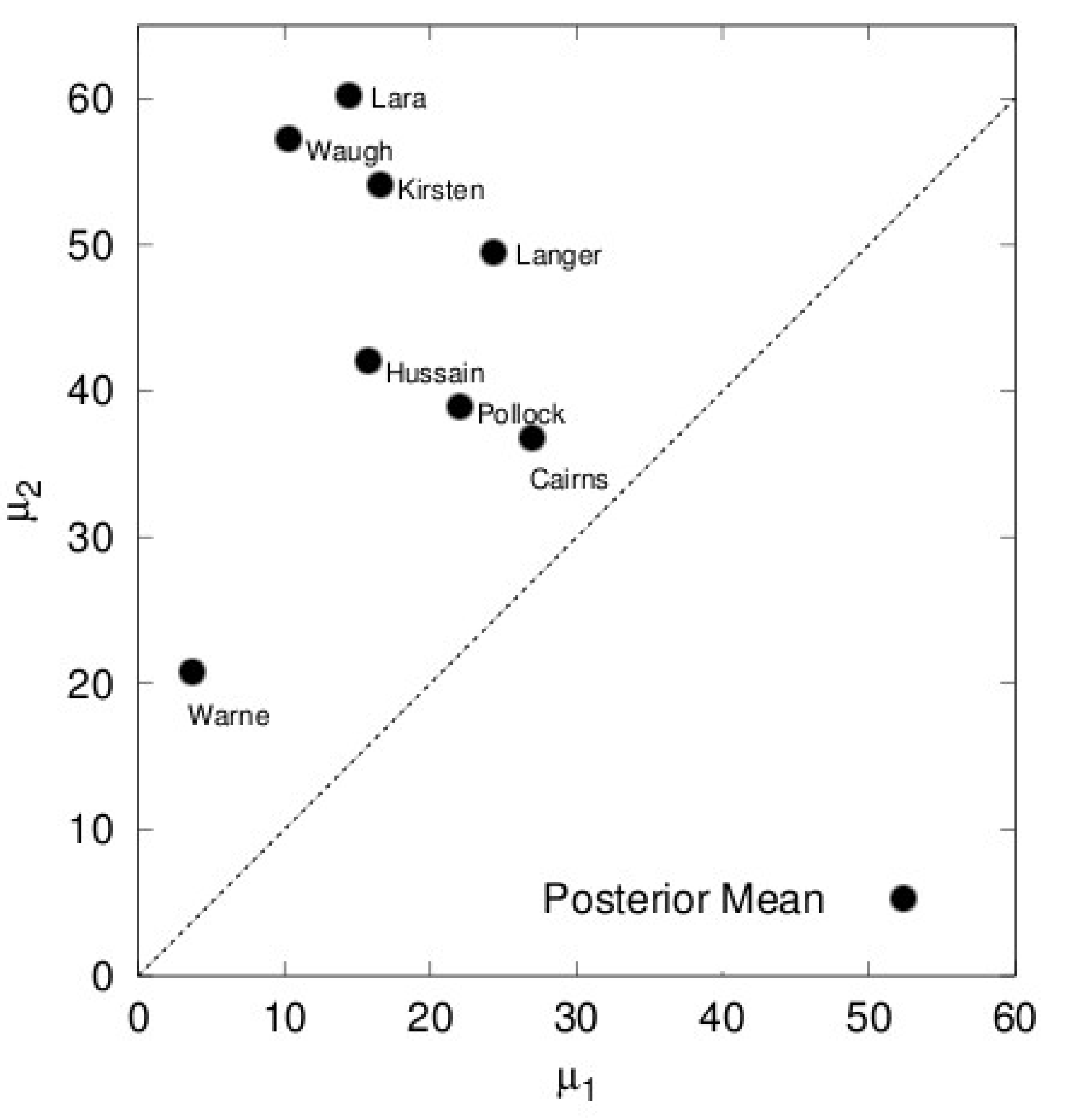
Posterior for Steve Waugh



I know you're not meant to do this in talks...

Table 1: Summaries for the sample of players.

Player	μ_1	μ_2	τ	L
Cairns	26.9 ± 9.2	36.7 ± 5.5	14.5 ± 17.7	3.1 ± 3.0
Hussain	15.6 ± 9.1	42.1 ± 4.4	5.2 ± 7.1	2.2 ± 1.0
Kirsten	16.6 ± 9.3	54.1 ± 5.7	7.3 ± 5.5	2.9 ± 2.4
Lara	14.5 ± 8.3	60.2 ± 4.7	5.1 ± 2.9	2.8 ± 1.8
Langer	24.3 ± 11.5	49.6 ± 4.9	8.9 ± 14.3	2.8 ± 2.9
Pollock	22.1 ± 7.7	38.9 ± 5.4	9.7 ± 9.3	3.1 ± 2.9
Warne	3.5 ± 2.0	21.1 ± 2.0	1.1 ± 0.6	0.5 ± 0.4
Waugh	10.5 ± 5.5	57.3 ± 4.4	1.8 ± 1.6	0.8 ± 1.2
Prior	32.9 ± 17.4	32.9 ± 17.4	20.0 ± 20.0	3.0 ± 3.0



Lara
Waugh
Kirsten
Langer
Hussain
Pollock
Cairns
Warne
Posterior Mean
μ_2
μ_1
0
10
20
30
40
50
60

- All rounders seem to require less “warming-up”

Q: Is this true in general or a fluke of my sample?

A: No idea, I had a PhD to write.

Segue

- Thinking of probabilities as subjective degrees of plausibility can help clarify traditionally confusing topics
- For example, the 2^{nd} law of thermodynamics: some phenomena go one way only
- Unscrambling an egg is harder than scrambling it

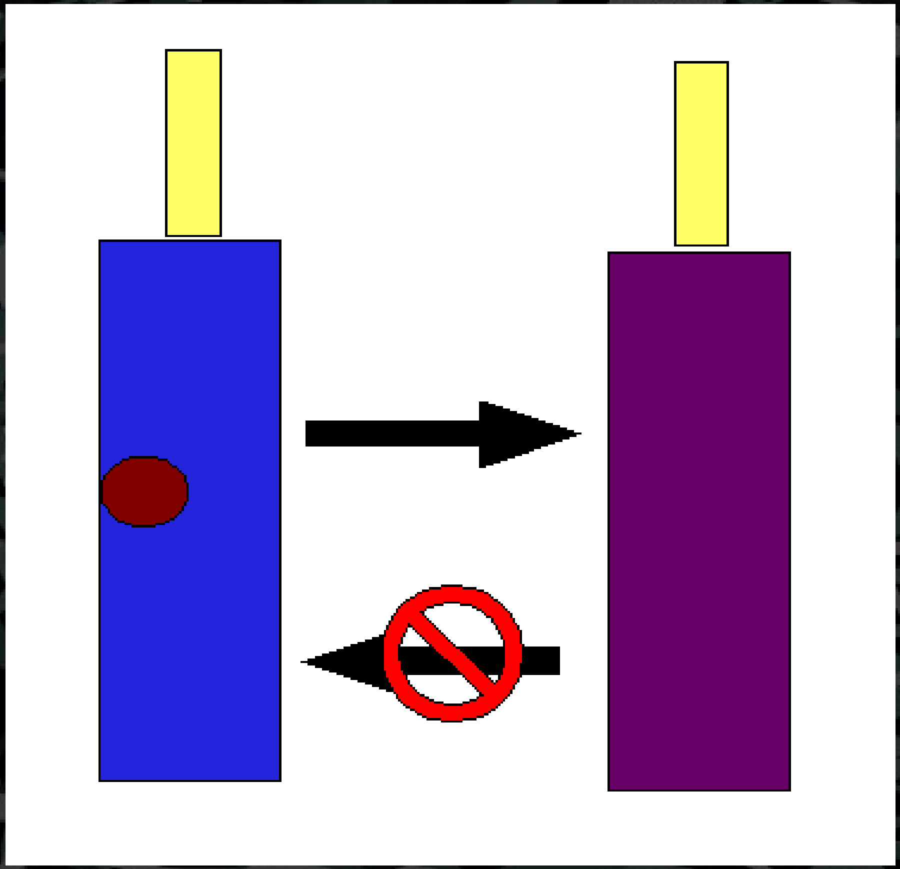
Hot Spot

- Hot spots fade away with time
- They do not spontaneously appear in the bat
- That's why hot spot is (arguably) useful

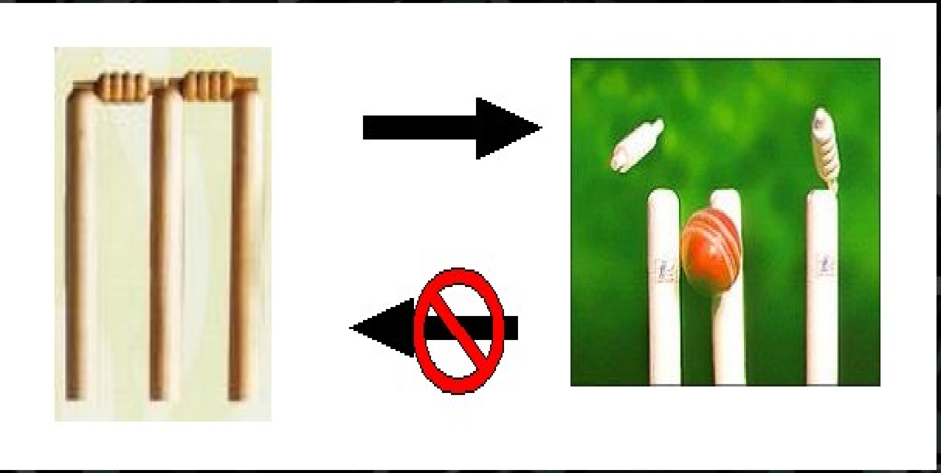
Bowled 'im

- Note that the umpire always has to straighten the stumps and put the bails back on carefully
- As opposed to throwing the ball and the bails at the stumps in order to reset them :)

Need a new principle – the 2nd law



- Defined in terms of "vague" macroscopic properties like temperature



But Mechanics is Time-Reversible!!

- Reversing the velocities in any system sends the positions back to their original state!
- Therefore these “impossible” reversed scenarios are actually possible.
- How can this be so?

Entropy (probability version)

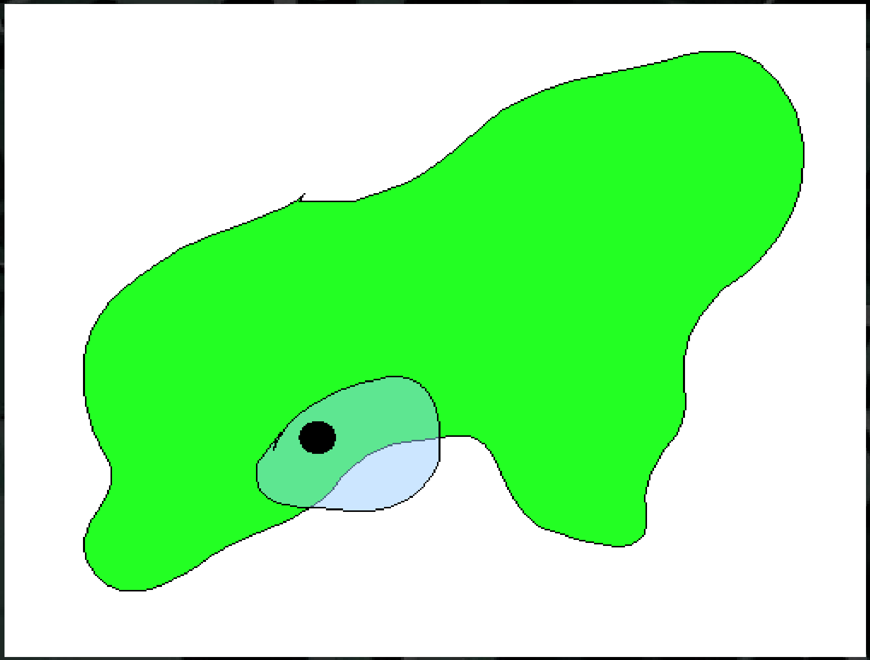
- In probability, there's a concept called entropy (Shannon 1949)

$$H = -\sum_i p_i \log p_i$$

- Measures "amount of uncertainty" in a discrete probability distribution
- For *N* equally probable, mutually exclusive hypotheses, entropy = log(*N*)

Entropy of Probability Distributions

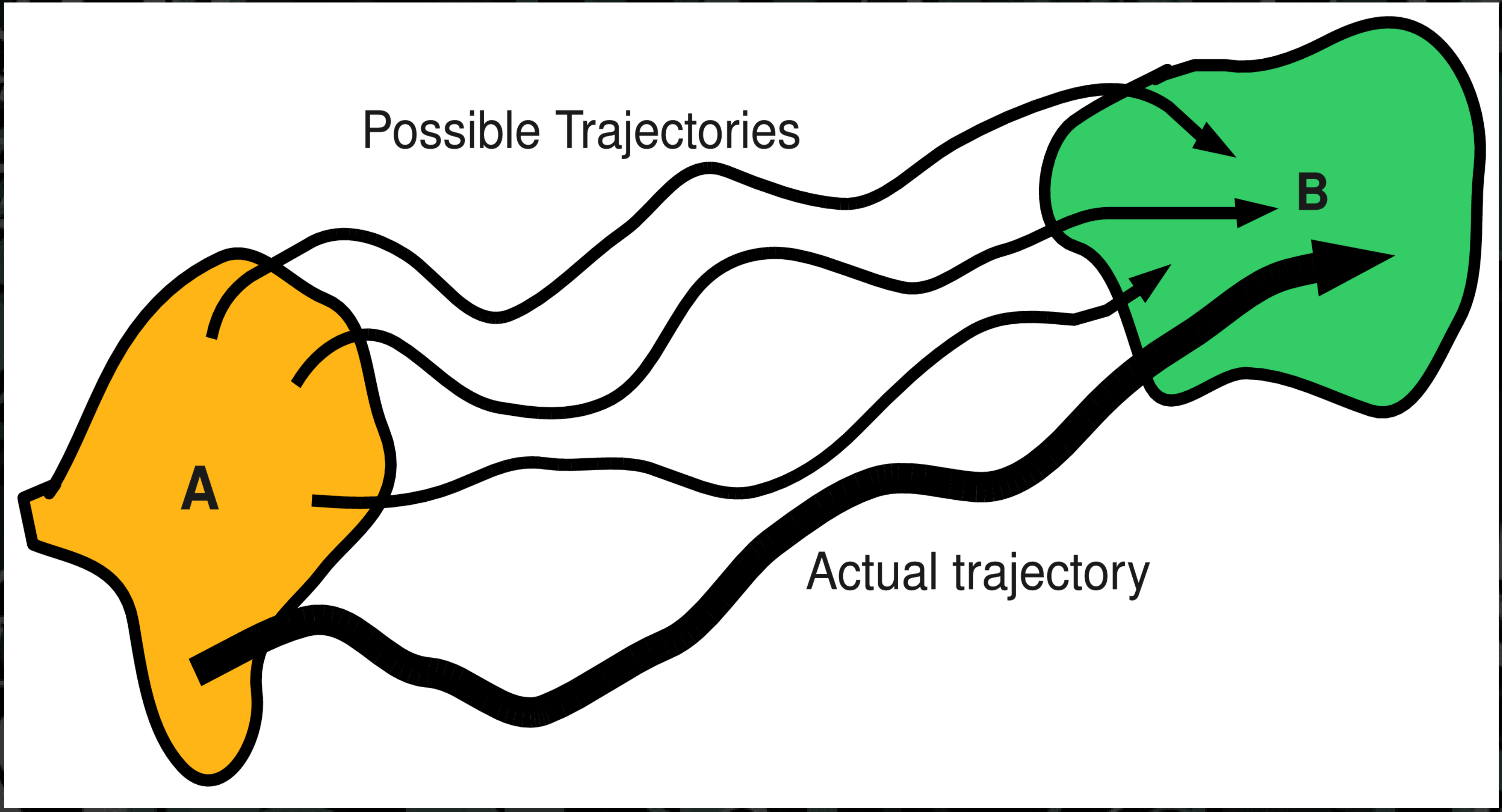
- Continuous version: $$H = -\int_X f(x) \log \frac{f(x)}{m(x)} dx$$
- log(volume of the region of uncertainty)
- Green = a high entropy distribution, blue = a low entropy distribution, black dot = the truth



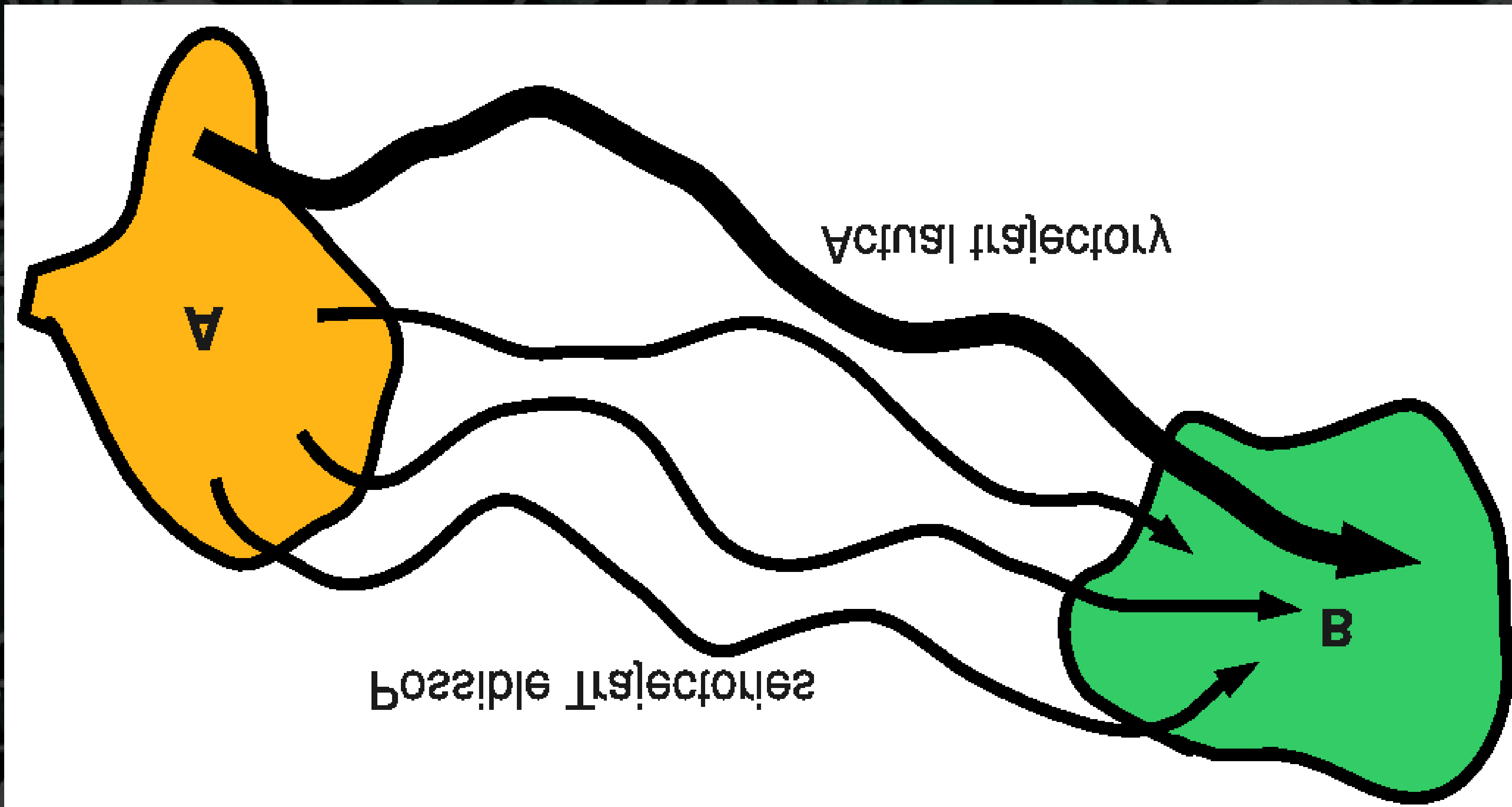
Statistical Mechanics

- The laws of mechanics fully determine the evolution of a system, given the initial conditions
- But if we don't know the initial conditions, we can't predict the state of the system at a different time.
- Describe knowledge of initial conditions by a probability distribution over phase space (set of possible states of a system – all positions and velocities)

Our knowledge evolves with time but (information) entropy is CONSERVED (does not increase!!!!)

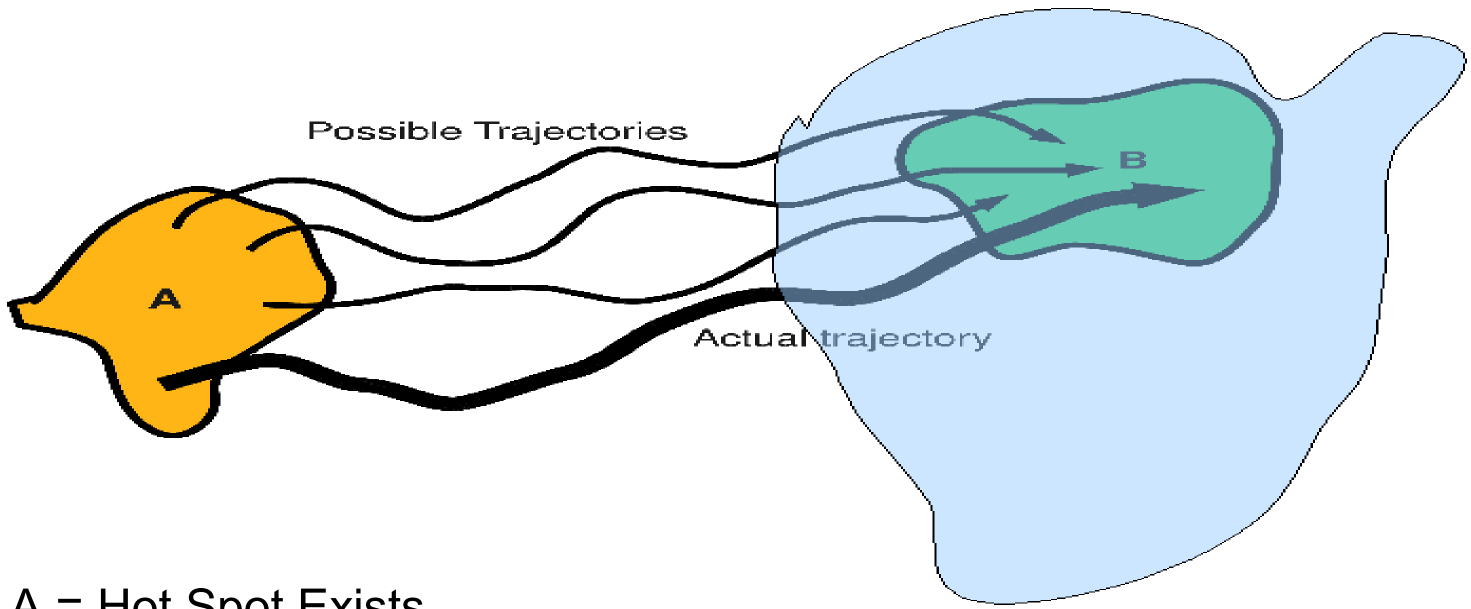


Oh, and reverse the arrows...

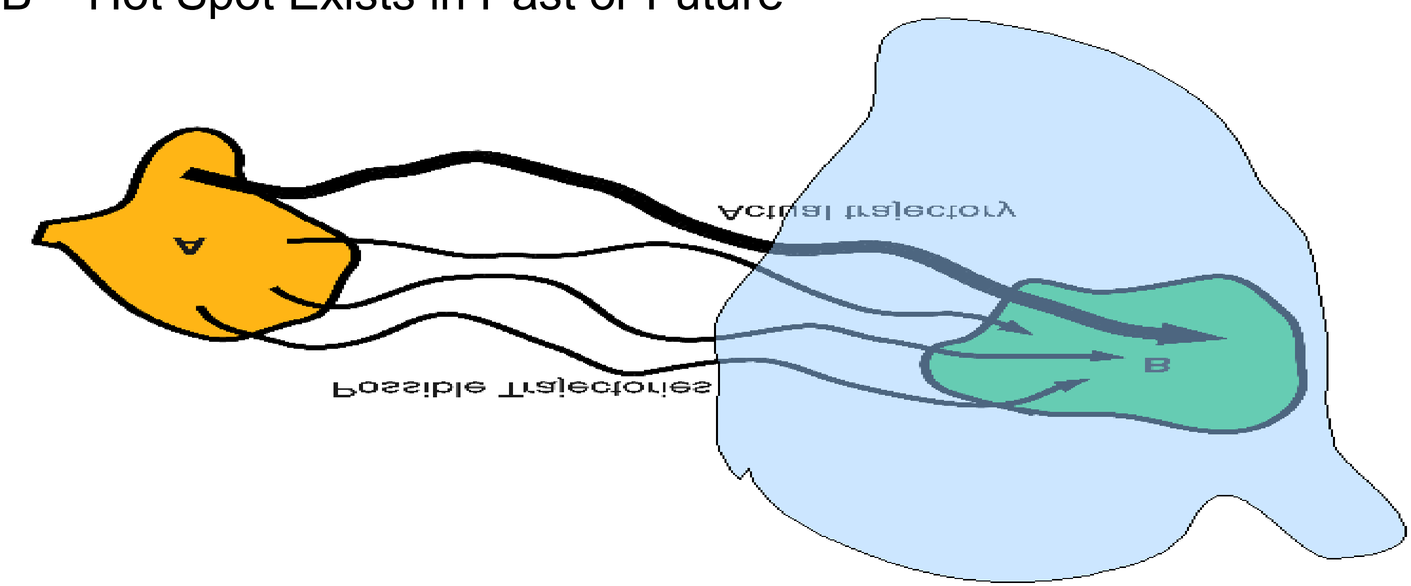


An Obvious Statement

- Big things can't fit inside small things
- Small things can fit inside big things



A = Hot Spot Exists
Light Blue = No Hot Spot
B = Hot Spot Exists in Past or Future



Putting the bails back on

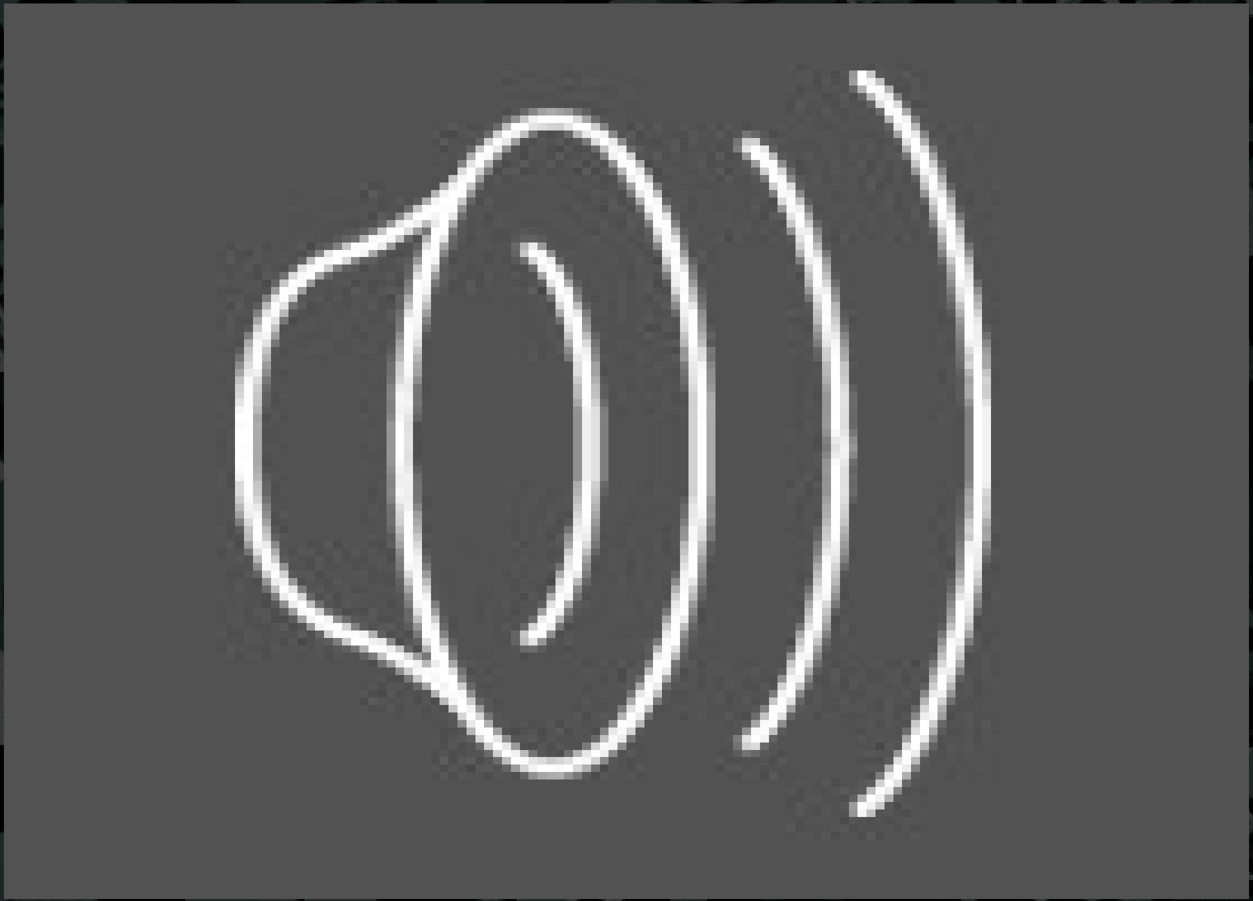
- Would require throwing the ball and the bails so precisely to land them all back in place
- Need to set the stumps wobbling exactly the reverse of how they were, and reverse thermal motions in the top of the stumps, to stop the bails when they land.

“Warney could do it…” - Matt Francis

Rules of Thumb

- Any *vague statement* V about a system* implies a probability distribution for the actual microscopic state (positions+velocities)
- Therefore any vague statement V has an entropy H(V)
- If $H(V_1) >> H(V_2)$ then it cannot be typical/reproducible for a system to evolve from V_1 to V_2 – but the reverse is permitted.
- All because entropy (uncertainty) is **conserved** by the equations of classical mechanics.
- * Conventionally called a macrostate.

Cosmological N-Body Simulations of Structure Formation
Credit: Volker Springel, Max-Planck-Institute for Astrophysics
Simulation code: Hydra
By varying initial conditions and concentrating on typical properties, simulators are already taking into account the "2nd law"



Fun Polemical Quote!

Glib, unqualified statements to the effect that "entropy measures randomness" are in my opinion totally meaningless, and present a serious barrier to any real understanding of these problems.

Conclusion

- Can use probability to model uncertainty – states of mind, not states of the world
- It makes problems conceptually easy
- Mike Hussey is not the 2nd best batsman ever
- Bradman did not “deserve” to average 100
- In hindsight, the 2nd law of thermodynamics should be obvious
- Any questions?