Beyond the Power Spectrum

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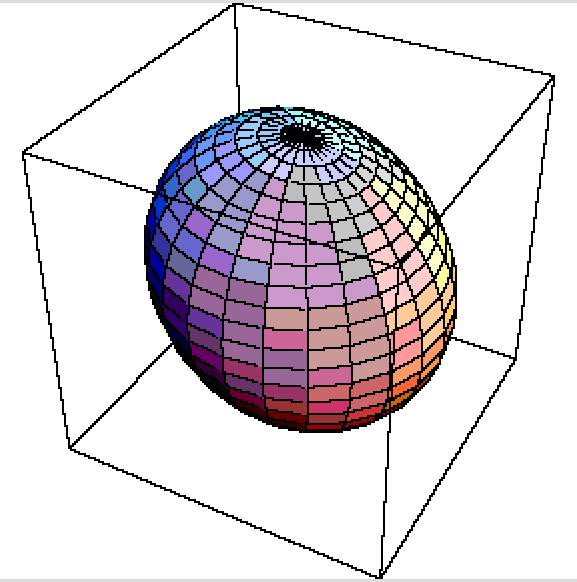
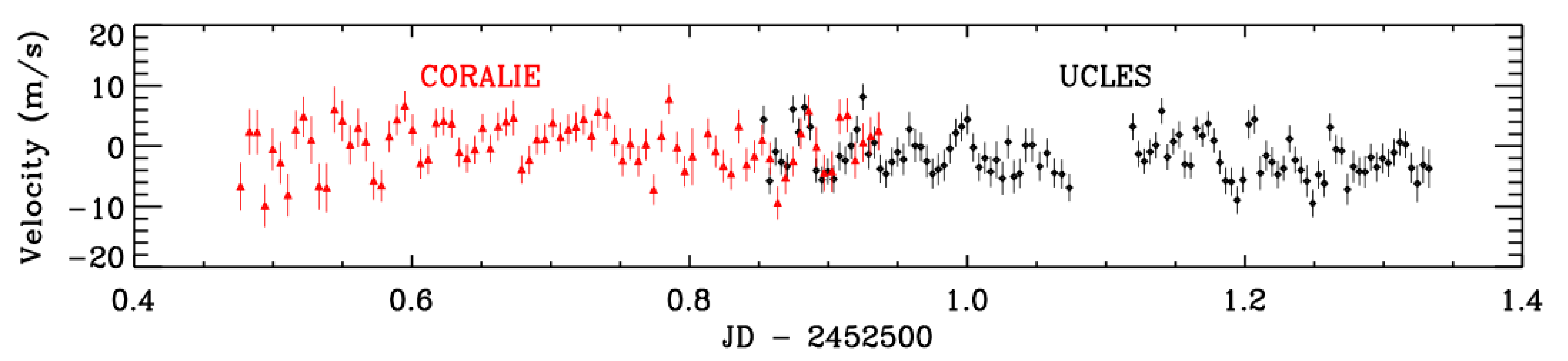
What We Want

- Stellar models give predictions about the frequencies of oscillation eigenmodes
- If we can measure those frequencies, we learn which stellar models are more plausible



What We've Got

- We actually measure a time series
- What frequencies could have produced this data?
- No unique answer (data aren't perfect)

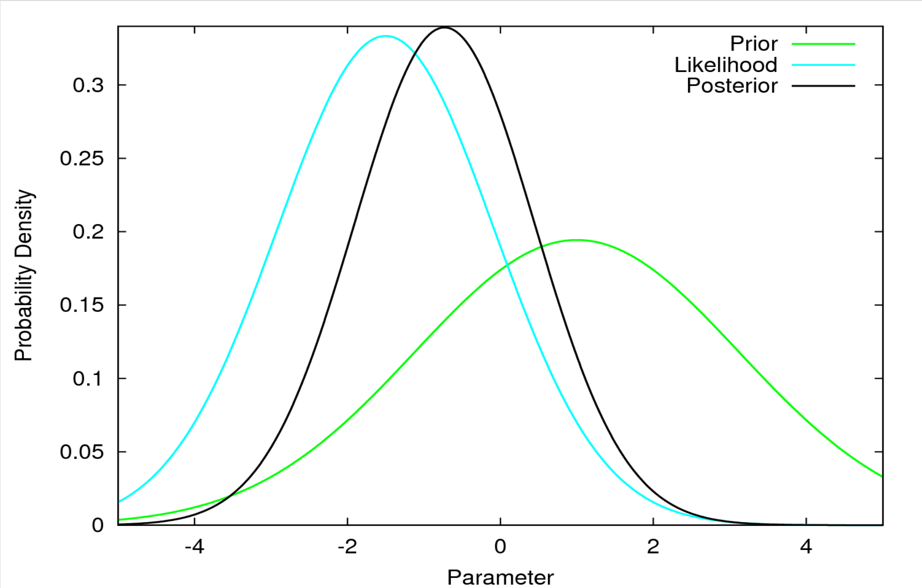


Bedding, T.R., et al, 2006, ApJ, 647, 558-563.
ν Indi, metal poor subgiant

Bayesian Inference

- Probability = 'degree of plausibility'
- Probability distribution for unknown parameters changes from `prior distribution' to (hopefully narrower!) `posterior distribution' when we get extra data *D.*

$$p(\theta|D) \propto p(\theta)p(D|\theta)$$



For Oscillation Data...

- Given the number of modes *M*, their frequencies and their amplitude and phase $\{A_i, B_i, \upsilon_i\}$, what would we predict about the observed time series $\{y_1, \ldots, y_n\}$?
- First calculated predicted noise-free signal *f(t)*, then add Gaussian noise

$$f(t_i) = \sum_{j=1}^{M} \left(A_j \sin(2\pi\nu_j t_i) + B_j \cos(2\pi\nu_j t_i) \right)$$

$$y_i = f(t_i) + \epsilon_i$$

Probability Distribution for Data

$$p(\{y_i\}|\{A_j, B_j, \nu_j\}_{j=1}^{M}) = \prod_{i=1}^{N} \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y_i - f(t_i)}{\sigma_i}\right)^2\right)$$

$$\propto \exp\left(-\frac{1}{2}\chi^2(M, \{A_j, B_j, \nu_j\}_{j=1}^{M})\right)$$

- Simple relationship between Bayesian Inference and methods based on χ^2. Minimising χ^2 is equivalent to finding the peak of the posterior distribution, for a ~flat prior distribution.

A Way of Thinking About It

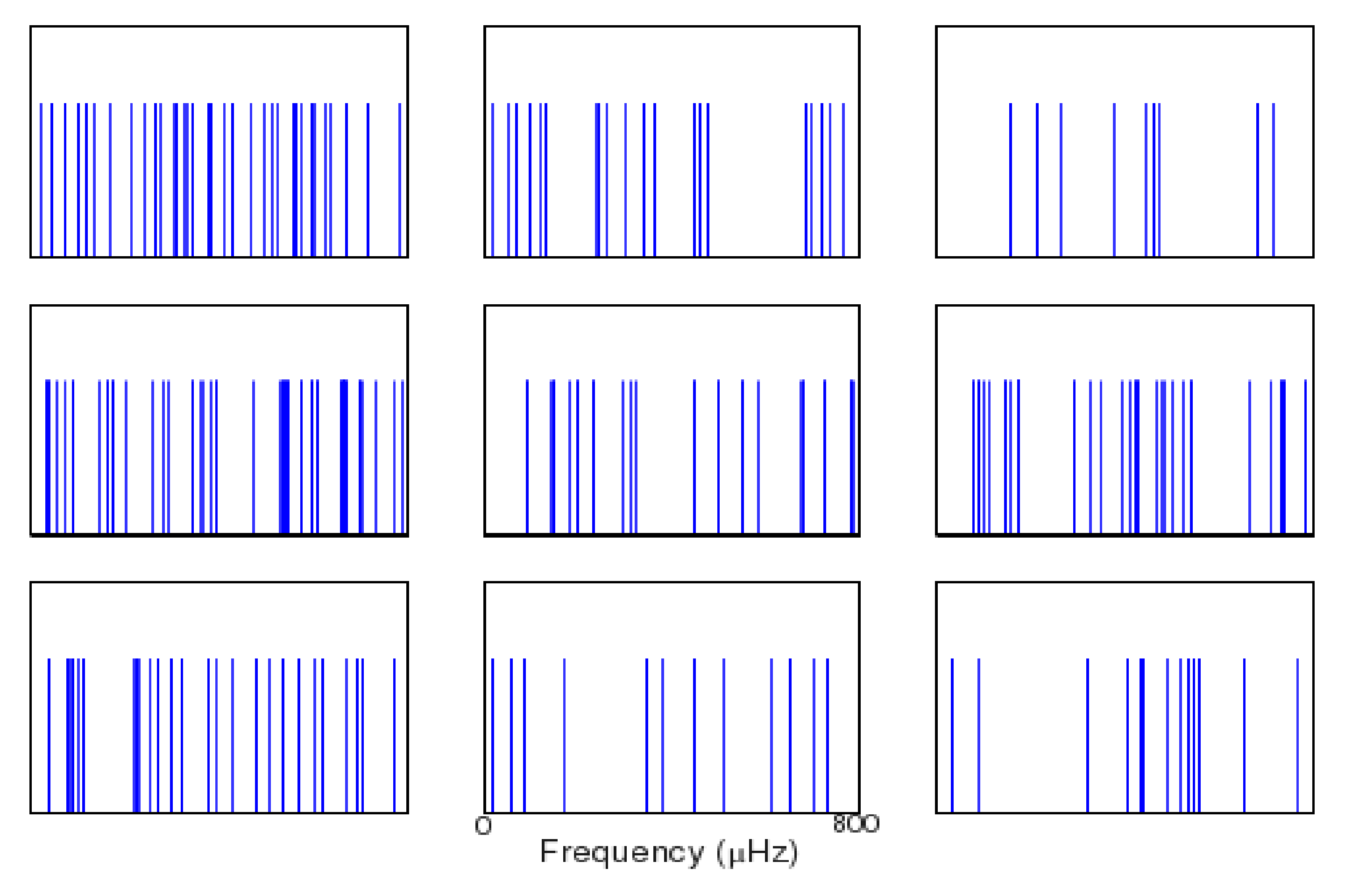
- Imagine generating a large sample of possible models (values for *M*, frequencies)
- Test each one against the data by calculating the likelihood $\exp(-1/2\ \chi^2)$, but χ^2 can only be calculated if we knew the amplitudes as well
- Average the likelihood over all possible values the amplitudes could have; this gives the *marginal likelihood* for the frequencies only.

The Power Spectrum

- Bretthorst (1988) proved that, *after some approximations*, the likelihood function depends on the data only through the power spectrum (periodogram)
- All model functions (sine and cosine curves) must be orthogonal when summed over the data timestamps.
- Only true if time series is long and continuous, and if we are not interested in closely spaced frequencies.

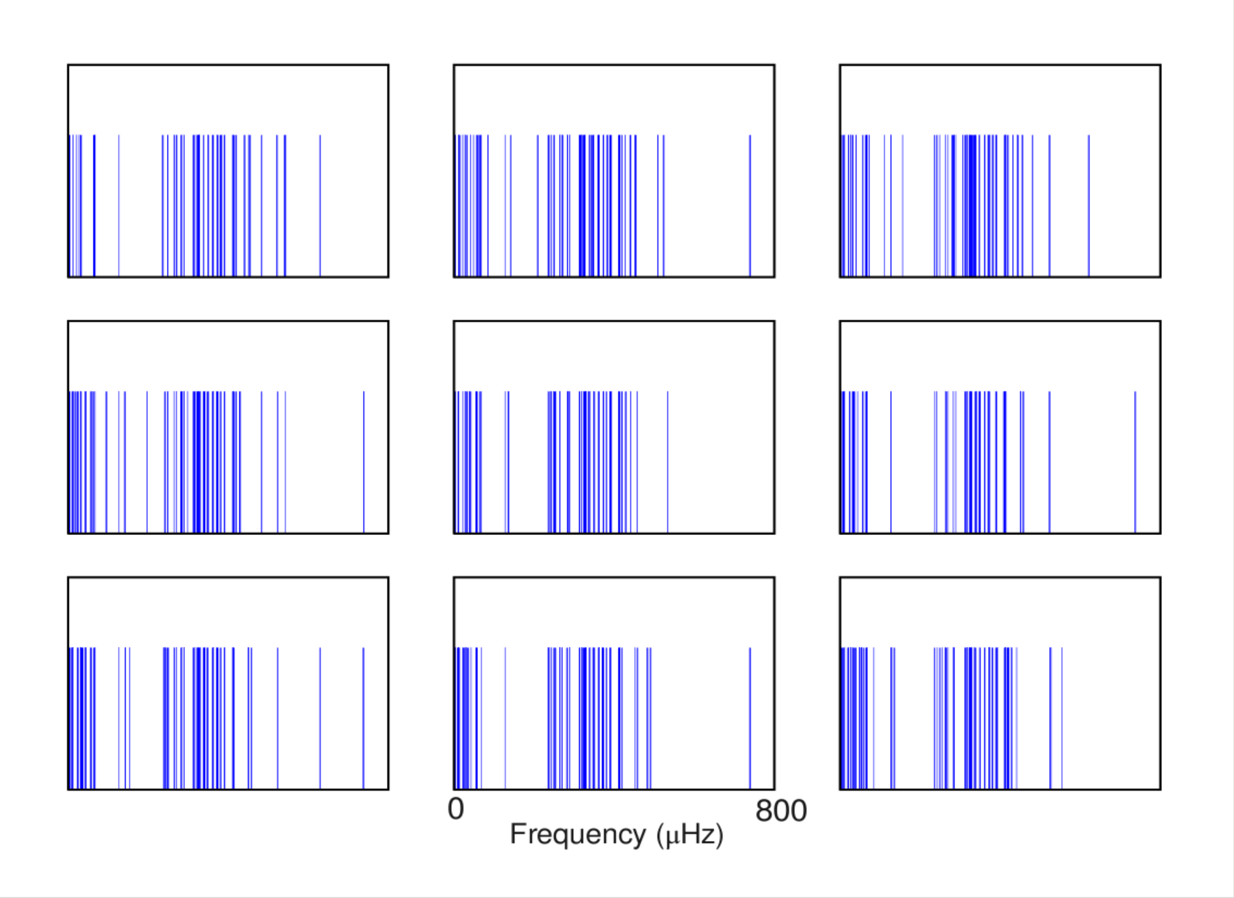
Some Random Models, Before Data

- Sampled from Prior.

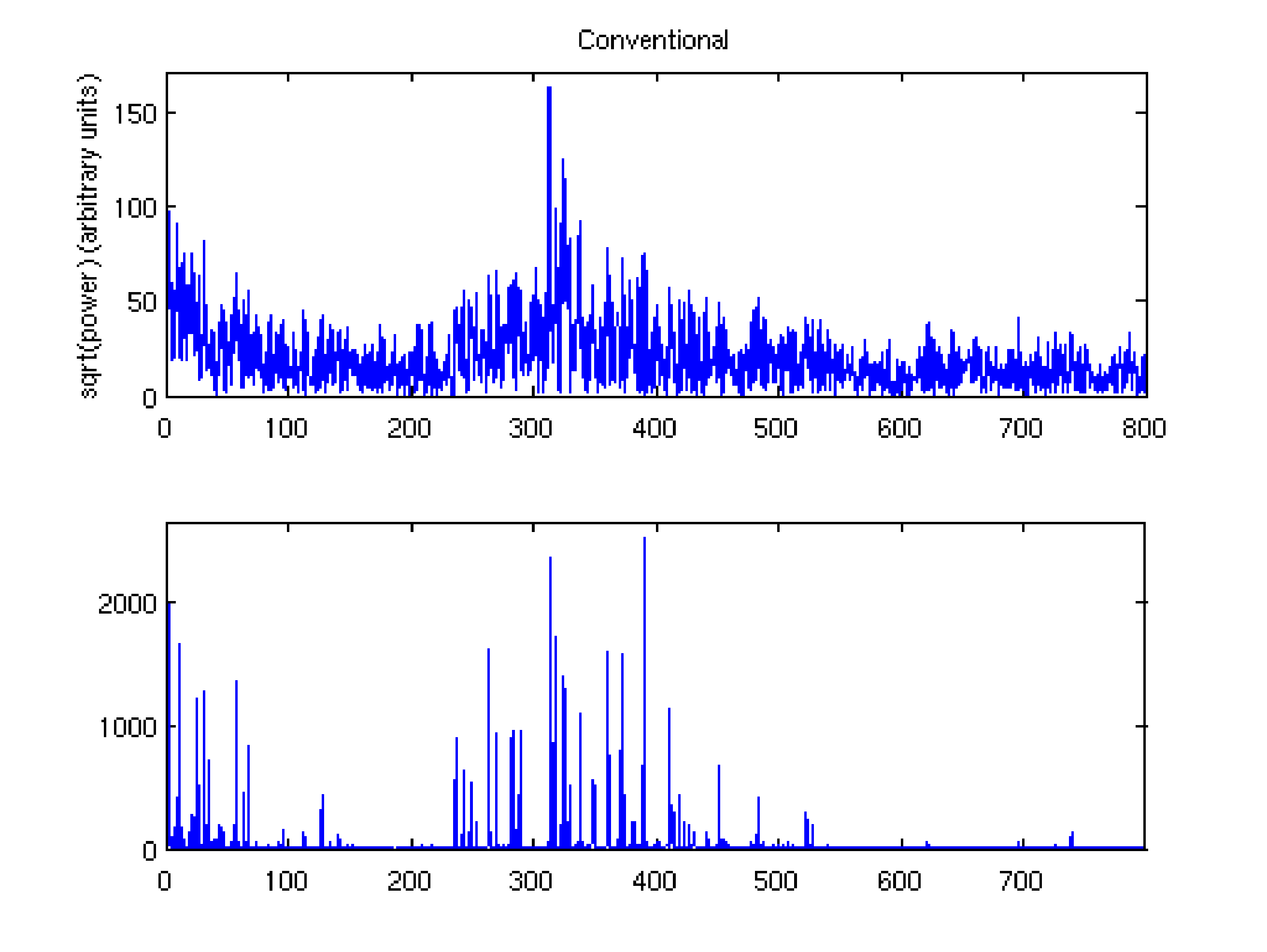


Some Random Models, After Data

- Sampled from Posterior via Markov Chain Monte Carlo.



Automatic Dealiasing (where possible)



But we never expected perfect sinusoids...

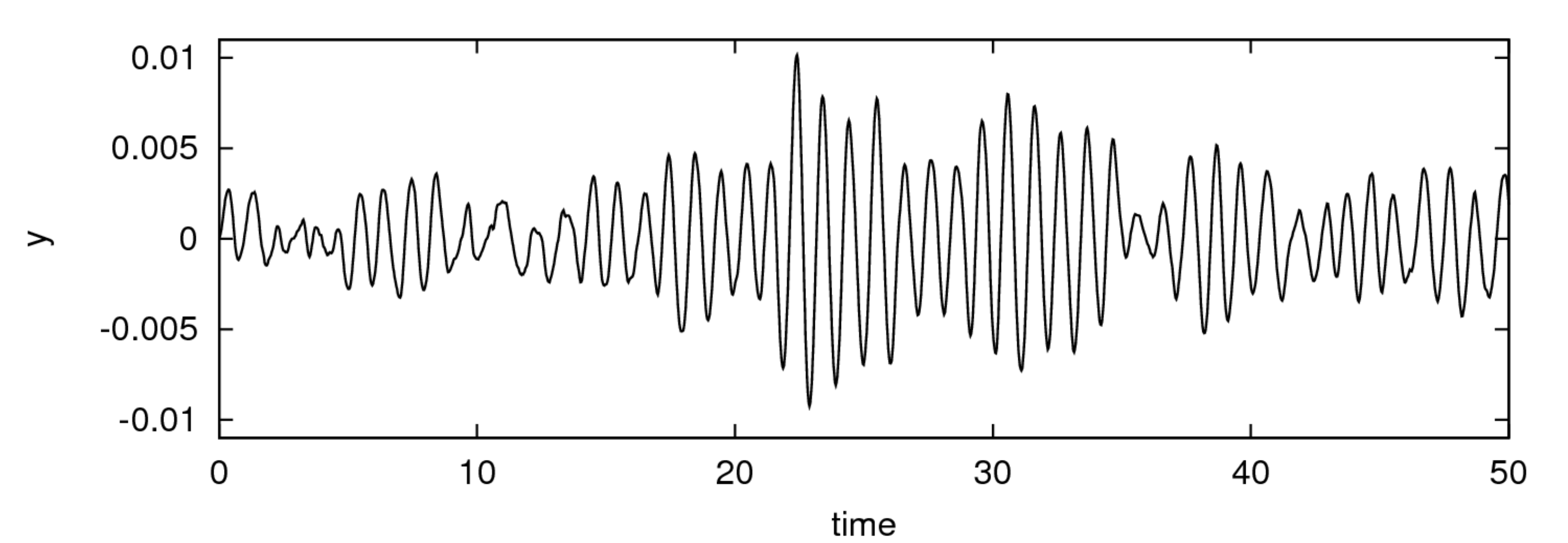
- This method still assumes that the signal is sinusoidal
- In solar-like oscillations, there is damping and stochastic excitation of modes
- With any other mechanism, the signal is still not going to be exactly sinusoidal
- Can we make a method that takes this into account?

Damped, Stochastically Excited Oscillator

$$\frac{d^2y}{dt^2} = -(2\pi\nu)^2 y - \frac{2}{\tau}\frac{dy}{dt} + \beta f(t)$$

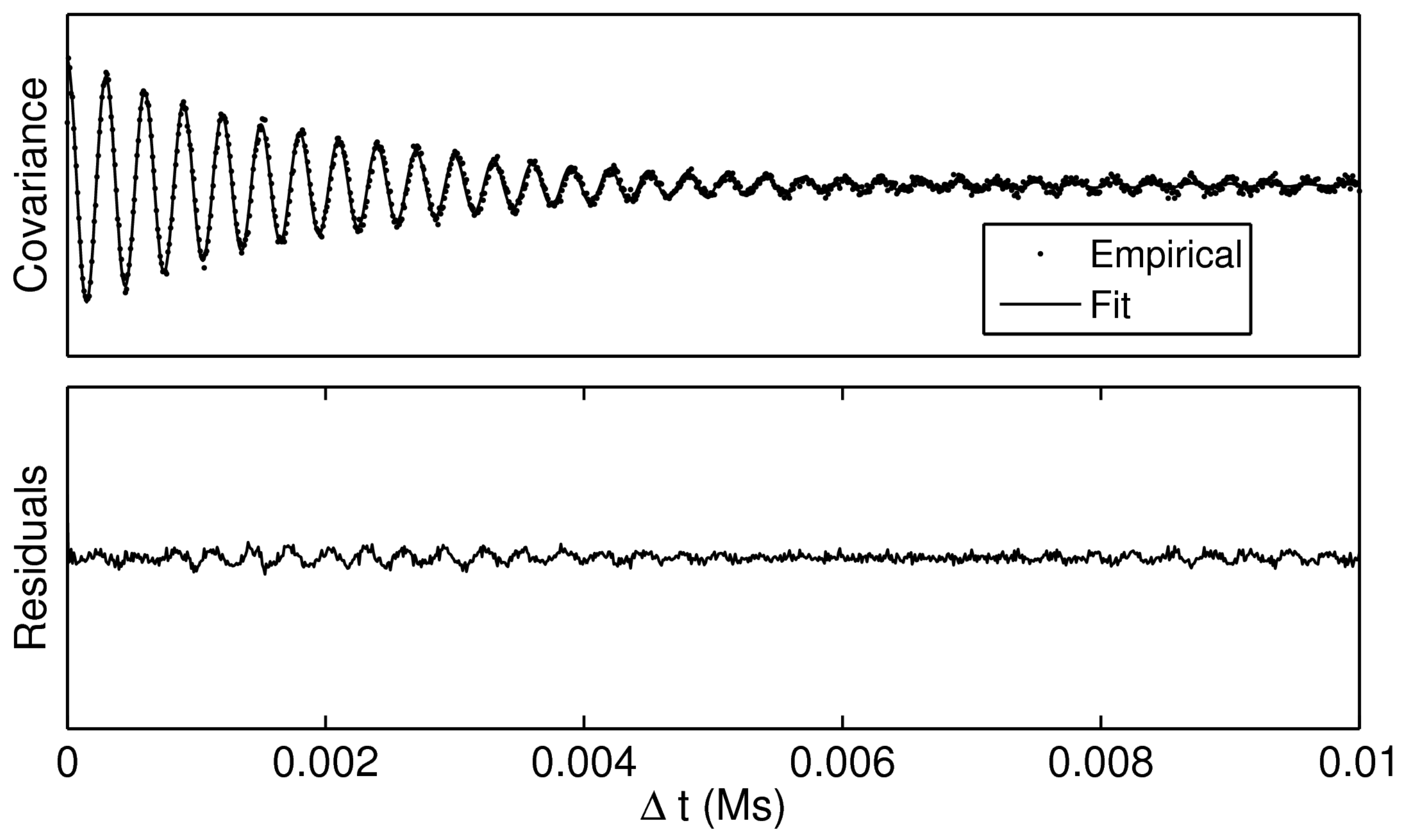
- $f(t)$ is white noise.
- Solution is a random function
- **Can still define probability density for data given parameters!**

Brewer and Stello 2008, in prep

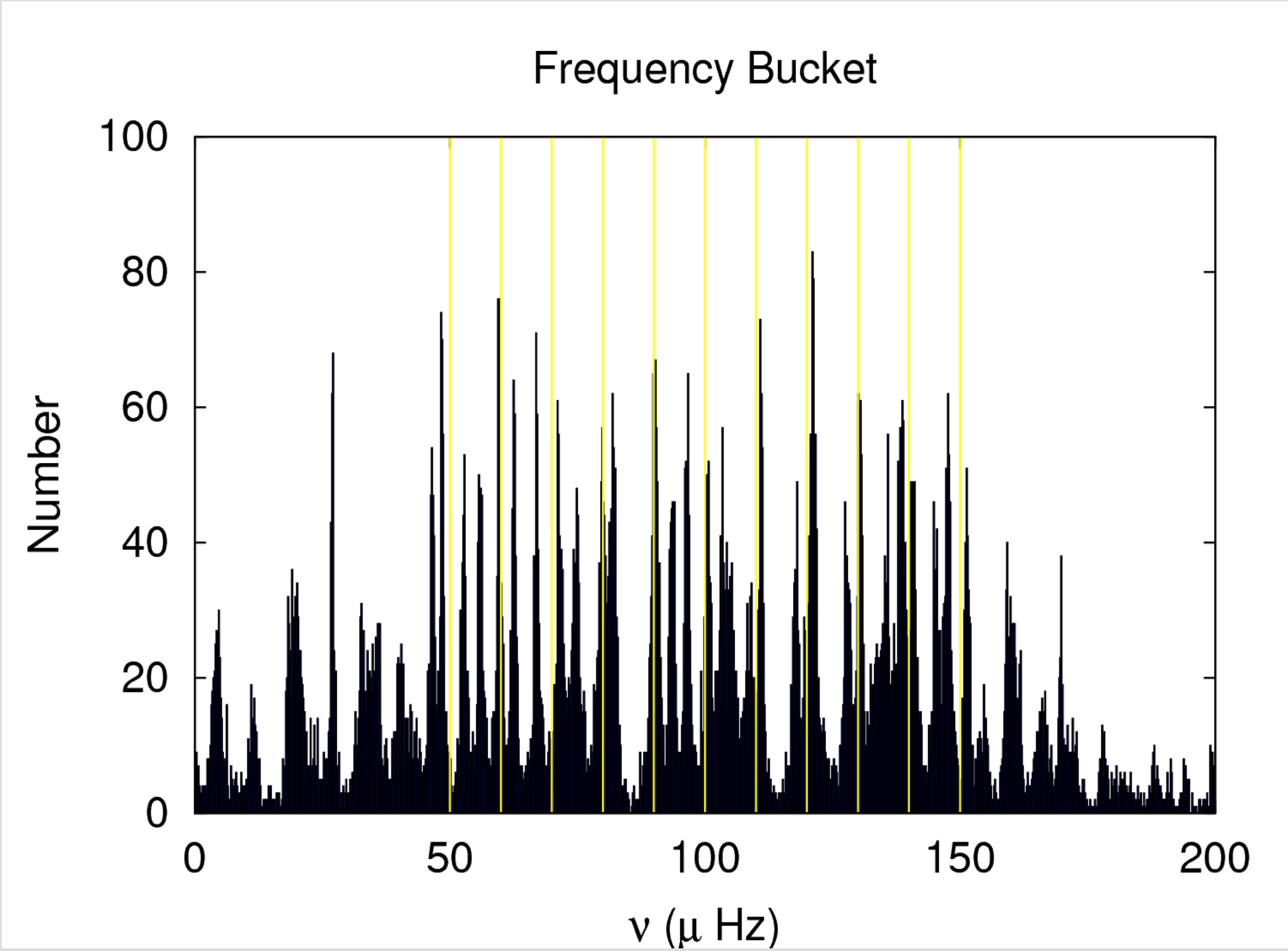


Covariance Function for Quasi-Oscillating Functions

- Exponentially decaying cosine

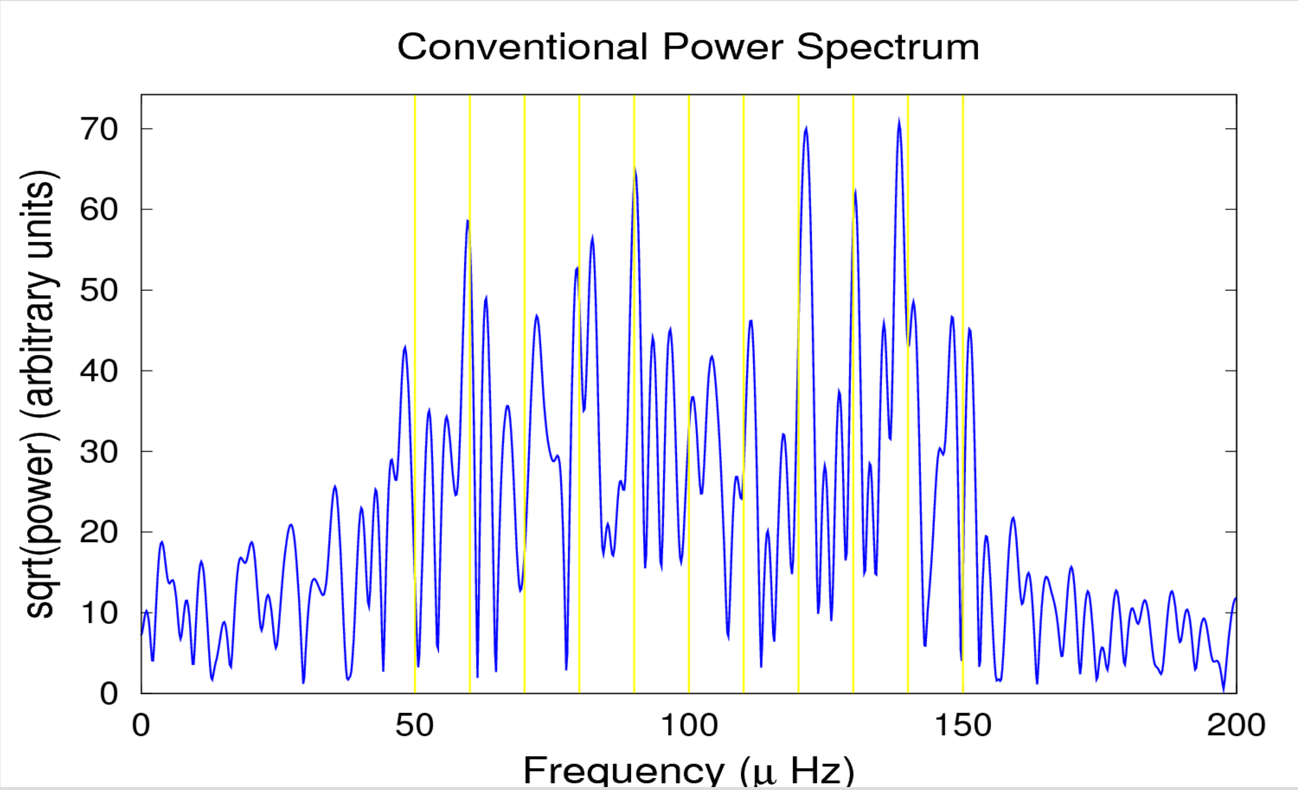


- Analysed simulated data (433 points) with sinusoidal assumption
- Finds too many `modes'

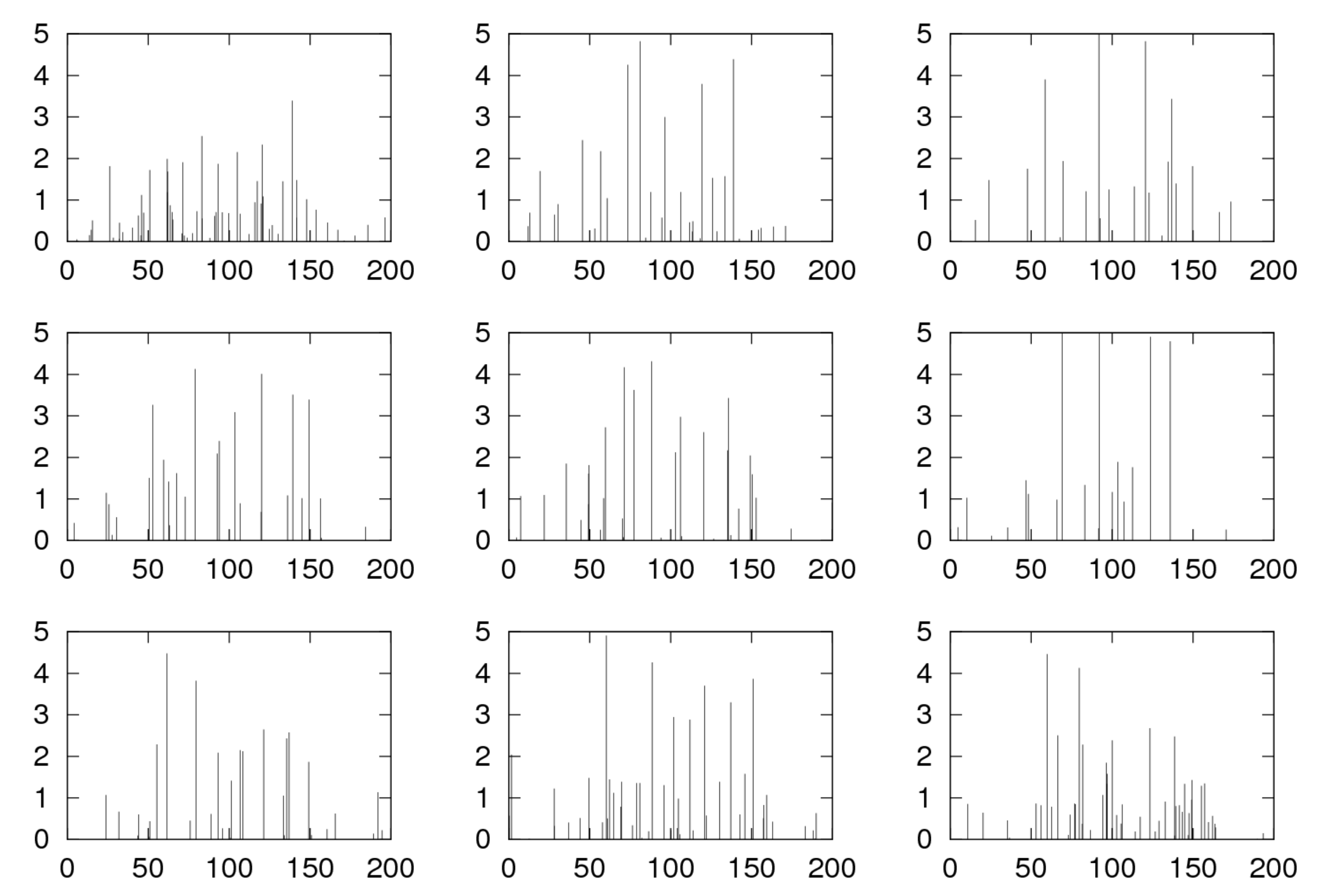


Power Spectrum

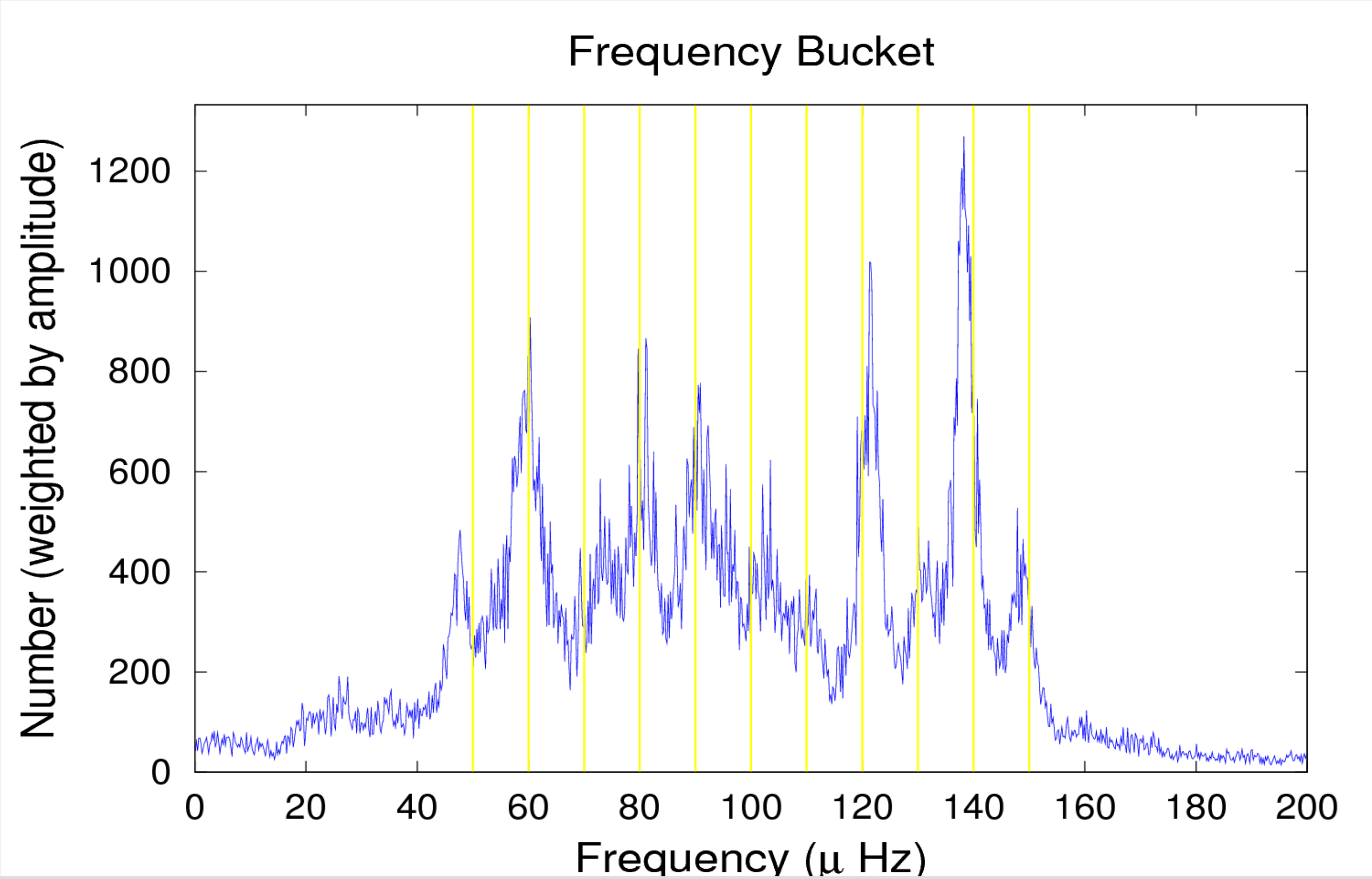
- Also has too many spurious peaks



Sample From Posterior, non-sinusoidal model

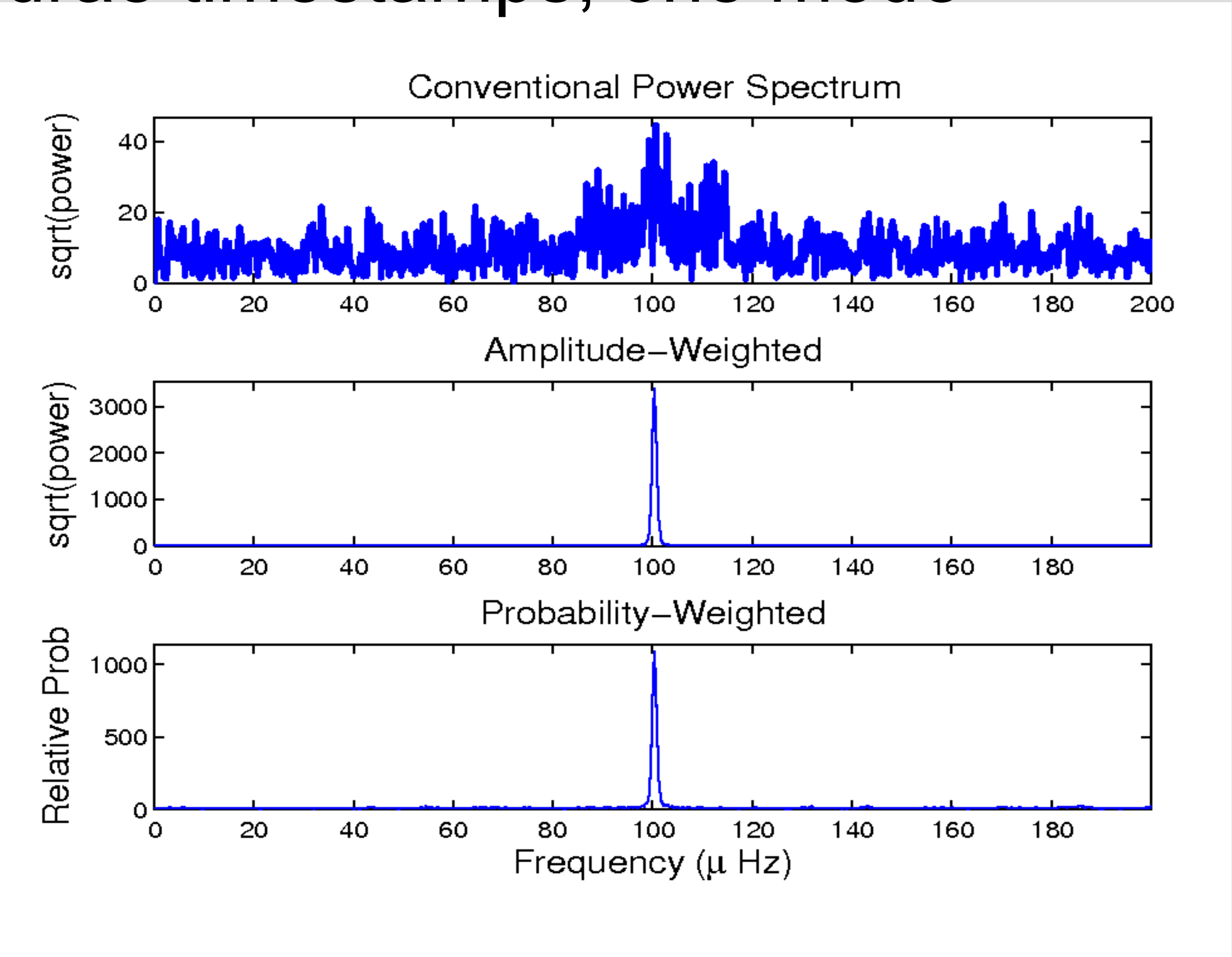


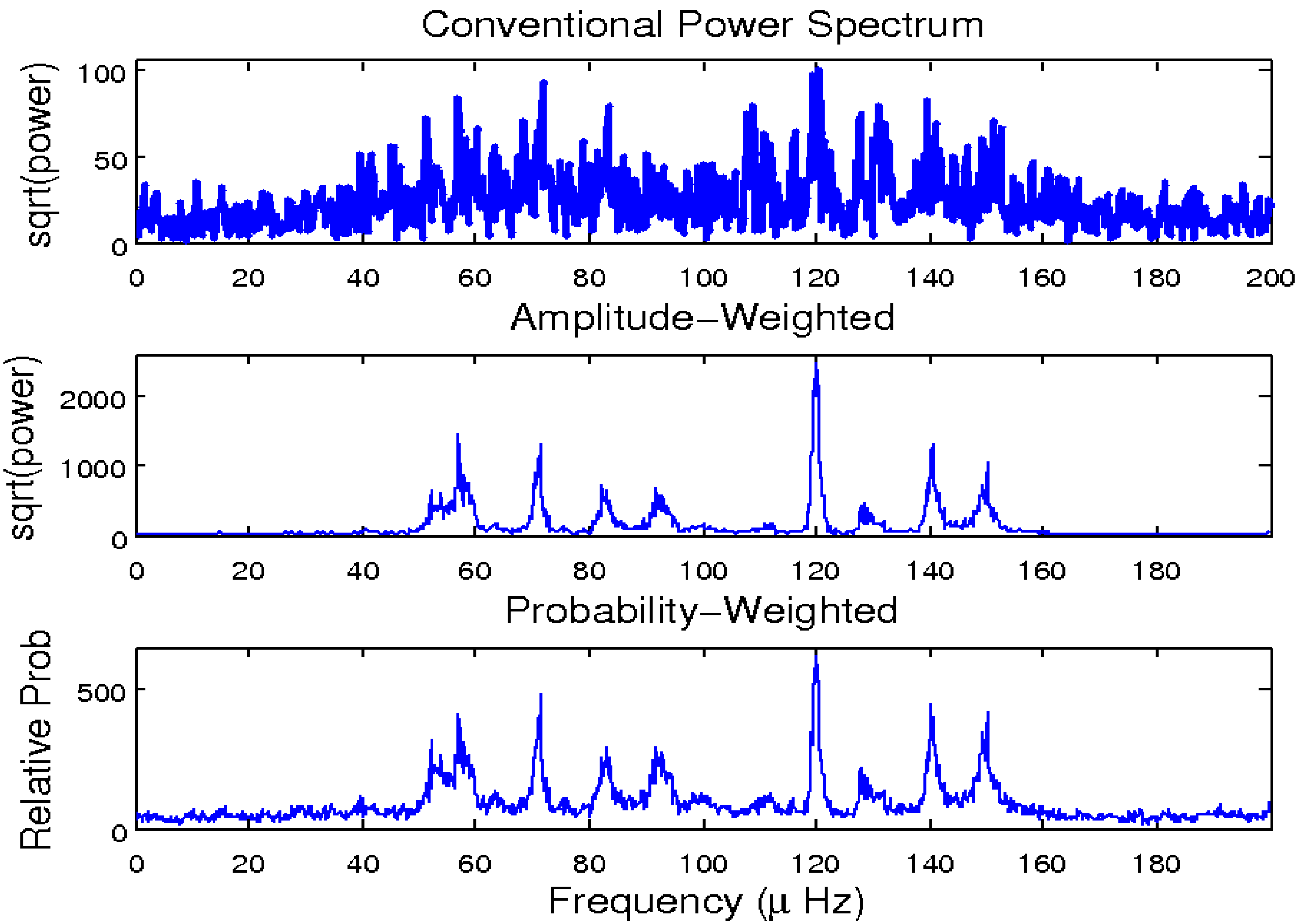
- Result
- Can also estimate mode lifetime (it's just another parameter)



Data with Gaps

- ξ Hydrae timestamps, one mode



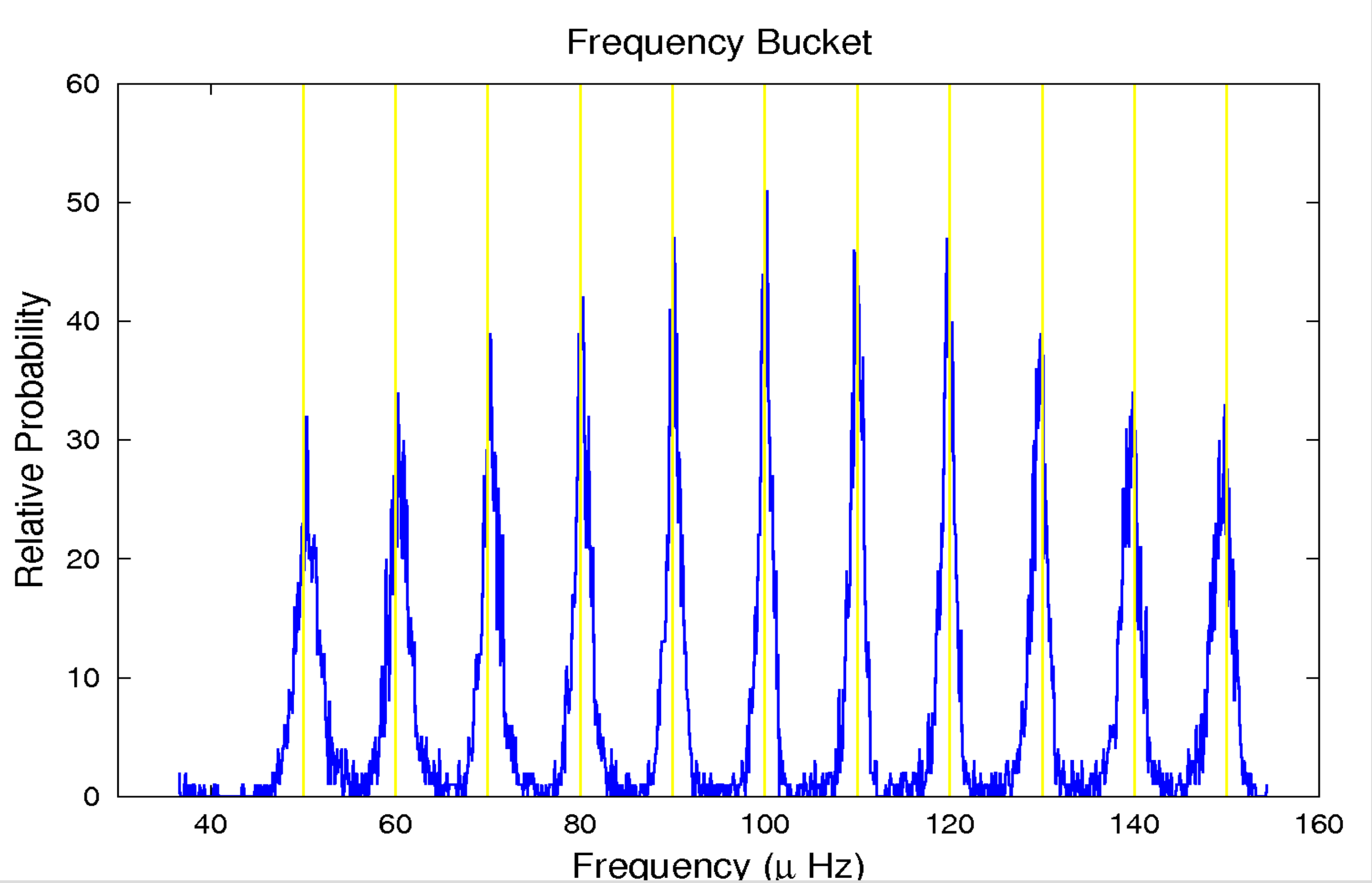


Conventional Power Spectrum
sqrt(power)
100
50
0
0 20 40 60 80 100 120 140 160 180 200
Amplitude−Weighted
sqrt(power)
2000
1000
0
0 20 40 60 80 100 120 140 160 180
Probability−Weighted
Relative Prob
500
0
0 20 40 60 80 100 120 140 160 180
Frequency (μ Hz)

Prior Information about the Frequency Pattern

- If we know that they are evenly spaced in advance, this greatly restricts the possible solutions
- Can make more realistic assumptions than ”they're evenly spaced”. e.g. Use theoretical models
- Makes a big difference!

Makes a Big Difference!



Conclusions

- For long, complete time series and sinusoidal oscillations, the power spectrum is excellent
- Bayesian sine-wave fitting provides incremental improvements and uncertainties on all conclusions
- *Explore parameter space, don't optimise!*
- It is possible to model quasi-sinusoidal signals and take into account theoretical prior information