So we saw that the rate limiting step is the \( p + ^{14}N \) reaction with \( E_g = 48.1 \) MeV. So 24 MeV are released each \( t = \frac{1}{n_p \langle \sigma v \rangle} \) per seed nucleus. And

\[
\varepsilon = \frac{1}{n_p n_s} \langle \sigma v \rangle \geq 28 \text{ MeV}
\]

where \( S = 2.75 \text{ keV} \cdot \text{Barn} \)

\[
\langle \sigma v \rangle = 1.52 \times 10^{-15} \frac{1}{T_7^{1/3}} e^{-72.19/T_7^{1/3}}
\]

\[
\langle \sigma v \rangle = \frac{R}{n_p n_N} = \frac{6.48 \times 10^{-18} \text{(2.75 keV} \cdot \text{Barn}) (48.1 \times 10^3 \text{keV})^{3/4} e^{-72.19/T_7^{1/3}}}{1.58 \times 10^{-15} e^{-72.19/T_7^{1/3}} \text{ cm}^3 \text{s}^{-1}} \text{ Valid corrected (4.26)}
\]

so we have \( n_s \approx 10^{-3} n_p \), and

\[
\varepsilon = 10^{-3} \frac{n_p^2}{n_p} \langle \sigma v \rangle \geq 28 \text{ MeV}
\]

\[
\rho = \frac{n_p n_p}{n_p}
\]

\[
= 10^{-3} \frac{R}{n_p n_p} \langle \sigma v \rangle \geq 28 \text{ MeV}
\]

\[
= \frac{10^{-3}}{(1.47 \times 10^{-24})^2 \frac{2}{T_7^{1/3}}} e^{-72.28/T_7^{1/3}} (1.58 \times 10^{-15} \text{ cm}^3 \text{s}^{-1} (28 \times 10^3) (1.6 \times 10^{-24} \text{ cm}^3))
\]

\[
= 2.5 \times 10^{25} \frac{1}{T_7^{1/3}} \exp \left( -\frac{72.28}{T_7^{1/3}} \right) \text{ yr}^{-1} \text{ keV}^{-1}
\]
Now let's build (roughly) the high mass main sequence.

$$L_{\text{nuc}} = L_\odot (\frac{M}{M_\odot})^3$$

$$= (2.5 \times 10^{25} \text{ erg sec}) \frac{1.99 \times 10^{33} \text{ g}}{9\text{ sec}} \frac{\rho}{T^{2/3}} \exp \left( \frac{-72.20}{T^{1/3}} \right) = 4 \times 10^{33} \left( \frac{M}{M_\odot} \right)^3$$

$$= (5.0 \times 10^{58} \text{ erg sec}) \frac{\rho}{T^{2/3}} \exp \left( \frac{-72.20}{T^{1/3}} \right) = 4 \times 10^{33} \left( \frac{M}{M_\odot} \right)^2$$

$$\ln \frac{5 \times 10^{58}}{4 \times 10^{33}} = \frac{-72.20}{T^{1/3}} = 0$$

$$T^{1/3} = \frac{72.20}{57.79} \approx 1.25$$

$$T = 1.2 \times 10^7 \text{ K}$$

Expand $\varepsilon$ at $T_7 = 2$

$$\frac{d\varepsilon}{dT} = \frac{(\varepsilon_0)}{4\pi T^2} \varepsilon = \frac{2}{19} \frac{\varepsilon}{T}$$

$$\varepsilon = 2 \left( \frac{T}{2 \times 10^7} \right)^{19} \rho$$

So

$$L_{\text{nuc}} \approx \left( \frac{T}{2 \times 10^7} \right)^{19} \rho^2 \frac{M}{M_\odot} \approx M^3$$
The Main Sequence

But we know that

\[ \frac{M}{R} \propto T \text{ or } R \propto \frac{M}{T} \propto T^{-1/2} \]

\[ M^2 \frac{T^{-3}}{M^3} \propto \frac{T^{-9}}{M^3} \propto M^{9/11} \propto M^{0.61} \text{ often quoted.} \]

Also

\[ R \propto M^{9/11} \propto M^{0.61} \]

Thus

\[ L \propto M^{3} \propto R^{2} T^{4} \]

\[ T_e \propto M^{3} \frac{T_e}{R^2} \propto \frac{M^{3}}{M^{18/11}} \propto M^{15/11} \]

\[ T_e \propto M^{15/11} \propto M^{0.34} \]

So from 1-10 M_☉ there is only a factor of 2 difference in temperature.

So now that we understand how fusion occurs, we have finally set the scale for the radius of a star.

(Again)

\[ T_e = 10^{7} = \frac{\frac{1}{2} \frac{G M M_p}{R L}}{R L} \Rightarrow \frac{M}{R} \propto \text{constant} \]

\[ L = 4\pi R^2 \sigma T^4 \]

\[ T = \left( \frac{L}{4\pi R^2 \sigma} \right)^{1/4} \]

But \( L \propto L_p (M/M_☉)^3 \) for massive stars.

\[ T \propto \left( \frac{M^3}{M^2} \right)^{1/4} \propto M^{1/4} \]
Slope of $M - S$

Thompson $R$

\[ R \propto \text{CNO} \]

1. \[ \frac{d\epsilon_{13}}{dT/1T} = \left( \frac{E_G}{4kT} \right)^{1/2} \sim 19 \]

2. \[ L \propto E_M \times L_0 \left( \frac{M}{M_0} \right)^3 \]

\[ L \propto \frac{T^{12}}{M^{11/2}} \]

\[ L \propto \frac{M^2}{R^3} T^{19} \]

\[ T \propto \frac{M}{R} \]

\[ R \propto \frac{M}{T} \]

\[ L \propto \frac{M^2}{R^3} T^{19} \]

\[ \alpha \frac{T^{22}}{M} \sim M^3 \Rightarrow \frac{T_{ce}^{22}}{M} \sim M^{9/11} \]

3. \[ T_{\text{eff}} \]

\[ L \propto T_{\text{eff}}^4 R^2 \]

\[ R \propto \frac{M}{T} \times \frac{M^{2/11}}{M^{9/11}} \]

\[ L \propto T_{\text{eff}}^4 M^{12/11} \sim M^3 \]

\[ T_{\text{eff}} \propto \frac{M^{3/11}}{M^{12/11}} \]

\[ T_{\text{eff}} \propto M^{1/3} \]

\[ T_{\text{eff}} \propto M^{13/2} \]

\[ T_{\text{eff}} \propto M^{39/2} \]

\[ L \propto T_{\text{eff}}^4 \left( \frac{M}{T} \right)^2 \sim T_{\text{eff}}^4 \left( \frac{M}{M_{0.34}} \right)^2 \]

\[ \sim (M_{0.34})^4 \left( \frac{M}{M_{0.34}} \right)^{2} \times M^{39/2} \]
\[ L = T^4 R^2 \times T^{\alpha/(\beta - \gamma)} \times T^{\frac{\gamma}{\alpha}} \left( \frac{T^{2.9}}{T} \right)^2 \times T^{\gamma} \left( \frac{T}{T^2} \right)^2 \times T^8 \]

3) Compare to diagram

i.e., \( T_e \approx M^{0.34} \)

So, \( M \approx T_e^{2.94} \)
Between 0.08 and 60 M\(_\odot\), many things change:

1. Opacity \(10^5 \rightarrow 10^4\)
2. Prog
3. PP cycle becomes less important at high M due to extreme temperature sensitivity.

**Immediate Implications**

1) Get some sensitivity to metallicity.

2) The cores become convective since most of the energy is released very near the core.
   All luminosity is generated near the center, so flux becomes high.

\(1-2) \ M_\odot \leq M \leq 60\)

Convective core + radiative envelope
All burning from CNO.

<table>
<thead>
<tr>
<th>(\frac{M}{M_\odot})</th>
<th>(\log \frac{L}{L_\odot})</th>
<th>(\log T_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(^{-6})</td>
<td>5.701</td>
<td>4.68</td>
</tr>
<tr>
<td>10(^{-7})</td>
<td>3.77</td>
<td>4.41</td>
</tr>
<tr>
<td>8\times10(^{-9})</td>
<td>0</td>
<td>3.75</td>
</tr>
<tr>
<td>0.10</td>
<td>-3.023</td>
<td>3.48</td>
</tr>
</tbody>
</table>
Now a crude age relation for $M > M_6$ is

$$L = L_6 \left( \frac{M}{M_6} \right)^{3.5}$$

$$T_{MS} = \frac{E_{\text{fuel}}}{L}$$

Typically the central region collapses when 10% of the fuel has burned, so

$$E_{\text{fuel}} = 0.10 \text{ M}_2 \text{ in 18 M}_\odot$$

$$= 1.3 \times 10^{51} \text{ erg}_2$$

$$T = \frac{1.3 \times 10^{51} \text{ erg}_2}{4 \times 10^{33}} \left( \frac{M_6}{M} \right)^{3.5} \left( \frac{1}{M_6} \right)$$

$$T = 1 \times 10^{10} \text{ yr} \left( \frac{M_6}{M} \right)^{2.5}$$

<table>
<thead>
<tr>
<th>$M/M_6$</th>
<th>$t_{MS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8 \times 10^9$</td>
</tr>
<tr>
<td>10</td>
<td>$2.5 \times 10^7$</td>
</tr>
<tr>
<td>40</td>
<td>$8 \times 10^5$</td>
</tr>
</tbody>
</table>

So the main sequence lifetime of a massive star can be less than the K-H contraction time of a low M star.

This strong mass dependence means that clusters of stars born simultaneously will have distinctive features.