Shear strain localization in elastodynamic rupture simulations

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We study strain localization as an enhanced velocity weakening mechanism on earthquake faults. Fault friction is modeled using Shear Transformation Zone (STZ) Theory, a microscopic physical model for non-affine rearrangements in granular fault gouge. STZ Theory is implemented in spring slider and dynamic rupture models of faults. We compare dynamic shear localization to deformation that is uniform throughout the gouge layer, and find that localized slip enhances the velocity weakening of the gouge. Localized elastodynamic ruptures have larger stress drops and higher peak slip rates than ruptures with homogeneous strain.

1. Introduction

The earthquake problem spans a vast range of length and time scales, from contacts between individual grains up through tectonic networks of faults. One of the primary modeling challenges is identifying the relevant physical instabilities, and accurately and efficiently propagating this information between scales. In this study, we investigate gouge-scale strain localization as a mechanism for enhanced velocity weakening and apply it to dynamic rupture propagation on faults. Strain localization has been observed in a variety of contexts, including numerical simulations of gouge [*Morgan and Boettcher*, 1999], experimental studies of laboratory faults [*Marone*, 1998], and field observations of faults [*Chester and Chester*, 1998].

The formation of shear bands is a dynamic instability that we resolve numerically by incorporating a gouge layer of finite width. This approach differs from the common practice of implementing a slip weakening or rate and state friction law on a planar fault. In our model, the dynamic behavior of the friction law determines how strain is distributed throughout the gouge layer. Within the gouge, the material is governed by Shear Transformation Zone (STZ) Theory, a microscopic physical model for localized plasticity in amorphous materials [*Falk and Langer*, 1998, 2000]. We study the formation of localized shear bands, contrasting our results with homogeneous deformation, where strain is uniform throughout the gouge layer. In shear bands, slip spontaneously localizes to an interface which is narrow even on the scale of the gouge. The dynamic localization instability enhances the velocity weakening, decreasing the sliding friction and increasing the peak slip rates in dynamic earthquake models. The results illustrate that gouge-scale strain localization has fault-scale consequences.

2. STZ Friction Law

STZ Theory is a continuum model for amorphous materials, suitable for predicting the constitutive behavior of fault gouge [*Falk and Langer*, 1998, 2000; *Lemaitre and Carlson*, 2004; *Lois et al.*, 2005; *Manning et al.*, 2008; *Daub and Carlson*, submitted manuscript]. A schematic of the fault is shown at the left in Fig. 1. A layer of fault gouge, which is modeled using STZ Theory, is sheared between elastic rocks. The plastic strain rate $\dot{\gamma}_{pl}$ can vary in the z-direction in the gouge (center picture in Fig. 1), though the shear stress τ is assumed to be uniform in the layer.

The motion of a collection of particles can be decomposed into an affine (elastic) component where the displacement is homogeneous and a non-affine (plastic) component where the displacement is heterogeneous. Non-affine motion frequently involves particles switching neighbors. Simulations show that in many types of disordered solids the non-affine deformation occurs when localized regions switch from one metastable orientation to another [*Falk and Langer*, 1998, 2000; *Lois et al.*, 2005]. These regions, or Shear Transformation Zones (STZs), are modeled as bistable groups of particles that flip in the direction of the applied shear stress. A single STZ reversal generates a fixed amount of plastic strain, and a minimum stress must be applied for the reversal to occur. Simulations show that the STZs flip only once, but as energy is dissipated new deformable zones are continually created and annhilated to sustain plastic flow. A schematic of an STZ switching orientation is shown at the right in Fig. 1.

The plastic strain rate is influenced by two properties of the gouge: the number of STZs, and the rate at which the STZs reverse due to the applied stress. The number of STZs follows a Boltzmann distribution, with an effective disorder temperature χ [*Langer*, 2008; *Langer and Manning*, 2008]. These can be represented as:

$$\dot{\gamma}_{pl} = \exp\left(-1/\chi\right) R\left(\tau\right). \quad (1)$$

The function $R(\tau)$ describes how the rate at which STZs change orientation depends on the applied shear stress. The physics behind our choice for $R(\tau)$ is an Eyring model [*Eyring*, 1936], which matches the logarithimic velocity dependence of laboratory faults [*Dieterich*, 1979]. However, the enhancement of velocity weakening that strain localization produces is independent of the details of $R(\tau)$.

Regions with higher effective temperature have more STZs, and undergo more plastic strain. The effective temperature is distinct from the thermal temperature, though we expect the effective temperature to exhibit dynamic behavior similar to thermal temperature. We therefore include shear heating and diffusion terms in the effective temperature governing partial differential equation. Heating and diffusion occur on a time scale that is slow compared to the stress equilibration time scale in the fault gouge, but similar to the time scale of coseismic slip. During the interseismic period, time dependent relaxation of the effective temperature can account for healing, but we do not consider re-strengthening for the single ruptures in this study. The friction equations and the parameters for our simulations can be found in Table 1. The details of the derivation can be found in *Manning et al.* [2007] and *Lemaitre and Carlson* [2004].

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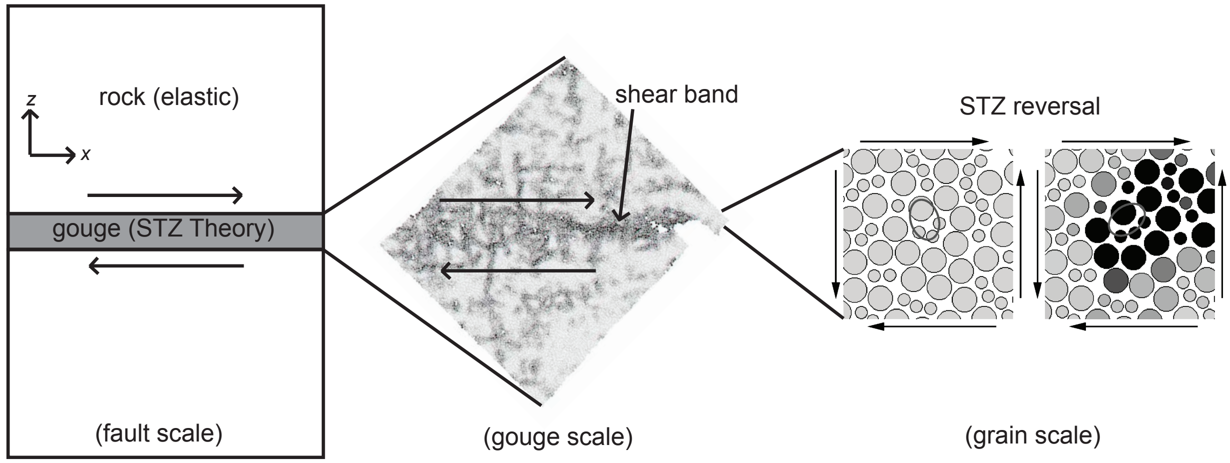

Figure 1. An illustration of the range of scales in the earthquake rupture problem. The scale moves to progressively smaller phenomena from left to right. Left: fault scale model, with a thin layer of gouge described by STZ theory sheared between elastic rocks. Center: close up of deformation inside the gouge, where shear strain develops into a localized shear band (dark regions). Shear band image taken from *Falk and Shi* [2002], and re-oriented to match the sense of shear of the fault and grains. Right: microscopic picture of the grain scale, with an STZ undergoing transformation from a "+" oriented zone (left) to a "−" oriented zone (right). As the gouge deforms plastically, the ellipse drawn through the particles flips its orientation. STZ diagram taken from *Falk and Langer* [1998].

3. Spring Slider Model

To investigate the dynamics of localization, we study the STZ friction law (Table 1) coupled to a non-inertial single degree of freedom elastic slider. A layer of gouge of width $2w$ separates two blocks, one of which is pulled by a spring of stiffness k at a velocity V_0. Stress evolves according to

$$\frac{d\tau}{dt} = k\left(V - \int_{-w}^{w} \dot{\gamma}_{pl} dz\right). \qquad (2)$$

We numerically solve for the stress and the effective temperature in the gouge layer using finite differences with an explicit two step time integration scheme. Boundary conditions on the effective temperature are periodic. We drive the slider from rest to a seismic slip rate of $V_0 = 1$ m/s with a spring of stiffness $k = 100000$ MPa/m. This approximates the rapid loading and slip acceleration that occurs during seismic slip. We set the half width of the gouge layer as $w = 1$ m and start with an initial shear stress of $\tau(t = 0) = 70$ MPa.

Table 1. Equations and parameters for the STZ constitutive law. The equations include the specific version of Eq. (1) and the partial differential equation governing the effective temperature.

Equations	
$\dot{\gamma}_{pl} = \exp(-1/\chi)(2\epsilon/t_0)\exp(-f_0)(1 - \tau_y/\tau)\cosh(\tau/\sigma_d)$	
$\partial\chi/\partial t = \left[\dot{\gamma}_{pl}\tau/(c_0\tau_y)\right]\left[1 - (\chi/\chi_w)\log(\dot{\gamma}_0/\dot{\gamma}_{pl})\right] + \dot{\gamma}_{pl} D\left(\partial^2\chi/\partial z^2\right)$	
Parameter	**Description**
$\epsilon = 10$	Typical number of particles in an STZ
$t_0 = 10^{-6}$ s	STZ rearrangement time scale
$f_0 = 118$	STZ activation energy scaled by thermal energy
$\tau_y = 50$ MPa	Yield stress (below τ_y, $\dot{\gamma}_{pl} = 0$)
$\sigma_d = 0.5$ MPa	STZ rearrangement activation stress
$c_0 = 1$	Effective temperature specific heat
$\dot{\gamma}_0 = 80000$ s^{-1}	Strain rate at which χ diverges
$\chi_w = 0.5$	Stress weakens with strain rate if $\chi_w < 1$
$D = 10^{-5}$ m^2	Diffusion constant

A special case of the spring slider dynamics arises for homogeneous initial conditions for the effective temperature. In this case, by symmetry all subsequent deformation is homogeneous. A plot of stress as a function of displacement for homogeneous deformation with the uniform initial effective temperature $\chi(t = 0) = 0.018$ is illustrated in Fig. 2, which shows that stress weakens with displacement in a manner similar to the laboratory-based Dieterich-Ruina friction law [*Dieterich*, 1979], and the STZ Free Volume law considered by Daub and Carlson (submitted manuscript).

In general, we expect the initial conditions for effective temperature, which describes the structural disorder of the gouge, to reflect the heterogeneity of fault zones [*Chester and Chester*, 1998]. A localized shear band dynamically forms in this case, due to the response of the friction equations.

While a narrow shear band results from any non-uniform initial conditions, we focus on an idealized scenario to illustrate the dynamic evolution of the shear band. We add a small amplitude ($\Delta\chi = 2 \times 10^{-7}$) symmetric step perturbation in the center of the gouge layer of half width 0.1 m to the otherwise uniform initial effective temperature ($\chi(t = 0) = 0.018$).

We compare the shear stress as a function of displacement for both dynamically localized strain and homogeneous strain in Fig. 2(a). Uniform strain requires about 0.3 m of displacement for the shear stress to stabilize to $\tau = 63.38$ MPa. Dynamic localization drops the sliding stress to $\tau = 62.14$ MPa much more rapidly, and illustrates that localization enhances the velocity weakening of the gouge.

Figures 2(b) and 2(c) show snapshots of the strain rate in the gouge layer for dynamically localizing strain at a series of representative points along the stress versus displacement curve. During the earliest stages of displacement, strain occurs nearly uniformly throughout the gouge layer (plot (1) in Fig. 2(b)). The effect of the initial perturbation is negligible, and the strain rate is uniform. The displacement before localized strain begins depends on the magnitude of the perturbation in the initial effective temperature. This is because a region with an elevated effective temperature also has a higher strain rate (Eq. (1)). The shear heating term in the effective temperature equation (Table 1) is proportional to the strain rate, so regions of increased effective temperature heat more rapidly. Therefore, for larger initial perturbations, the local effective temperature increases more rapidly, and less displacement is needed before the strain dynamically localizes.

The strain rate profile once localization begins is shown in curve (2) in Fig. 2(b). The dynamic instability in the friction law causes strain to localize to a shear band, and friction weakens rapidly. This localization arises due to a feedback where energy dissipation increases the number of STZs, allowing more plastic deformation and energy dissipation. A narrower shear band (plots (3) in Fig. 2(b) and (4) in Fig. 2(c)) develops with further displacement. By symmetry, the narrow shear band occurs in the center of the gouge, and its width is determined by the diffusion constant $\sqrt{D}$. Once the strain localizes to this diffusion-limited shear band, the shear stress no longer changes with displacement. While the shear heating and diffusion terms do not exactly cancel each other, there is no noticeable change in the effective temperature on the time scale of an earthquake rupture.

4. Dynamic Ruptures

In this section, we investigate the impact of strain localization on the propagation of spontaneous elastodynamic ruptures. We model the fault gouge as a thin layer with half-width w between two homogeneous, isotropic, linear elastic solids. We solve for the elastodynamic response for 2D anti-

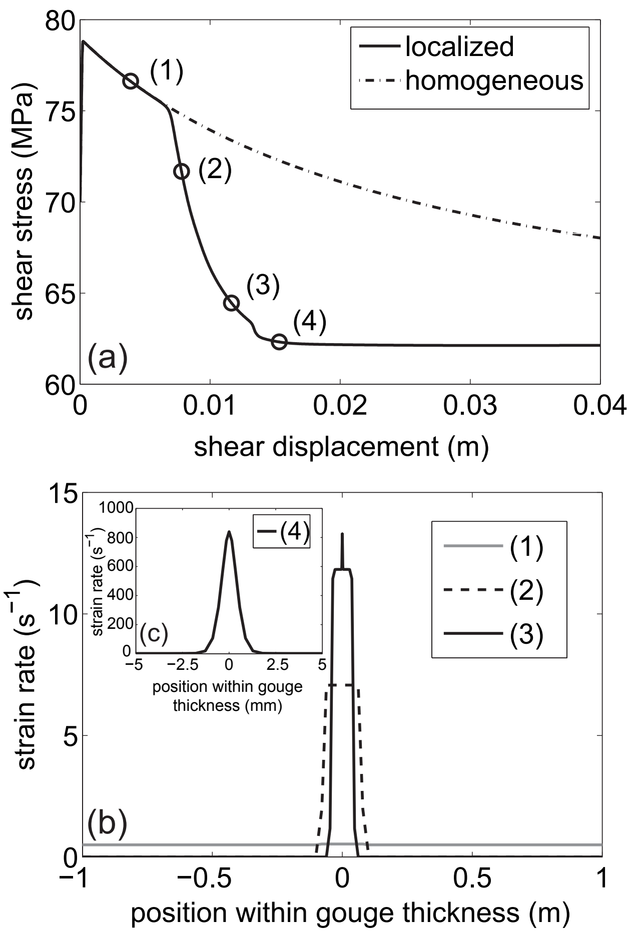

Figure 2. (a) Plot of shear stress as a function of shear displacement for the spring slider model. Comparison between dynamic strain localization and homogeneous deformation reveals that localized strain exhibits more rapid weakening. (b) Strain rate profiles for four representative shear displacements during localized strain, indicated in plot (a). (1) An initial period of broad deformation occurs before (2) strain dynamically localizes. (3) A narrower, diffusion-limited shear band develops with further shear. (c) Inset: The narrow shear band eventually accommodates all of the deformation.

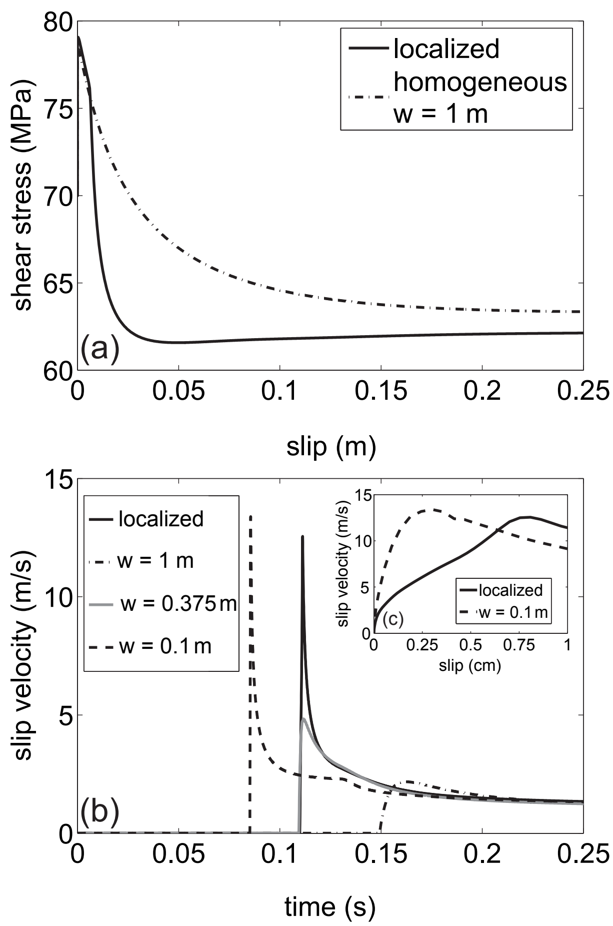

Figure 3. Dynamic rupture evolution at a point 0.35 km from the hypocenter. (a) Comparison of shear stress as a function of slip. Dynamic localization of deformation produces more rapid velocity weakening than the rupture with homogeneous strain. (b) Plot of slip rate as a function of time. The dynamic strain localization rupture is compared with a host of models with homogeneous strain. None of the values of the imposed gouge width w can match both the peak slip rate and rupture front arrival of the rupture with localized strain. (c) Inset: Slip rate as a function of slip for the localized and narrowest width homogeneous rupture. The more rapid acceleration of slip in the narrowest homogeneous rupture is distinct from the localized model.

plane slip using a spectral boundary integral method [*Perrin et al.*, 1995]. The elastodynamic equation is solved simultaneously with the friction law (Table 1).

The spectral boundary integral method can only accommodate explicit time steps. The diffusion term in the effective temperature equation imposes a time step restriction that makes implementation in a rupture code impractical. However, ignoring diffusion in the spring slider model leads to nearly the same stress response as a function of displacement. Therefore, as a first effort to incorporate localization into dynamic rupture models, we omit the diffusion term in the effective temperature evolution equation.

We consider a simple fault 2 km in length along strike, with a gouge half width of 1 m. The friction parameters in Table 1 are spatially uniform along strike and throughout the gouge width (not including the effective temperature, which we solve for dynamically). The shear stress on the fault is initially $\tau(t = 0) = 70$ MPa, except for a small patch of length 0.2 km at the center of the fault where the stress is 79 MPa to initiate rupture. We consider two different rupture scenarios, analogous to those in the spring slider section: one where shear strain localizes dynamically, and one where deformation is homogeneous. The initial effective temperature does not vary along strike, and within the gouge layer it is the same as for the corresponding spring slider model for each rupture scenario.

A plot comparing how stress weakens with slip for the two ruptures is shown in Fig. 3(a). During the initial stages of slip, the curves are indistinguishable. For the later stages of slip, shear stress weakens much more rapidly due to the dynamic instability of localization, increasing the stress drop. Homogeneous deformation and dynamic localization produce very different slip rates, as can be seen in Fig. 3(b). The rupture front arrives earlier and has a higher peak slip rate when strain localization occurs.

To illustrate the importance of the dynamic instability, we contrast our results for localized shear with two additional homogeneous ruptures of different fixed gouge widths w chosen to match particular aspects of the localized rupture. Slip velocity as a function of time is plotted at a point 0.35 km from the hypocenter for all models in Fig. 3(b). The properties that we compare are the peak slip velocity and the time at which slip initiates. The rupture front in the intermediate model ($w = 0.375$ m) matches the arrival time of the localized rupture, but the peak slip rate is smaller. For the narrowest gouge thickness ($w = 0.1$ m), we see peak slip rates similar to the localized rupture but earlier arrival. Figure 3(c) plots the slip rate as a function of slip for the homogeneous rupture with $w = 0.1$ m and the localized rupture. This clearly shows that the initial broad deformation in the localized rupture does not simply delay the rupture, but also lessens the slip acceleration during the earliest stages of slip.

5. Discussion

The main effect of dynamic strain localization in our simulations is to provide a mechanism for enhanced velocity weakening. The dynamic instability that produces localization has two effects on the shear stress: a reduced dynamic sliding stress, and a very rapid decrease in stress with slip. Allowing for enhanced weakening makes our faults more unstable – both the stress drop and the peak slip rates increase for dynamic localization relative to homogeneous slip.

The width of the narrow shear band in our simulations is determined by the diffusion length scale $\sqrt{D}$. For different driving rates, other shear band widths are possible.

Other weakening mechanisms have been discussed in the context of rapid seismic slip, including pore fluid pressurization [*Lachenbruch*, 1980], flash heating at asperity contacts [*Tullis and Goldsby*, 2003], production of a thixotropic silica gel [*Di Toro et al.*, 2004], and frictional melting [*Di Toro et al.*, 2006]. Most of these are due to (regular) thermal effects. Localization complements these, as localization occurs before significant change in the thermal temperature. Rupture models that employ both could examine how thermal weakening and shear localization interact.

The enhanced weakening effect of dynamic localization occurs regardless of the choice of the function $R(\tau)$ in Eq. (1). Other forms of $R(\tau)$ will still exhibit more rapid weakening during localized deformation. This indicates that shear localization can affect a variety of materials, and have consequences for other geophysical systems.

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