FLUID MECHANICS
Fluid Compressibility

- Consider an “ideal gas” of N particles:
  - Particles collide but do not attract/repel each other
  - Pressure is proportional to density: \( \Delta p = \left( \frac{B}{\rho_0} \right) \Delta \rho \)

- Real atoms – attract/repel depending on range
  - Particles get closer together → PE is converted into KE
  - KE leaves system (collisions with walls and surroundings)
  - Density increases → B increases even more

- In “dense” state (i.e. liquid or solid)
  - Inter-particle forces far outweigh pressure forces → large B
  - Typical values for B in Pa: gas \(10^5\) liquid \(10^9\) solid \(10^{10}\)
  - Relative to gases – liquids/solids are incompressible \(\Delta \rho \approx 0\)
Pascal's Principle

- Information about pressure changes
  - Moves through a fluid at speed of sound
  - In incompressible fluids:
    - Pressure equalizes very quickly

- Pascal's Principle
  - Incompressible fluid can “transmit” force
    - External force → pressure immediately changes in whole fluid

- Hydraulic Systems
  - Force multiplier using incompressible fluid
    - Energy conservation → large force, small distance
Gravity vs. Pressure

- Gravity pulls down on fluid (like everything else!)
  - Deeper fluid must support weight of fluid above
  - Result: pressure increases as depth increases

Newton's 2nd Law:

\[
\frac{d}{dz} p = -g (\rho (z))
\]

Compressible Fluid:

\[
\rho (z) \sim p (z) \quad \rightarrow \quad p (z) = p_0 e^{-\frac{z}{z_0}}
\]

Incompressible Fluid:

\[
\rho (z) \sim \rho_0 \quad \rightarrow \quad p (z) = p_0 + \rho_0 g (z_0 - z)
\]

Ocean pressure increases by \( \approx 1 \text{ atm} \) for every 10 meters of depth.
Archimedes' Principle and Buoyancy

• Consider a solid object submerged in fluid
  – Pressure at bottom is greater than pressure at top
  – Fluid exerts upward “buoyant force” on object

• Archimedes' Principle
  – Fluid level raises due to object
  – Thus, can determine $F_{buoyant}$

$$F_{buoyant} = \rho_{fluid} \ V_{displaced} \ g$$

• If object is less dense than fluid:
  – Equilibrium reached with $V_{displaced} < V_{object} \rightarrow$ object floats
  – If object more dense than fluid $\rightarrow F_{buoyant} < Weight \rightarrow$ sinks
Examples

• About 10% of iceberg volume is above water
  – Calculate density of iceberg relative to water
  – To “float” a human on air – estimate size of helium balloon
  – Car brakes – which way does a helium balloon inside move?

• Water has unusual thermal expansion properties
  – Expands when it freezes – water is most dense at 4º C
  – Why do lakes and ponds take months to freeze in winter?

• Barometer – measures atmospheric pressure
  – Flip fluid-filled tube into fluid-filled dish
  – Calculate height of fluid if atmospheric pressure = $p_0$
  – Why is mercury used instead of water?
Surface Tension

- Consider molecules at the surface of a liquid:
  - Intermolecular forces pull inward
  - Creating a very thin layer of higher density
  - Acts like a “loose balloon” confining the rest of the fluid

- Surface Tension – Units: (Force/Length)
  - Liquid attempts to minimize its surface area
  - External forces can cause non-minimal surface
  - “Surfactants” – reduce surface tension
  - Example: Soaps and detergents
Capillary Action

• Intermolecular forces in liquid – “cohesion”
  - Liquid molecules interacting w/ another material – “adhesion”

• In many situations: adhesion > cohesion
  - Liquid “creeps up” walls of container (creating a “meniscus”)
  - Surface tension pulls up on liquid to minimize surface area
  - Process continues until weight of fluid balances adhesion

• Capillary Action
  - Height of fluid depends on surface / volume ratio
  - Thinner tubes → higher lift
  - Example: How tall are the tallest trees?
  - Can atmospheric pressure lift water this high?
Fluid Flow

- Fluids are commonly moved through pipes or tubes
  - Using pressure differences to push the fluid
  - Examples: hoses, drinking straw, blood vessels

- Width of pipes/tubes can change
  - Example: garden hose attachment
  - Water speed is increased

- Equation of continuity
  - Compressible: $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$
  - Incompressible: $A_1 v_1 = A_2 v_2$
Bernoulli Equation

• Typical forms of energy in fluids:
  - “Elastic” PE due to pressure gradient
  - KE due to bulk motion of the fluid
  - Gravitational PE due to fluid being at a non-zero height

• Bernoulli Equation – conservation of “energy density”

\[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{Constant} \quad (\text{everywhere along a “streamline”}) \]

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

• Examples:
  - Reproduce pressure/depth equation from Bernoulli's equation
  - Area of pipe decreases → what happens to water's momentum?
Viscosity

- **Ideal fluids** – no resistance to “shear”
  - Intermolecular forces in **real** fluids → “friction” inside fluid
  - Felt by neighboring “layers” of fluid moving at **different** speeds

- **Viscosity**
  - Resistance to “sheared flow” in fluid
  - **Examples**: honey, syrup

- **Adhesion to pipe walls** → sheared flow
  - Viscosity **resists** flow through pipes
  - Need a pressure **difference** to push fluid through pipe

- Poiseuille flow: \[ \Delta p = \left( \frac{8 \eta L}{R^2} \right) v_{\text{avg}} \] (thin pipe → large \( \Delta p \))
Turbulence

• Consider 2 fluids – one moving through the other
  - At low speed – fluid shear at edges is small
  - So viscous forces are small
  - Flowing fluid keeps its shape – laminar flow

• Above some critical speed:
  - Viscous forces become large enough to deform flowing fluid
  - Known as turbulent flow → fluid shape becomes chaotic
  - Analogy: front brake on bicycle – low speed ok, high speed fall

• Turbulence can exert quasi-periodic forces
  - Which can excite resonances in objects
Aerodynamics / Hydrodynamics

- **Applications** of Bernoulli / viscosity / turbulence
  - Diverting flows of air / water to exert or minimize forces
  - Governing equations are exceptionally difficult to solve
  - **Common practice:** Computational physics and “wind tunnels”