Exercise 24.2

The plates of a parallel-plate capacitor have a separation of d, and each has an area of A. Each plate carries a charge of magnitude Q. The plates are in vacuum.

Part A

What is the capacitance?

The equation for the capacitance of a parallel plate capacitor is:

$$C = \epsilon_0 \frac{A}{d} \quad \text{(cf. p. 817, Young and Freedman)}$$

Part B

What is the potential difference between the plates?

$$C \equiv \frac{Q}{V} \qquad \text{(this is the definition of capacitance)}$$

Thus $V = \frac{Q}{C}$

Part C

What is the magnitude of the electric field between the plates?

For a constant electric field E, $V = Ed$ (this comes from $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l} = \int_0^d E\,dx = Ed$)

Thus $E = \frac{V}{d}$

Problem 24.53

Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit, the flash lasts for a time interval of Δt with an average light power output of P_{ave}.

Part A

If the conversion of electrical energy to light has an efficiency of e % (the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash?

The required energy for a flash is $U_{flash} = P_{ave} \Delta t$. It is related to the energy stored in the capacitor by the efficiency: $U_{flash} = \frac{e}{100} U_{capacitor}$.

Solving for $U_{capacitor}$:

$$U_{flash} = \frac{e}{100} U_{capacitor}$$

$$U_{capacitor} = \frac{100}{e} U_{flash}$$

$$U_{capacitor} = \frac{100}{u} P_{ave} \Delta t$$

Part B

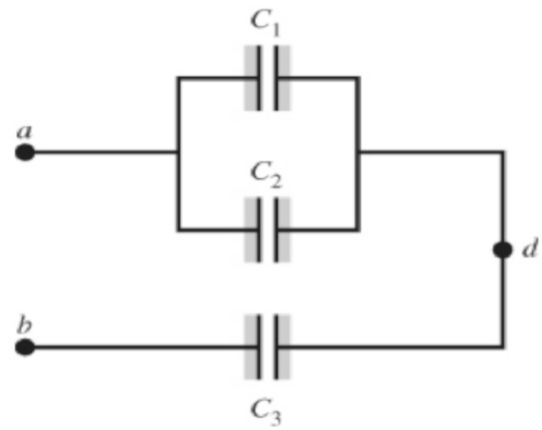
The capacitor has a potential difference between its plates of V when the stored energy equals the value calculated in part A. What is the capacitance?

The energy stored in a capacitor is $U = \frac{1}{2} C V^2$ (cf. p. 824)

Thus $C = \frac{2U}{V^2}$

A Simple Network of Capacitors

In the figure are shown three capacitors with capacitances C_1 , C_2 , C_3 . The capacitor network is connected to an applied potential V_{ab}. After the charges on the capacitors have reached their final values, the charge on the second capacitor is Q_2 .



Part A

What is the charge Q_1 on capacitor C_1?

The voltage over both capacitors C_1 and C_2 is V_{ad}. Thus $Q_1 = V_{ad} C_1$ and $Q_2 = V_{ad} C_2$. We can write $V_{ad} = \frac{Q_2}{C_2}$ so that:

$$Q_1 = V_{ad} C_1 = \left(\frac{Q_2}{C_2}\right) C_1$$

Part B

What is the charge on capacitor C_3?

Because capacitors 1 and 2 are in series with capacitor 3, the charge on capacitor 3 must be equal to the total charge on 1 and 2. That is,

$$Q_3 = Q_1 + Q_2$$

Part C

What is the applied voltage, V_{ab}?

The total voltage is $V_{ab} = V_{ad} + V_{db}$, where $V_{ad} = \frac{Q_3}{C_1 + C_2}$ and $V_{db} = \frac{Q_3}{C_3}$. This is because the region between a and d is equivalent to a single capacitor with charge $Q_1 + Q_2 = Q_3$ and capacitance $C_1 + C_2$.

Thus $V_{ab} = \frac{Q_3}{C_1 + C_2} + \frac{Q_3}{C_3}$

Energy of a Capacitor in the Presence of a Dielectric

A dielectric-filled parallel-plate capacitor has plate area A, plate separation d and dielectric constant k. The capacitor is connected to a battery that creates a constant voltage V. Throughout the problem, use $\epsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$.

Part A

Find the energy U_1 of the dielectric-filled capacitor.

The energy stored in any capacitor is $U = \frac{1}{2}CV^2$. The capacitance of a parallel plate capacitor with a dielectric is $C = k\epsilon_0\frac{A}{d}$ (cf. p. 830 of the book).

Thus $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(k\epsilon_0\frac{A}{d}\right)V^2$

Part B

The dielectric plate is now slowly pulled out of the capacitor, which remains connected to the battery. Find the energy U_2 of the capacitor at the moment when the capacitor is half-filled with the dielectric.

The situation is equivalent to having two capacitors in parallel, one with $k' = 1$ (air) and one with $k' = k$. Each has area $\frac{A}{2}$.

$C_1 = \epsilon_0\left(\frac{A}{2}\right)\frac{1}{d} \qquad C_2 = k\epsilon_0\left(\frac{A}{2}\right)\frac{1}{d}$

$C_{eq} = C_1 + C_2 = \frac{1}{2}(1+k)\,\epsilon_0\frac{A}{d}$

Thus $U_2 = \frac{1}{2}C_{eq}V^2 = \frac{1}{4}(1+k)\epsilon_0\frac{A}{d}V^2$

Part C

The capacitor is now disconnected from the battery, and the dielectric plate is slowly removed the rest of the way out of the capacitor. Find the new energy of the capacitor, U_3.

The charge on the plates remains constant through this process.

Initially, $U_2 = \frac{1}{2}QV$ so that $Q = \frac{2U_2}{V}$

The final energy U_3 can be written solely in terms of the final capacitance and charge:

$U_3 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\left(\frac{2U_2}{V}\right)^2\frac{1}{C}$

The final capacitance is just the capacitance of a dielectric-free parallel plate capacitor:

$C = \epsilon_0\frac{A}{d}$

Thus $U_3 = \frac{1}{2}\left(\frac{2U_2}{V}\right)^2\left(\frac{d}{\epsilon_0 A}\right) = \frac{2U_2^2 d}{\epsilon_0 A V^2}$

Part D

In the process of removing the remaining portion of the dielectric from the disconnected capacitor, how much work W is done by the external agent acting on the dielectric?

By energy conservation the work done is equal to the difference in energy between the initial and final states of the capacitor.

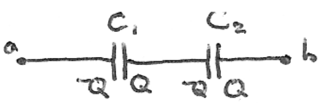
That is, since $U_3 = U_2 + W$, $W = U_3 - U_2$

Exercise 24.16

Two capacitors are connected in series. Let C_1 be the capacitance of first capacitor, C_2 the capacitance of the second capacitor, and V_{ab} be the potential difference across the system.

Parts A and B

Calculate the charge on each capacitor.



Since the section in between the capacitors is essentially insulated, the total charge must equal zero

Thus $Q_1 = Q_2 = Q$

The voltage drops V_1 and V_2 over the capacitors must sum to V_{ab}.

$$V_{ab} = V_1 + V_2$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} \quad \left(\text{using } V = \frac{Q}{C} \text{ for each capacitor}\right)$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad \text{so} \quad Q = \frac{V_{ab}}{\frac{1}{C_1} + \frac{1}{C_2}} \text{ for both capacitors}$$

Parts C and D

Calculate the potential difference across each capacitor.

Using Q from parts A and B, the voltages are just:

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

Exercise 24.25

A C F, parallel-plate, air capacitor has a plate separation of d and is charged to a potential difference of V.

Calculate the energy density in the region between the plates.

Energy density is defined as energy per unit volume.

Thus the energy density is the total energy in the field between the plates, $U = \frac{1}{2}CV^2$, divided by the total volume between the plates, Ad. That is, $u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$. The area of the plates is given by $C = \frac{\epsilon_0 A}{d}$ so that $A = \frac{Cd}{\epsilon_0}$.

Thus, $u = \frac{\frac{1}{2}CV^2}{\left(\frac{Cd}{\epsilon_0}\right)d} = \frac{\epsilon_0}{2}\frac{V^2}{d^2}$

Capacitors with Partial Dielectrics

Consider two parallel-plate capacitors identical in shape, one aligned so that the plates are horizontal (Intro 1 figure) , and the other with the plates vertical (Intro 2 figure) .

The horizontal capacitor is filled halfway with a material that has dielectric constant K. What fraction f of the area of the vertical capacitor should be filled (as shown in the figure) with the same dielectric so that the two capacitors have equal capacitance?

The horizontal capacitor is equivalent to two parallel capacitors with separation distance $\frac{d}{2}$ connected in series, one with dielectric constant $k=1$ (air) and the other with $k=K$.

$C_1 = \epsilon_0 \frac{A}{(d/2)}$ and $C_2 = K\epsilon_0 \frac{A}{(d/2)}$

The equivalent capacitance is:

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{1}{\frac{2\epsilon_0 A}{d}} + \frac{1}{\frac{2K\epsilon_0 A}{d}}} = \frac{2\epsilon_0 A}{d}\frac{1}{1+\frac{1}{K}} = \frac{2\epsilon_0 A}{d}\frac{K}{K+1}$$

The vertical capacitor is equivalent to two parallel capacitors with separation distance d connected in parallel, one with dielectric constant $k=1$ and area $(1-f)A$, and the other with $k=K$ and area fA.

$C_1' = \epsilon_0 \frac{[(1-f)A]}{d}$ and $C_2' = K\epsilon_0 \frac{(fA)}{d}$

The equivalent capacitance is:

$$C_{eq}' = C_1' + C_2' = \epsilon_0 \frac{(1-f)A}{d} + K\epsilon_0 \frac{fA}{d}$$

$$= \frac{\epsilon_0 A}{d}(1-f+Kf)$$

We want to find the value of f such that $C_{eq} = C_{eq}'$:

$$\frac{2\epsilon_0 A}{d}\frac{K}{K+1} = \frac{\epsilon_0 A}{d}(1-f+Kf)$$

$$\frac{2K}{K+1} = 1 + f(K-1)$$

$$f(K-1) = \frac{2K}{K+1} - 1$$

Thus $f = \frac{\frac{2K}{K+1} - 1}{K-1}$