APS March Meeting 2007

50 Years of BCS Theory

“A Family Tree”

Ancestors
BCS
Descendants

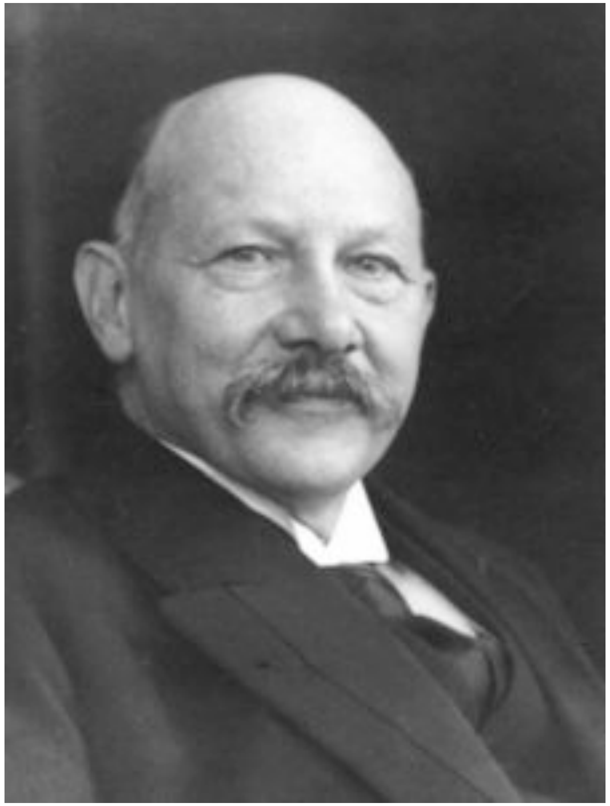
D. Scalapino: Ancestors and BCS

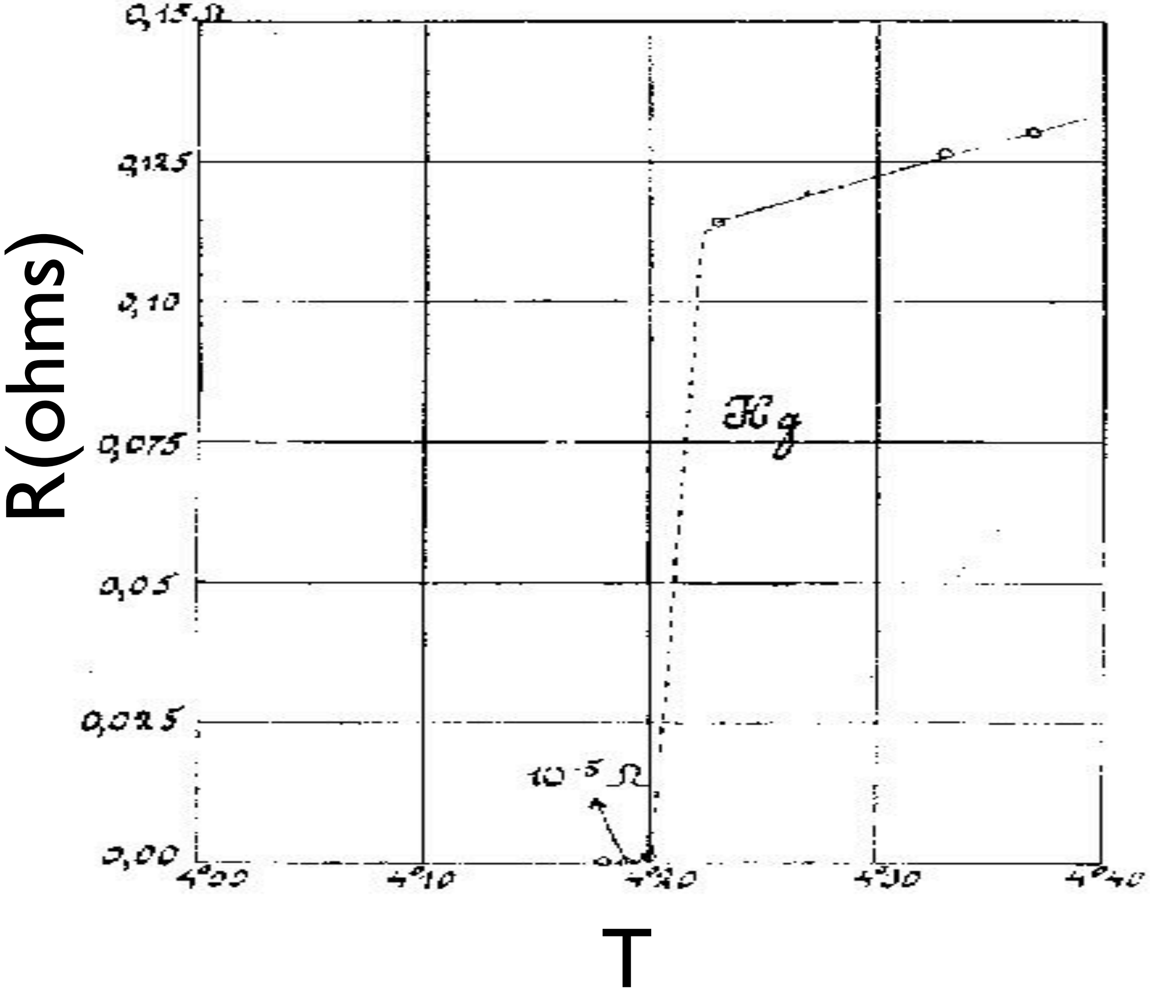
J. Rowell : A “tunneling” branch of the family

G. Baym: From Atoms and Nuclei to the Cosmos

Supraconductivity

1911 H. Kamerlingh Onnes ʻ(Gilles Holst) finds a sudden drop in the resistance of Hg at ~ 4.2K.

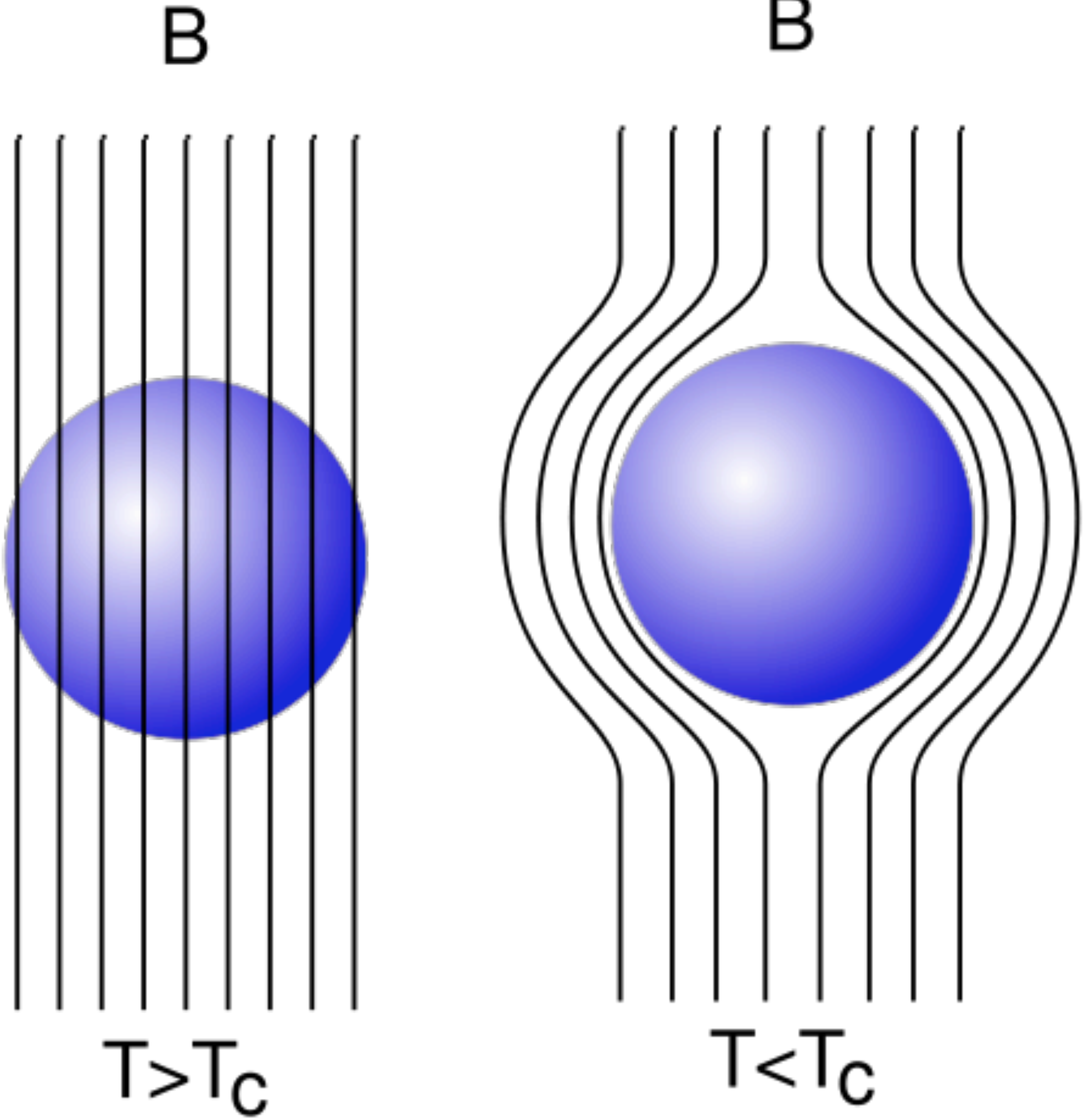




1933 Meissner and Ochsenfeld discover that superconductors are perfect diamagnets --flux expulsion

Walther Meissner
(1882-1974)

Robert Ochsenfeld 1901 - 1993



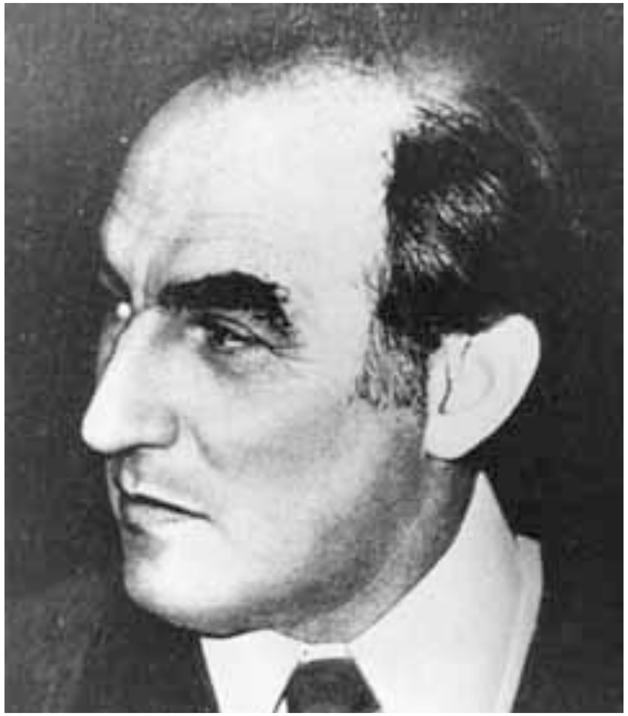
Phenomenology

- 1934 Casimir and Gorter 's two-fluid phenomenological model of thermodynamic properties.
- 1934 Heinz and Fritz London's phenomenological electrodynamics. F. London's suggestion of the rigidity of the wave function.
- 1948 Fritz London, "Quantum mechanics on a macroscopic scale, long range order in momentum."

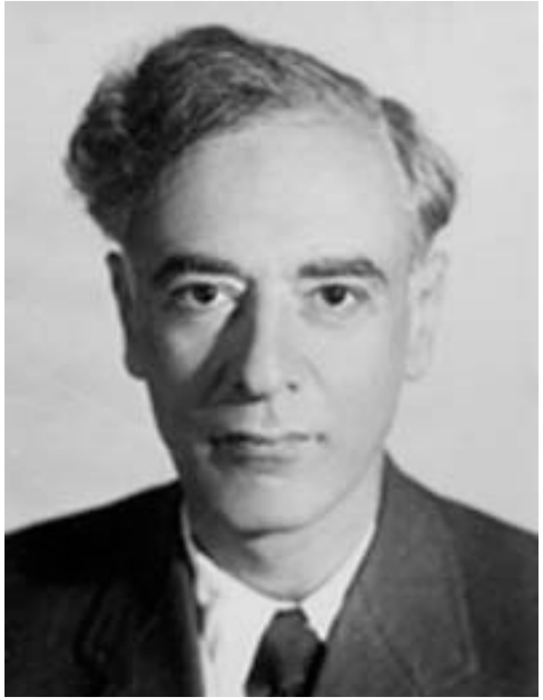
Fritz London (1900-1954)

1950 Ginzburg-Landau Theory

$$f(x) = \frac{n^*}{2m^*}|\frac{\hbar}{i}\nabla\Psi(x) + \frac{e^*}{c}\mathbf{A}(x)\Psi(x)|^2$$
$$+\alpha|\Psi(x)|^2 + \frac{\beta}{2}|\Psi(x)|^4$$



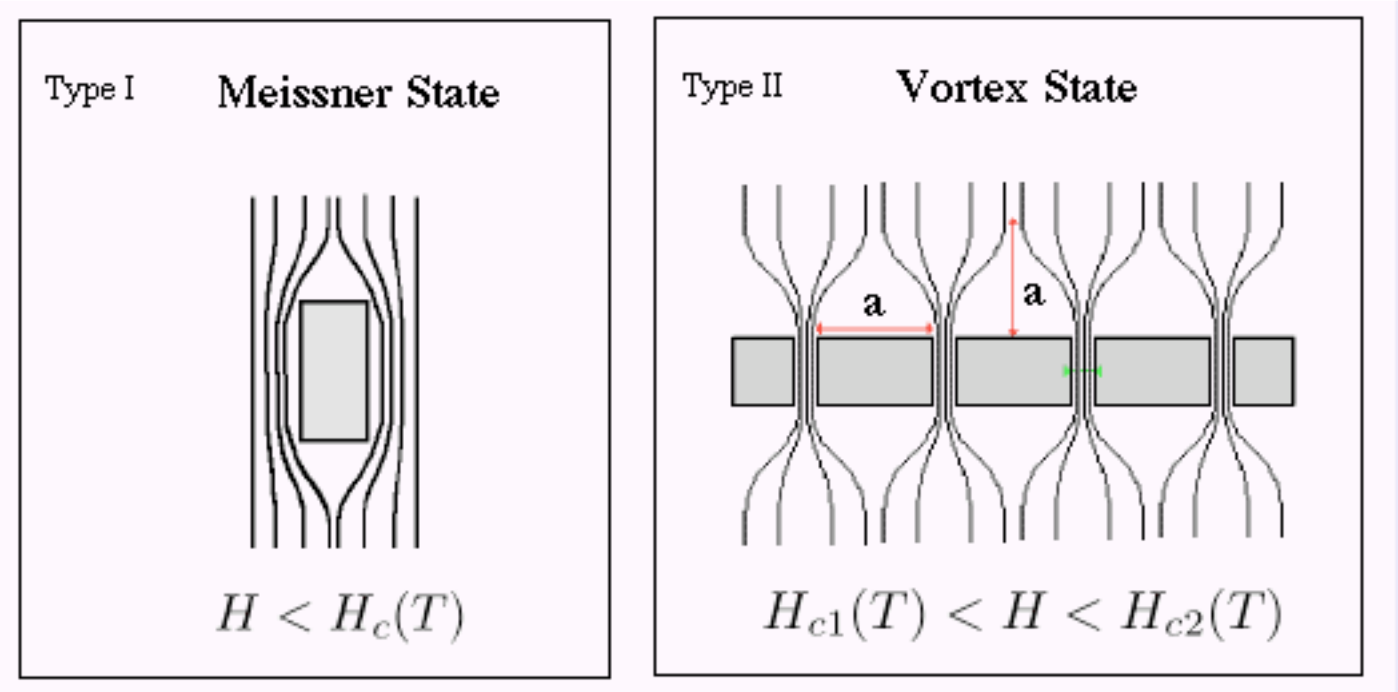
V. Ginzburg



L. Landau

1957 Type II Superconductivity

Aleksei Abrikosov



But the question remained:
“How does it work?”

R.P. Feynman ,1956 Seattle Conference

But the question remained: “How does it work?”

A long list of the leading theoretical physicists in the world had taken up the challenge of developing a microscopic theory of superconductivity.

A.Einstein,“Theoretische Bemerkungen zur Supraleitung der Metalle”

Gedenkboek Kamerlingh Onnes, p.435 (1922)

translated by B. Schmekel cond-mat/050731

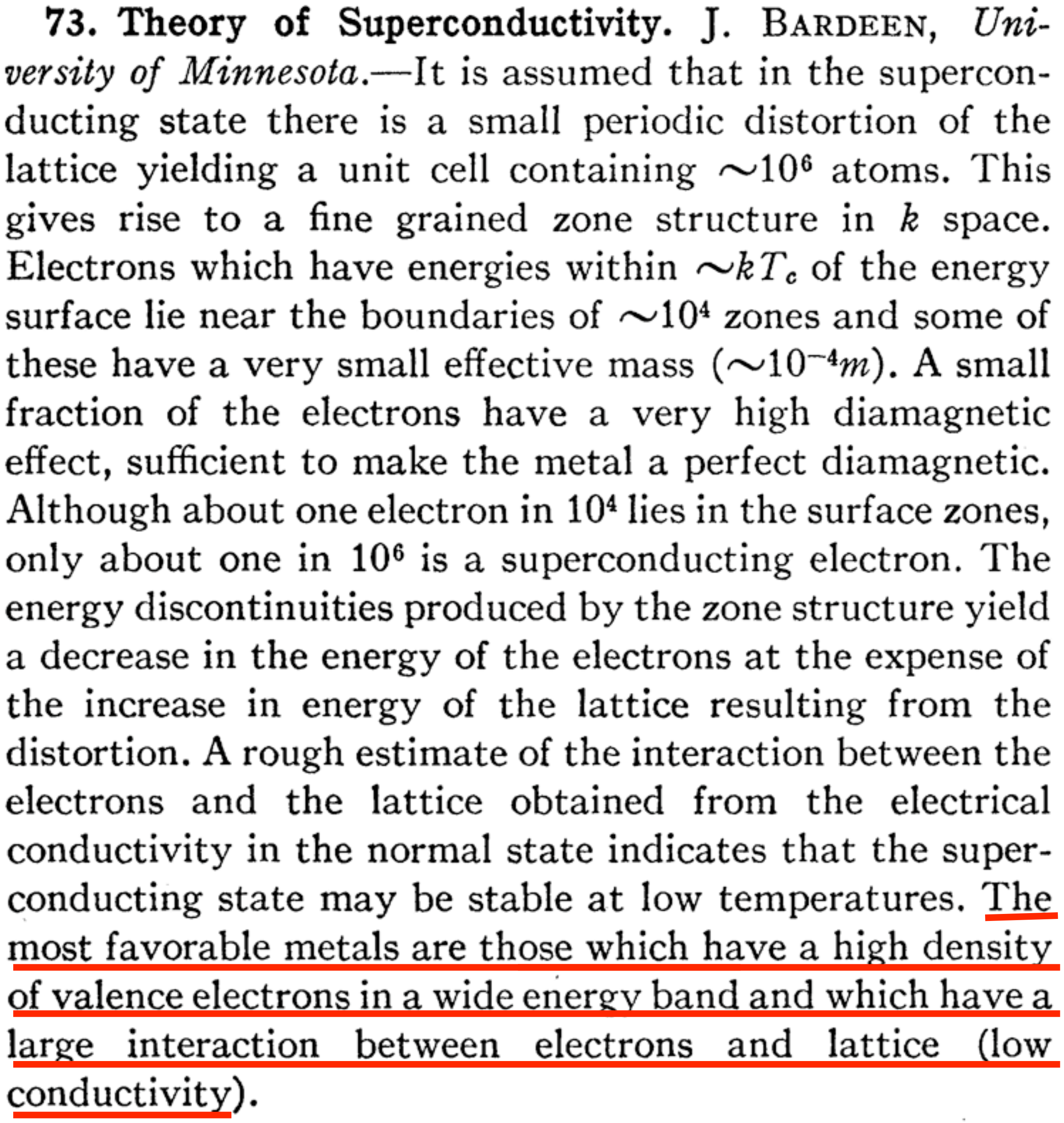
“...metallic conduction is caused by atoms exchanging their peripheral electrons. It seems unavoidable that supercurrents are carried by closed chains of molecules”

“Given our ignorance of quantum mechanics of composite systems, we are far away from being able to convert these vague ideas into a theory.”

Felix Bloch
(1905-1983)

Felix Bloch is said to have joked that ”superconductivity is impossible”.

Washington DC APS Meeting
May 1-3,1941



73. Theory of Superconductivity. J. Bardeen, *University of Minnesota.*—It is assumed that in the superconducting state there is a small periodic distortion of the lattice yielding a unit cell containing $\sim 10^6$ atoms. This gives rise to a fine grained zone structure in k space. Electrons which have energies within $\sim kT_c$ of the energy surface lie near the boundaries of $\sim 10^4$ zones and some of these have a very small effective mass ($\sim 10^{-4}m$). A small fraction of the electrons have a very high diamagnetic effect, sufficient to make the metal a perfect diamagnetic. Although about one electron in 10^4 lies in the surface zones, only about one in 10^6 is a superconducting electron. The energy discontinuities produced by the zone structure yield a decrease in the energy of the electrons at the expense of the increase in energy of the lattice resulting from the distortion. A rough estimate of the interaction between the electrons and the lattice obtained from the electrical conductivity in the normal state indicates that the superconducting state may be stable at low temperatures. The most favorable metals are those which have a high density of valence electrons in a wide energy band and which have a large interaction between electrons and lattice (low conductivity).

Bose-Einstein Condensation of Trapped Electron Pairs. Phase Separation and Superconductivity of Metal-Ammonia Solutions

RICHARD A. OGG, JR.
Department of Chemistry, Stanford University, California

March 2, 1946

The probable explanation of the above phenomena is to be found in the behavior of trapped electron pairs, recently demonstrated[3] to be a stable constituent of fairly dilute metal-ammonia solutions. . . . Because of their zero angular momentum, such pairs must obey Bose-Einstein statistics. If the effective mass does not exceed twice the electron mass by an extremely large factor, then the calculated degeneration temperature[5] at the concentrations in question is relatively high—of the order of a few hundred degrees absolute. It is postulated that the liquid-

In a superconductor,the exchange interaction associated with the Coulomb field leads to a spatial ordering of the electrons and the ordering of metastable current treads.

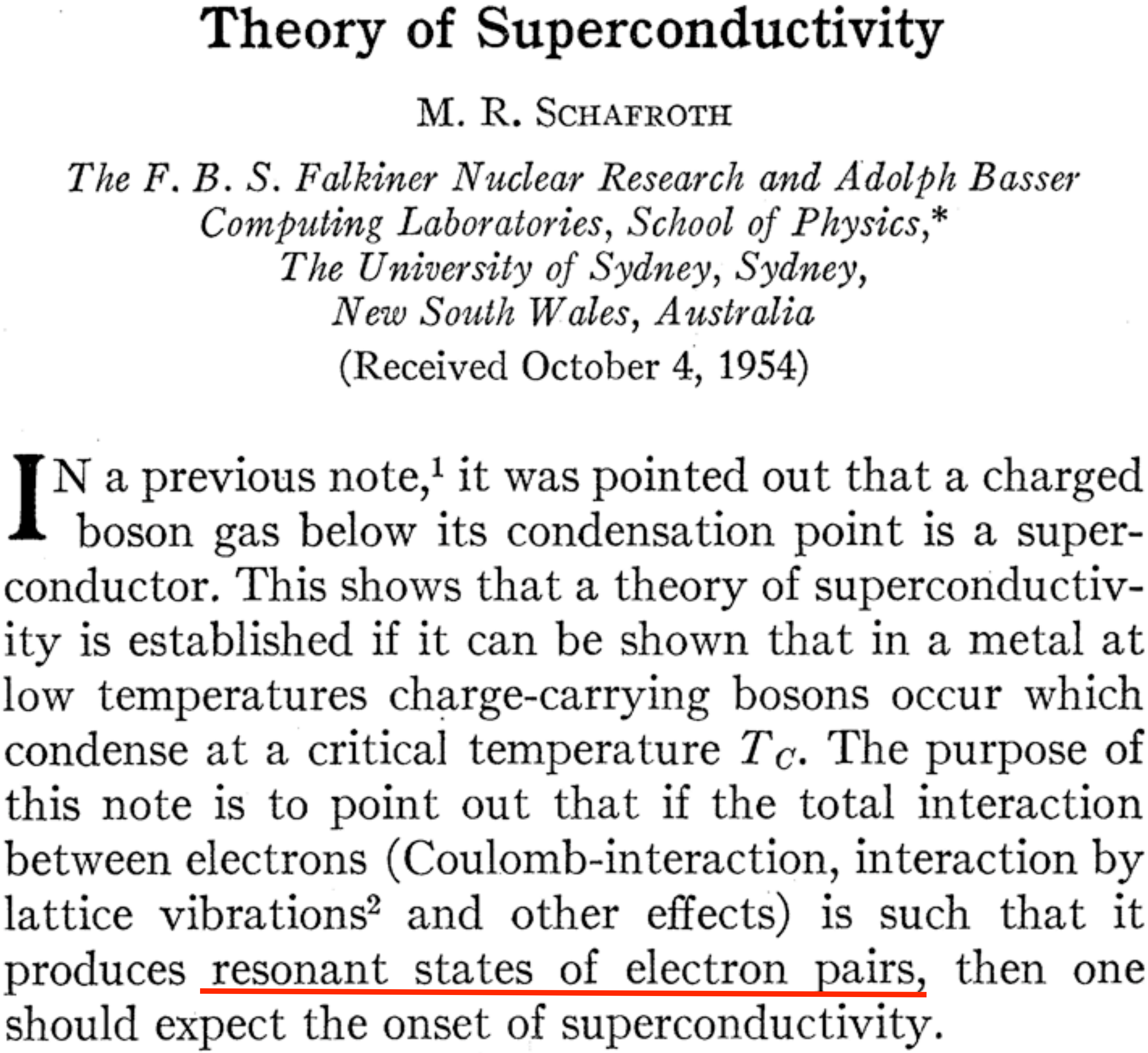
W. Heisenberg, Zeits. f. Naturkunde 2a, 185 (1947)

“In contrast to a recent attempt of Heisenberg, superconductivity is characterized not as a state of electronic lattice order in ordinary space but rather as a kind of condensed state in momentum space.”

“it is most probably the exchange interaction associated with the Coulomb field which is responsible for this condensation in momentum space.”

F. London, Phys. Rev. 74,
562 (1948)

Frohlich’s and Bardeen’s electron-phonon self-energy calculations ~ 1950



Theory of Superconductivity

M. R. Schafroth

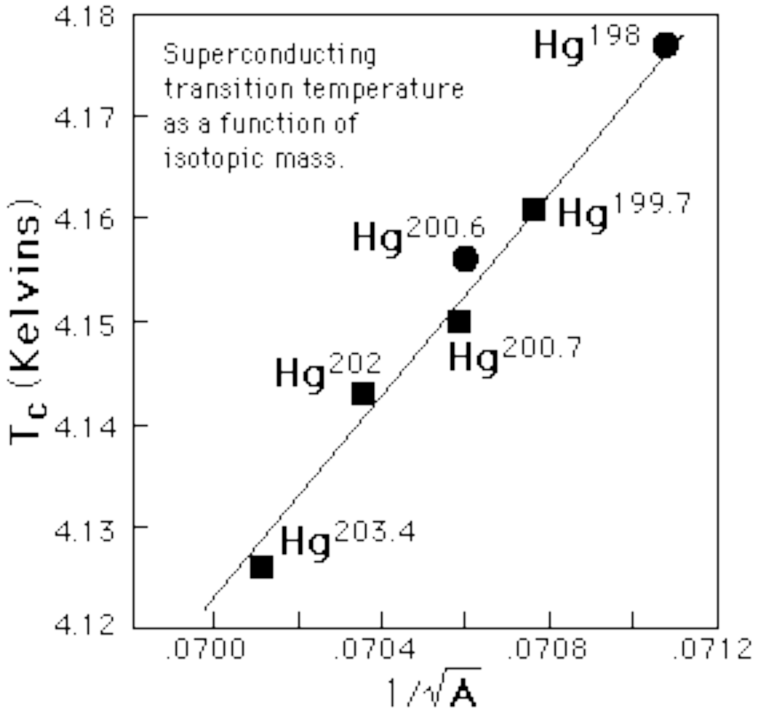
The F. B. S. Falkiner Nuclear Research and Adolph Basser
*Computing Laboratories, School of Physics,**
The University of Sydney, Sydney,
New South Wales, Australia

(Received October 4, 1954)

In a previous note,[1] it was pointed out that a charged boson gas below its condensation point is a superconductor. This shows that a theory of superconductivity is established if it can be shown that in a metal at low temperatures charge-carrying bosons occur which condense at a critical temperature T_C. The purpose of this note is to point out that if the total interaction between electrons (Coulomb-interaction, interaction by lattice vibrations[2] and other effects) is such that it produces resonant states of electron pairs, then one should expect the onset of superconductivity.

Important experimental results were coming out during this time.

An important clue “The Isotope Effect”

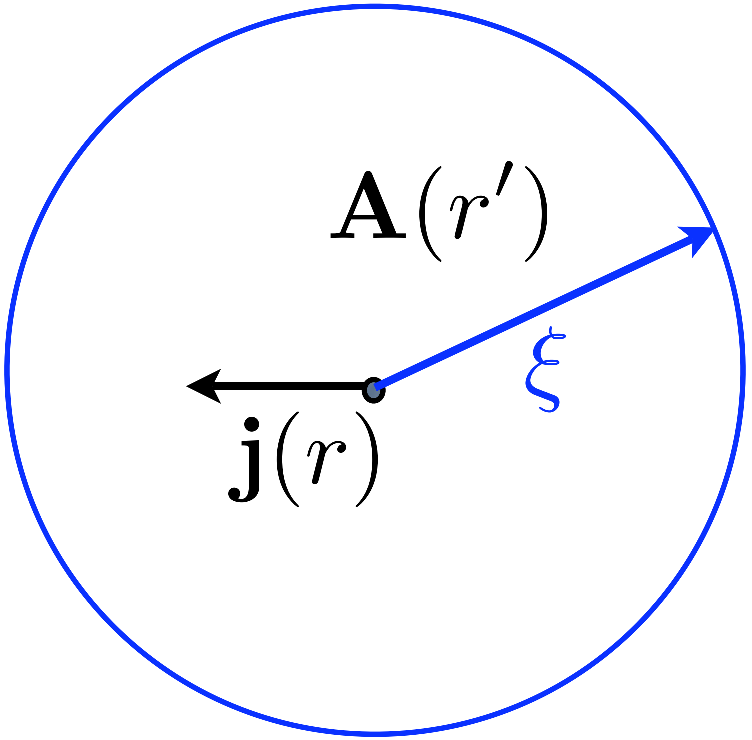


$$T_c \sim M^{-\alpha}$$

$$\alpha \approx 0.5$$

- 1950, E.Maxwell and Reynolds, Serin, Wright and Nesbitt

1953 Pippard coherence length



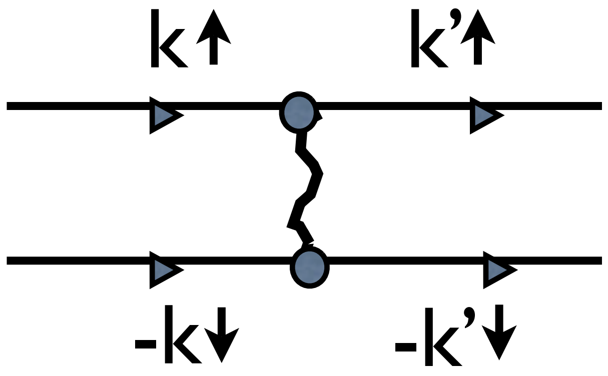
$$\xi \approx \xi_0 = \hbar v_f / \pi \Delta$$

Brian Pippard

The Electron-Phonon Interaction

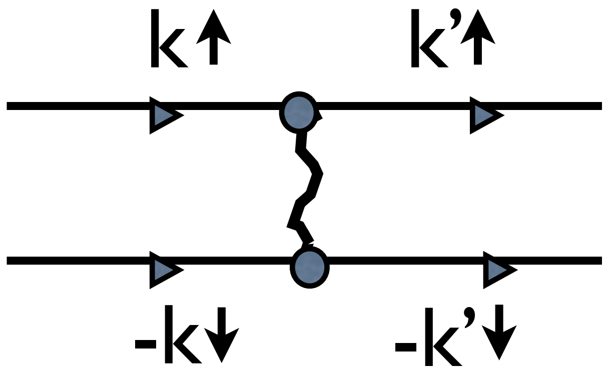
- 1950 Frohlich electron-phonon model
- 1954 Nakajima included the screened Coulomb interaction
- 1955 Bardeen and Pines effective screened electron-phonon and Coulomb interaction--an attractive interaction for electrons near the fermi surface.

Electron-phonon interaction



$$V_{el-ph}(k',k) = \frac{2|M(k',k)|^2\omega(k'-k)}{(\epsilon(k')-\epsilon(k))^2-\omega^2(k'-k)+i\delta}$$

Electron-phonon interaction



$$V_{el-ph}(k',k) = \frac{2|M(k',k)|^2\omega(k'-k)}{(\epsilon(k')-\epsilon(k))^2-\omega^2(k'-k)+i\delta}$$

$$\sim -\frac{2|M(k',k)|^2}{\omega(k'-k)}$$ attractive

for $|\epsilon(k')-\epsilon(k)| < \omega(k'-k)$

R.P. Feynman , International Congress on Theoretical Physics, Seattle,Sept. 21,1956

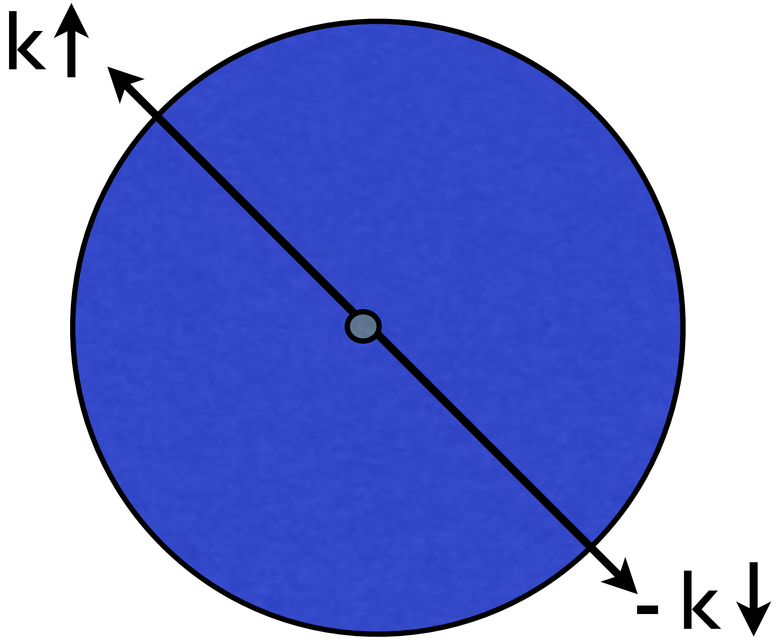
"The only reason that we cannot do this problem of superconductivity is that we haven't got enough imagination."

An answer was about to be given

- 1956 L. Cooper "Bound Electron Pairs in a Degenerate Fermi Gas"
- 1956 J. Bardeen, L.N. Cooper, J.R.Schrieffer "Microscopic Theory of Superconductivity"
- 1957 J.Bardeen, L.N. Cooper, J.R.Schrieffer "Theory of Superconductivity"

- 1956 L. Cooper “Bound Electron Pairs in a Degenerate Fermi Gas”

Two electrons outside a frozen fermi sea, interacting through an arbitrarly weak attractive force, will bind.



$$\psi(r_1, r_2) = \sum_{\mathbf{k} > \mathbf{k}_f} a_{\mathbf{k}} e^{i(\mathbf{r}_1 - \mathbf{r}_2)*\mathbf{k}} (\alpha(1)\beta(2) - \alpha(2)\beta(1))$$

$$\Delta E_B \sim \omega_c e^{-\frac{1}{N(0)|V|}}$$

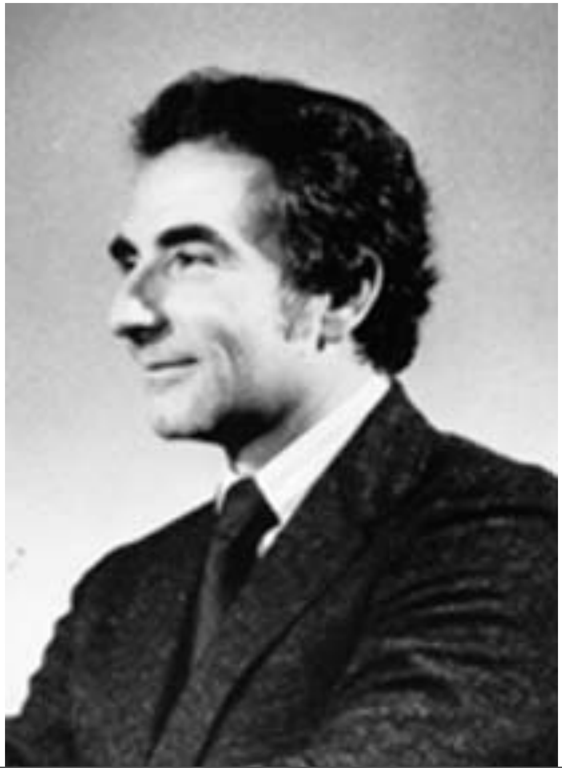
PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

A theory of superconductivity is presented, based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference between the electrons states involved is less than the phonon energy, $\hbar\omega$. It is favorable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy than the normal state by amount proportional to an average $(\hbar\omega)^2$, consistent with the isotope effect. A mutually orthogonal set of excited states in one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch states and by using the rest to form a linear combination of virtual pair configurations. The theory yields a second-order phase transition and a Meissner effect in the form suggested by Pippard. Calculated values of specific heats and penetration depths and their temperature variation are in good agreement with experiment. There is an energy gap for individual-particle excitations which decreases from about $3.5kT_c$ at $T=0°\text{K}$ to zero at T_c. Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.



From the BCS abstract:

"the interaction between electrons resulting from the virtual exchange of phonons is attractive when the energy difference between the electron states involved is less than the phonon energy"

"It is favorable to form a superconducting phase when this attractive interaction dominates the screened Coulomb interaction."

“It is favorable to form a superconducting phase when this attractive interaction dominates the screened Coulomb interaction.”

$$< \frac{-2|M_\kappa|^2}{\omega_\kappa} + \frac{4\pi e^2}{\kappa^2} > \; < 0$$

“The ground state of a superconductor is formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs.”

“The ground state of a superconductor is formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs.”

$$\Psi_{BCS} = \Pi_k[(1-h_k)^{1/2} + h_k^{1/2} b_k^*]\Phi_0$$

$$= \Pi_k[u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger]|0>$$

Schrieffer’s anzatz

BCS Quasi-particles

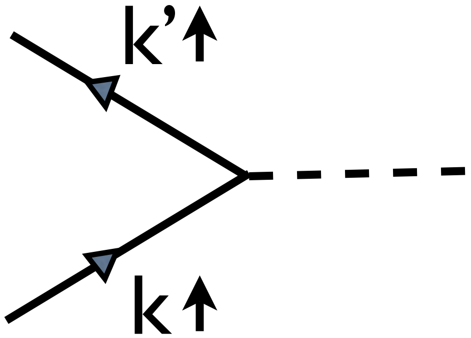
$$\gamma^\dagger_{k\uparrow} = u_k c^\dagger_{k\uparrow} - v_k c_{-k\downarrow} \qquad E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$$

BCS Quasi-particles

$$\gamma^\dagger_{k\uparrow} = u_k c^\dagger_{k\uparrow} - v_k c_{-k\downarrow} \qquad E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$$

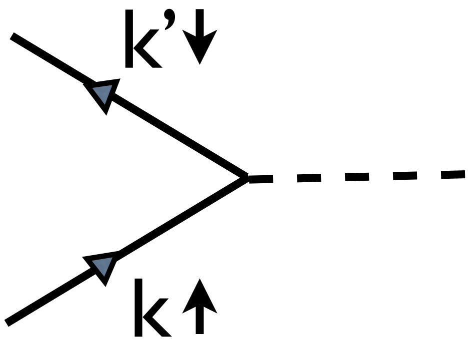
Coherence factors

ultrasonic attenuation



$$(1 - \frac{\Delta_k \Delta_{k'}}{E_k E_{k'}})$$

nuclear spin lattice
relaxation rate



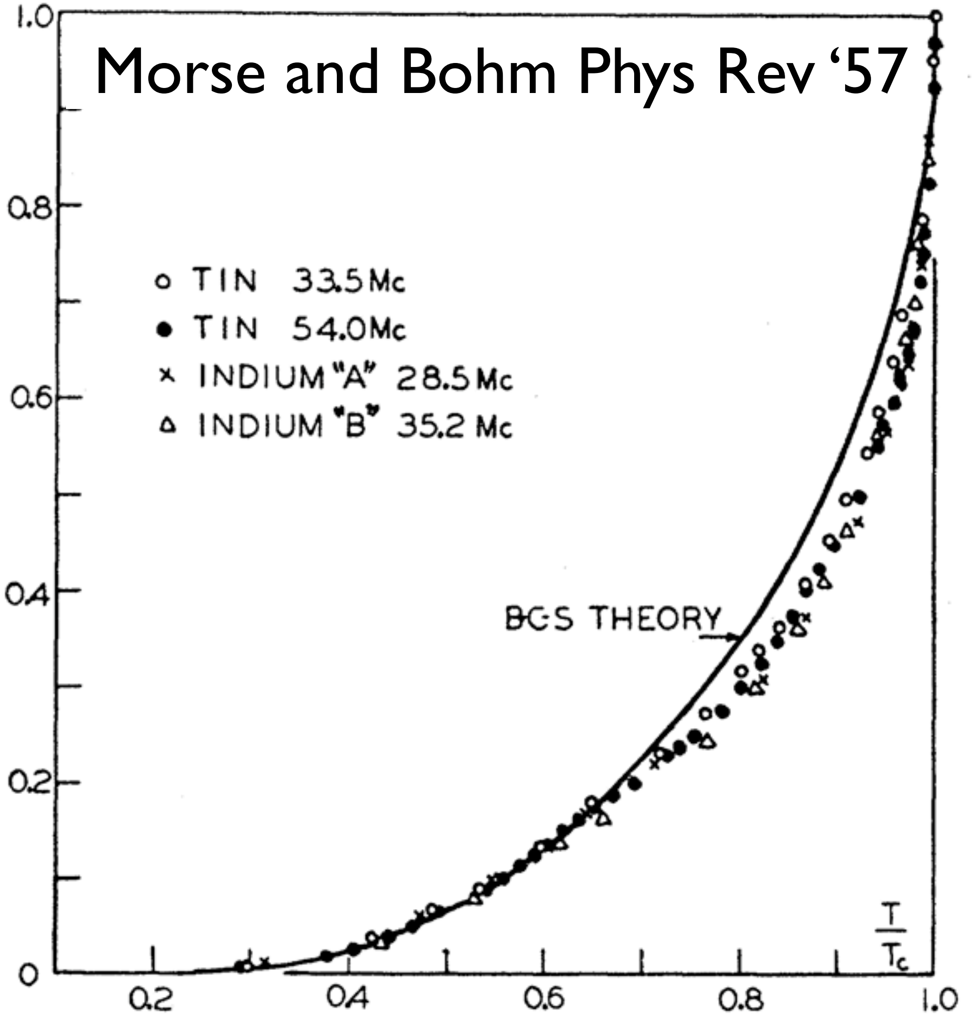
$$(1 + \frac{\Delta_k \Delta_{k'}}{E_k E_{k'}})$$

The coherence factors

ultrasonic attenuation

$$(1-\frac{\Delta_k\Delta_{k'}}{E_kE_{k'}})\sim 0$$

$\alpha(T)/\alpha(T_c)$



Morse and Bohm Phys Rev '57

FIG. 2. Measured values of α_s/α_n compared with the theoretical variation of Bardeen, Cooper, and Schrieffer assuming $\epsilon_0(0) = 1.75kT_c$. For tin $T_c = 3.71$ °K and for indium $T_c = 3.40$ °K.

The coherence factors

Hebel and Slichter, Phys Rev ’57 and ‘59

nuclear spin-lattice relaxation

$$(1+\frac{\Delta_k\Delta_{k'}}{E_kE_{k'}}) \sim 2$$

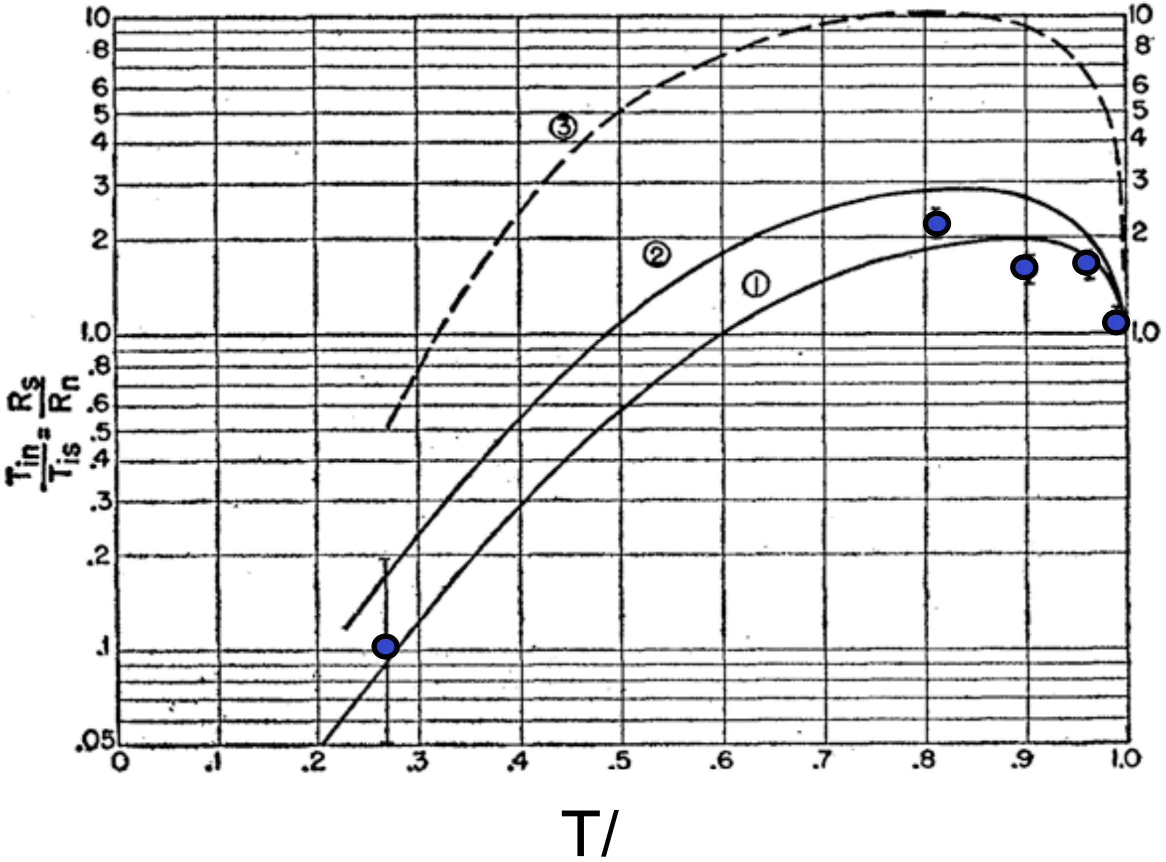


FIG. 3. Relaxation rate in a superconductor, R_s, relative to the zero-field value extrapolated from the normal state, $R_n(0)$, *versus* reduced temperature T/ The three theoretical curves using BCS theory are described in the text.

BCS THEORY

- The BCS theory yielded a second-order phase transition
- a temperature dependent energy gap $\Delta(T)$
- specific heat C(T), critical field $H_c(T)$ and the penetration depth $\lambda(T)$
- the Meissner effect (in a transverse field)
- matrix element coherence factors for calculating transport properties

Conclusion

“Although our calculations are based on a rather idealized model, they give a good account of the equilibrium properties of superconductors. ... This quantitative agreement as well as the fact that we can account for the main features of superconductivity is convincing evidence that our model is essential correct.”

1959 L.Gor'kov showed how the Ginzburg-Landau equations followed from the BCS theory.

$$\Psi(x) \sim \Delta(x) \qquad e^* = 2e$$

1960 I. Giaever single-particle electron tunneling measurement of the gap.

1960 G. M. Eliashberg theory of strong-coupling superconductors

1962 B. Josephson

$$I = I_1 sin(\phi) \qquad \dot{\phi} = 2eV/\hbar$$

The Nobel Prize in Physics 1972

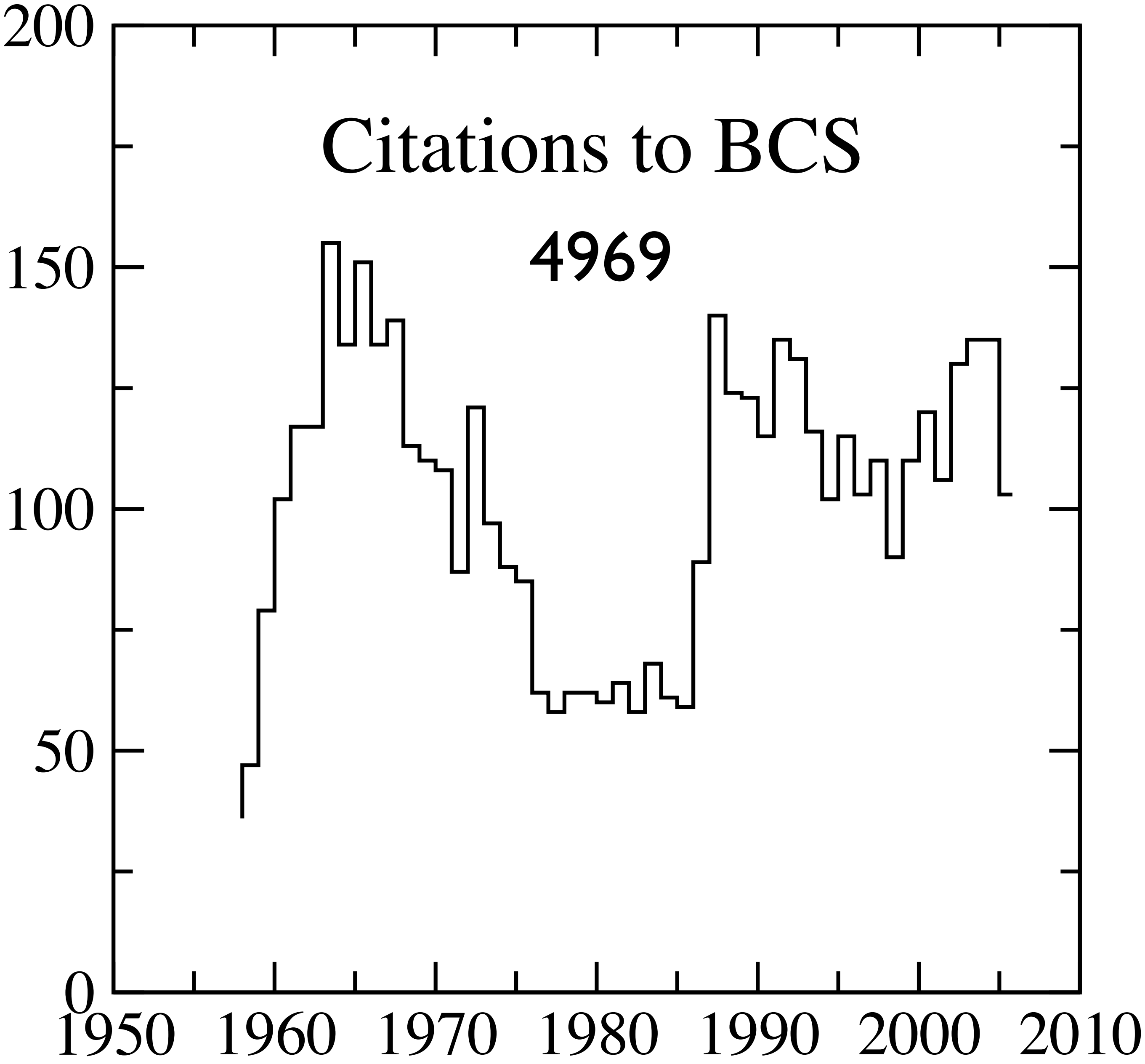
“for their jointly developed theory of superconductivity, usually called the BCS-theory"

John Bardeen

Leon N. Cooper

J. Robert Schrieffer

Impact of the BCS Theory



Google: BCS Theory of Superconductivity 407,000

Google: BCS Theory of Superconductivity 407,000

: BCS Football 1,230,000

A.B. Pippard-- Concluding remarks Colgate Conference on Superconductivity 1963

“The dominant impression has been the overwhelming success of the BCS theory not only in explaining what was known about superconductivity but in providing a framework for new developments.”

A.B. Pippard-- Concluding remarks Colgate Conference on Superconductivity 1963

However I would ask several questions:

1. Are phonon interactions the only interactions that can cause superconductivity?

2. How high can Tc go?

Impact in Condensed Matter

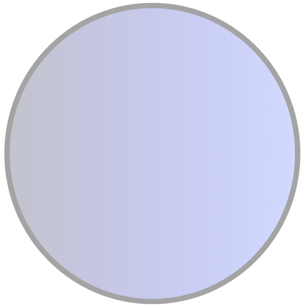
The BCS theory provided an explanation of the superconductivity:

~ 50 elements Hg Pb Nb S Ca Li

thousands of compounds Nb_3Ge MgB_2

fullerenes Cs_3C_{60}

graphite intercalation compounds CaC_6



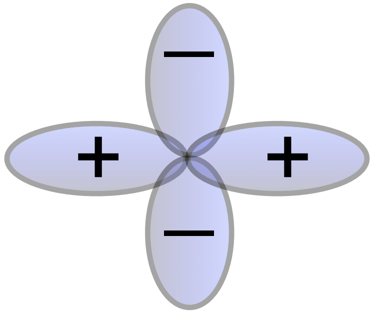
electron-phonon superconductors

s-wave

Non s-wave superconductivity

heavy fermion $CePt_3Si \quad PuCoGa_5$

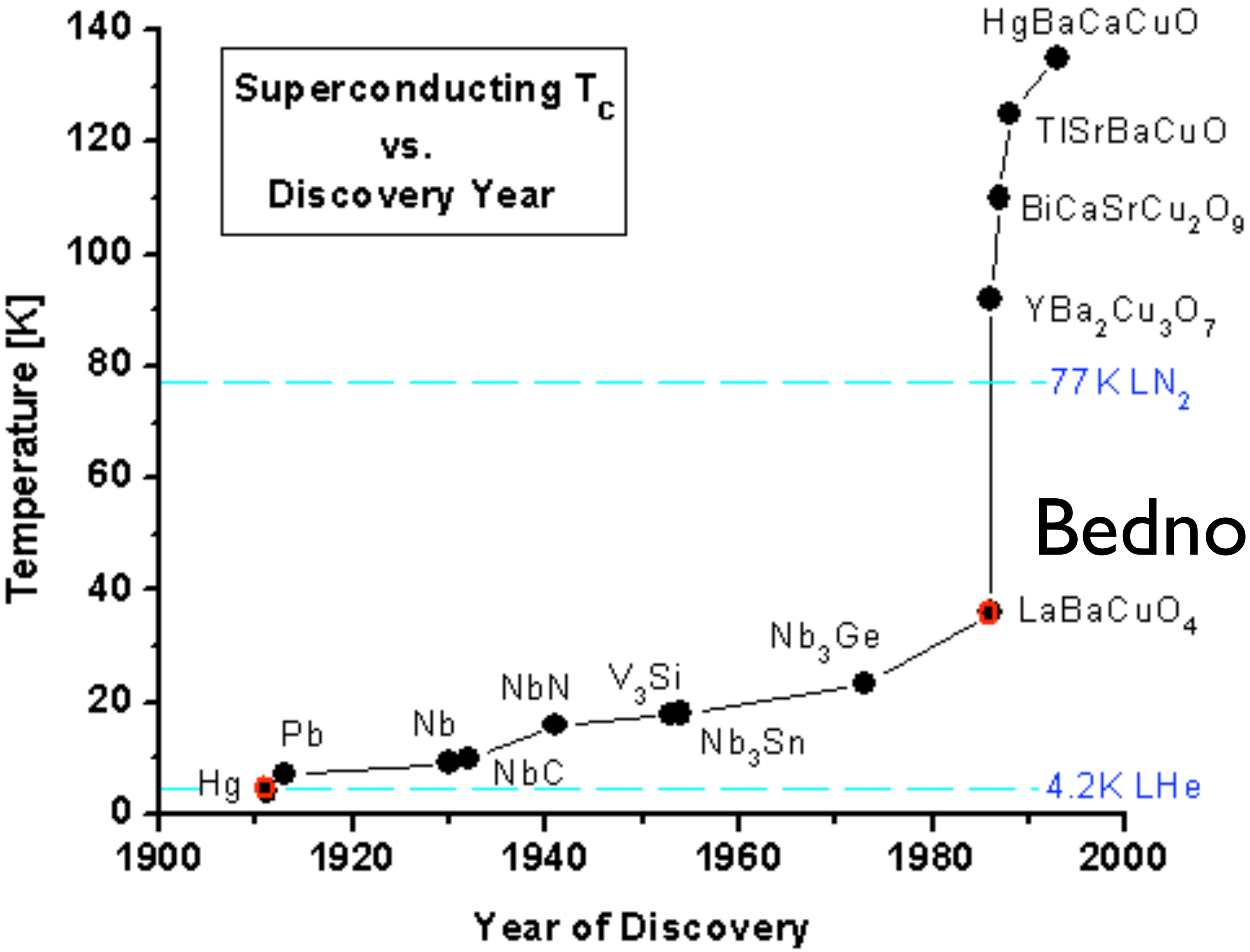
~50 cuprates $YBa_2Cu_3O_{7-x}$



d-wave

ruthenates Sr_2RuO_4 $(p_x + ip_y)$-wave

Superfluid He^3 (p-wave)



Chu and Wu

Bednorz and Muller

To answer Pippard's questions:

1. BCS theory is certainly not limited to s-wave electron-phonon pairing. Nor is it limited to condensed matter systems as Gordon Baym will discuss.

2. We do not know how high Tc can go.

BCS changed the way we think about condensed matter physics:

The BCS many-body wavefunction captured the essence of a new state of matter.

BCS found an important instability of a fermi liquid and the new non-perturbative state it lead to.

This theory provided a key example of symmetry breaking and phase transitions in an interacting fermi system.

It contained the important concept of "off diagonal long range order".

It would provide the basis whole new areas of condensed matter physics, such as the tunneling "branch" that John Rowell will tell us about.

It was a model of how experiment and theory would be intertwined in the developing area of "condensed matter physics".

Beyond this, as Gordon Baym will tell us, the BCS theory has provided essential new insights and understanding of physics that reaches from nuclei to neutron stars as well as broadly into the central problems in particle physics.

BCS ~ 1961