Physics 103 Review Problems
Summer 2015

Instructor: Keith Fratus
TA: Michael Rosenthal

Due Date: Optional!

These review problems are optional - they are for those of you who want to make sure you remember the basics from your previous physics courses.

1. An object moves with **constant speed** v along a circle of radius R centered on the origin. At time $t = 0$, the object is at the position $\boldsymbol{r}(t = 0) = (R, 0)$.

   (a) What is the period of the object's motion? That is, how long does it take for the object to come back to its original position?

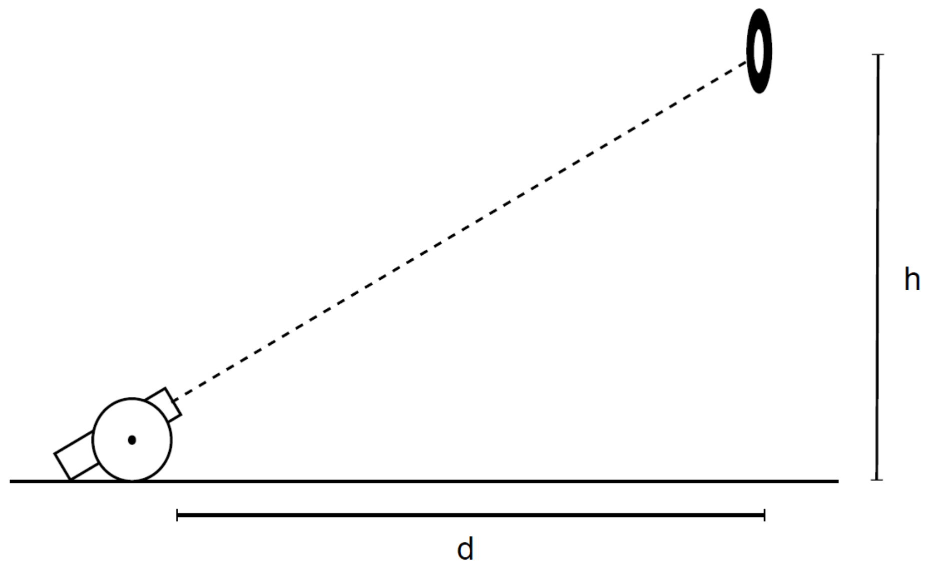
   (b) Write down the position vector $\boldsymbol{r}(t)$ of the object as a function of time. *Hint: What is the **angle** that the position vector makes with the x-axis? How is it related to the **arc length** of the circular path, and how does the arc length change with time?*

   (c) By differentiating $\boldsymbol{r}(t)$ with respect to time, show that the acceleration of the object is

   $$\boldsymbol{a} = -\frac{v^2}{R}\,\hat{r}$$

   where $\hat{r}$ is the unit vector pointing from the origin to the particle. Thus, the particle experiences a uniform acceleration directed radially inwards: this is the *centripetal acceleration* of an object in uniform circular motion.

2. A cannon is aimed at a target some height h above the ground, a distance d away, as shown. The cannon is fired at the **same instant** (which for convenience we'll take to be $t = 0$) that the target is released and allowed to fall freely under the influence of gravity. The bullet leaves the cannon with speed v.



(a) Find the trajectory of the target after it's released, that is, find the position of the target as a function of time.

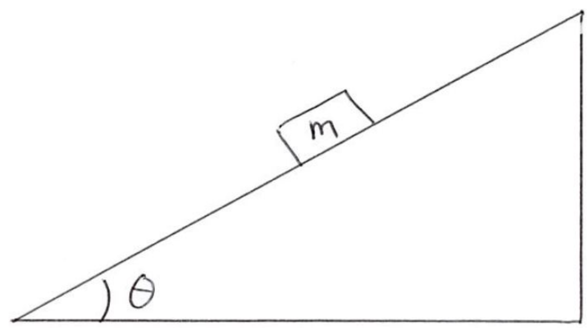
(b) Find the trajectory of the bullet after it's shot.

(c) Show that the bullet will *always* hit the target, no matter what h, d, or v are.

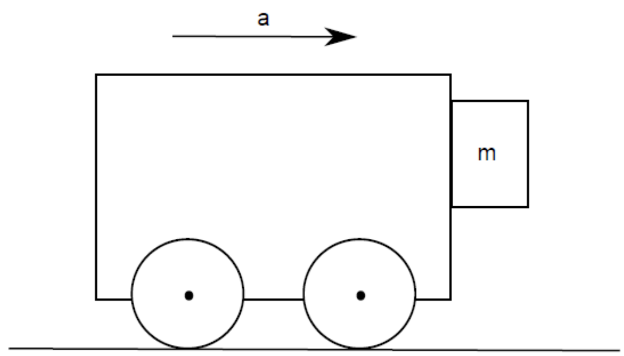
3. Consider a block of mass m, sitting on a ramp which is inclined at an angle θ, as shown.

   (a) If the coefficient of static friction between the block and ramp is μ_s, at what minimum angle must the ramp be oriented so that it will begin to slide? Your answer will involve the acceleration due to gravity, g.

   (b) Assume that the ramp has been oriented at an angle which is larger than the critical angle. If the coefficient of kinetic friction of the block is μ_k, solve for the motion of the block as a function of time.



4. A block of mass m is placed on the right side of a cart; the cart and the block are accelerating to the right with acceleration a, and the coefficient of static friction between the cart and the block is μ_s. The system is sketched below.



(a) Explain qualitatively why the block won't slip and fall if the acceleration of the cart is large enough.

(b) Calculate the minimum acceleration necessary to keep the block from falling. Start by drawing a free body diagram for the block and labeling all the forces on it; then apply Newton's second law in the x and y directions to calculate the minimum value of a that will prevent the block from falling.

5. A moving object subject to air resistance will feel a drag force proportional to, and in the opposite direction of, its velocity:

$$\boldsymbol{F}_{\text{drag}} = -b\boldsymbol{v}$$

where b is just some proportionality constant that depends on the size and shape of the object and the fluid the object is falling in. This is called *Stokes' law*. For simplicity, we'll work in one dimension.

   (a) Consider a freely falling particle under the influence of a drag force obeying Stokes' law (it can only move straight up or down, since we're only working in one dimension). Draw a free body diagram for the particle and label all the forces acting on it.

   (b) Using your free body diagram, calculate the acceleration of the particle using Newton's second law. What is the critical velocity v_T such that the velocity remains constant? This is called the *terminal velocity* of the particle. What is the terminal velocity in vacuum? (Hint: what is b for an object falling in vacuum?)

   (c) In physics class, it's drilled into your head that the time it takes an object to fall to the ground under the influence of gravity is independent of the object's mass. This is true in vacuum, of course, but we all "know" from everyday life that a feather falls to the ground much more slowly than a bowling ball. Using your result from part (b), explain why objects of different masses might take different times to fall to the ground.

6. (**Young & Freedman, problem 6.102**) An airplane in flight is subject to an air resistance force proportional to the square of its speed,

$$F_{\text{drag}} = \alpha v^2$$

where α is some constant. This force points opposite to the velocity. If we assume that the plane flies on a level path, this velocity points entirely in the horizontal direction, since the plane is not moving up or down. But there is an additional resistive force because the airplane has wings. Air flowing over the wings is pushed down and slightly forward, so from Newton's third law the air exerts a force on the wings and airplane that is up and slightly backward, as shown below. The upward force is the lift force that keeps the airplane aloft, and the backward force is called *induced drag.* At flying speeds, the induced drag is inversely proportional to v^2:

$$F_{\text{induced}} = \beta / v^2$$

where β is another constant. Thus the **total air resistance force** can be expressed by

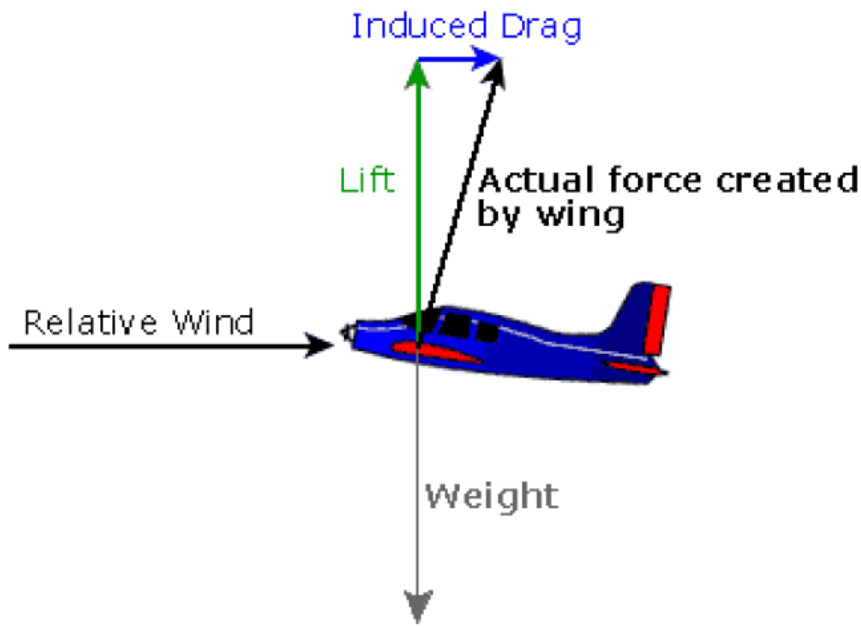
$$F_{\text{air}} = \alpha v^2 + \beta / v^2$$

In steady flight, the engine must provide a forward force that exactly balances the air resistive force.

(a) Calculate the speed at which the airplane will have the maximum ***range*** (that is, will travel the greatest distance) for a given quantity of fuel.

(b) Calculate the speed at which the airplane will have the maximum ***endurance*** (that is, will remain in the air the longest time) for a given quantity of fuel.

Hint: The amount of fuel that the engine uses during some amount of time is proportional to how much work the engine does in that amount of time. The amount of fuel the plane can carry is some finite amount, so the total amount of work the engine can perform is some finite amount. The rate at which the engine performs this work is the power. How does the power supplied by the engine relate to the force it supplies to the plane, and the velocity at which the plane travels? If the engine supplies some amount of power for some amount of time, how much work is done during that time? You'll need to read the last page of my notes on work and kinetic energy to find this information about power. Your final answers for parts a and b won't actually depend on the amount of fuel the plane is carrying with it.


Induced Drag
Lift
Actual force created
by wing
Relative Wind
Weight

7. The *ballistic pendulum* is a device used to measure the speed of a bullet. A bullet of mass m is fired into a block of mass M hanging on a pendulum of length L, into which it embeds itself; this causes the pendulum to swing up to some maximum angle θ, as shown. Calculate the initial speed of the bullet in terms of m, M, θ, L, and g. Hint: What conservation laws might you be able to apply in this problem? When can you apply each one, and when can you not?

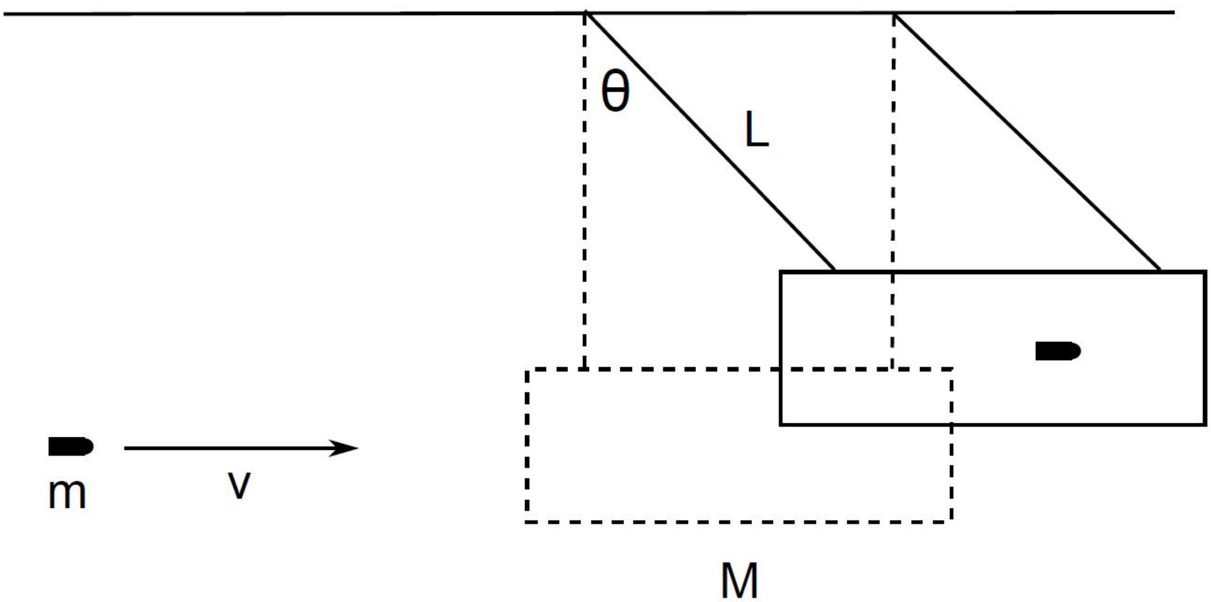


Figure 1: The operation of a ballistic pendulum. Special thanks to Sebastian Fischetti for the figure.

8. Consider a population of rabbits living on an island. The number of rabbits is a function of time, since as time passes, the rabbits will reproduce and die off at some rate. We'll label the population or rabbits, as a function of time, as $R(t)$.

   (a) An ecologist proposes that the population of rabbits should obey the differential equation
   $$\frac{dR}{dt} = kR,$$
   where k is a constant the does not change with time. Why do you think the ecologist might propose this as a reasonable equation that describes the rate of change of the rabbit population? In terms of the behaviour of the rabbit population, what reproductive facts or hypotheses do you think might be described by such a model? What do you think the meaning of the constant k is, and do you think it should be positive or negative, or could it be both?

   (b) Solve the differential equation using the method of separation mentioned in class. There should be an unknown constant, C, that results from doing an integral at some point.

   (c) Assume that the rabbit population does indeed obey the differential equation. Assume that you also know that at time $t = 0$, the rabbit population is equal to some particular value, $R(t = 0) = R_0$. How does this information help you find the value of the unknown constant C? What does the solution of the differential equation look like, written in terms of R_0 (instead of C)?

   (d) Do you think there is anything unreasonable about the solution? Is there anything about the solution that disagrees with your intuitive sense about how animal populations should behave?

   (e) A second ecologist proposes a different model for the population of rabbits, which looks like
   $$\frac{dR}{dt} = k\left(1 - \frac{R}{N}\right)R.$$
   We will now assume that the constant k is always positive. The number N is another constant, positive value, which does not change over time. What is the right side of the equation approximately equal to when R is much smaller than N? What is the right side approximately equal to when R is much BIGGER than N? How does the sign (positive or negative) of the right side change when we go from one case to the other? What happens when R = N?

   (f) Assume that the number of rabbits at time zero is some quantity $R(t = 0) = R_0$, and we want to know what the population is at some later time T, $R(t = T)$. Solve the differential equation using the method of separation discussed in class. This time, try performing a **definite** integral on both sides of the equation. What should the upper and lower **bounds** on the integrals be? Does this strategy allow you to avoid the unknown constant C? (You're not required to do the integrals by hand - you can use a calculator, or Wolfram Alpha, if you like).

(g) Assume that $N = 100$ and $k = 1$. Take your solution from the previous part, and make **two** plots of the function $R(t)$ - one of them for $R_0 = 10$, and the other one for $R_0 = 200$. **Make sure the plots extend up until at least $T = 10$.** What is the **major qualitative difference** between the two different plots? Does this shed any light on the meaning of the parameter N?

(h) **Why do you think the second model is a more reasonable model of population growth, as opposed to the first model?**