Vectors and Coordinate Systems

In Newtonian mechanics, we want to understand how material bodies interact with each other and how this affects their motion through space. In order to be able to make quantitative statements about this, we need a mathematical language for describing motion, which is known as kinematics. Part of the language of kinematics involves calculus, since we know that calculus lets us talk about how things change in a quantitative way. The other part of this mathematical language involves the notion of a vector, and the related concept of a coordinate system. Today we'll review the basic concepts of vectors and coordinate systems that we'll need, and then we'll add ideas from calculus in order to form the subject of kinematics.

The starting point for describing the location of objects in space is to pick an arbitrary location, known as the origin, and to describe every other position in space by the displacement vector between the origin and that point. This is shown in Figure 1 Because vectors allow us to talk about lengths and orientations in a quantitative way, they are a natural tool for describing motion. Much in the same way that I might say my house is 1.3 miles Northwest of campus, I can say that the location of some particle is a certain distance from the origin, at some angle. We won't belabour all of the basic properties of vectors (such as addition and subtraction), but if you are rusty on these subjects, now is the time to review!

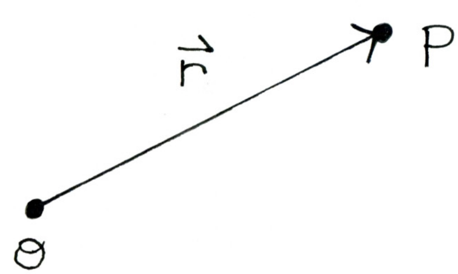

Figure 1: A point in space being indicated by its displacement vector from the origin. Image credit: Kristen Moore

As a practical matter, the easiest way to work with vectors is to set up a coordinate system, shown in Figure 2. In three dimensional space, a particularly common type of coordinate system, known as a *Cartesian coordinate system*, consists of the origin, along with three mutually perpendicular vectors, all with a length of one. Generally, we will call these three vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ (with the hats indicating that they have a length of one), although other notations exist (we'll introduce other notations later when they may be more convenient). Any other vector in three dimensional space, including the displacement vector describing the location of an object, can be written as a sum of these three vectors,

$$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}. \tag{1}$$

This expression tells us that if I travel a distance r_x in the direction parallel to $\hat{x}$, and then do similarly for the y and z directions, I will end up at the location described by the position vector $\vec{r}$. The numbers (r_x, r_y, r_z) are typically referred to as the *coordinates*, or alternatively the *components*, of the vector $\vec{r}$. Again, if this does not sound familiar to you, now is the time to review!

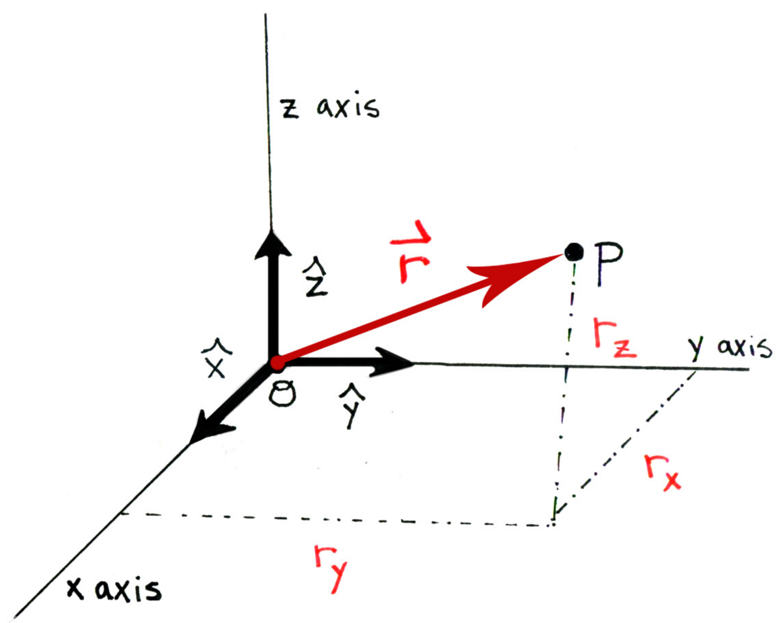

Figure 2: A displacement vector being represented with a particular choice of coordinate system. Image credit: Kristen Moore

The primary reason for writing vectors in this way is that their mathematical manipulations become especially simple. If I have another vector,

$$\vec{w} = w_x\hat{x} + w_y\hat{y} + w_z\hat{z}, \tag{2}$$

then the vector sum of $\vec{r}$ and $\vec{w}$ is given simply by adding components,

$$\vec{r} + \vec{w} = (r_x + w_x)\,\hat{x} + (r_y + w_y)\,\hat{y} + (r_z + w_z)\,\hat{z}. \tag{3}$$

Likewise, multiplying a vector by a number α is straightforwardly expressed in terms of components,

$$\alpha\vec{r} = (\alpha r_x)\,\hat{x} + (\alpha r_y)\,\hat{y} + (\alpha r_z)\,\hat{z}. \tag{4}$$

Additionally, two vectors are equal to each other if and only if all three of their components are equal,

$$\vec{r} = \vec{w} \;\Leftrightarrow\; r_x = w_x \;,\; r_y = w_y \;,\; r_z = w_z. \tag{5}$$

Of course we all remember from introductory freshman mechanics that the last equation is what allows us to solve Newton's second law, $\vec{F} = m\vec{a}$, one component at a time. Lastly, by using the Pythagorean theorem, the length of a vector can be written as

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2} \tag{6}$$

A word of caution is in order regarding the components of a vector. It is common to see a vector written in terms of its components as

$$\vec{r} = (r_x, r_y, r_z). \tag{7}$$

While most of the time we can get away with this kind of notation, it sometimes gives the false impression that a vector is literally equal to its components. This ignores the fact that we have to specify a coordinate system in order to determine the components of a vector. **If I had made a different choice of coordinate system, the components of the vector would change, even though the original vector is still the same**. Vectors themselves have a real physical significance to them, whereas components reflect an arbitrary choice of coordinate system, used to facilitate mathematical manipulations. While this distinction may seem overly pedantic, it will be important to remember later in the course when we discuss how to change between different coordinate systems, and in a few weeks when we discuss the subject of relativity.

Before moving on to the subject of kinematics, there are two other vector operations we should mention. While we have discussed how to add vectors and how to multiply them by numbers, we have not discussed the extent to which we can "multiply" two vectors together. Of course, we know that one of the ways we can multiply two vectors is through the dot product,

$$\vec{a} \cdot \vec{b} = ab\cos\theta, \tag{8}$$

where a and b are the lengths of the two vectors, and θ is the angle between them. The dot product is also commonly referred to as the inner product, or scalar product. Geometrically, the dot product can be understood as multiplying the lengths of the two vectors, reduced by a factor which measures how much the two vectors point in the same direction. This is shown in Figure 3. The scalar product distributes, in the sense that

$$\vec{a} \cdot \left(\vec{b} + \vec{c}\right) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}. \tag{9}$$

This fact can be proven from the original definition by using some geometry and trigonometry.

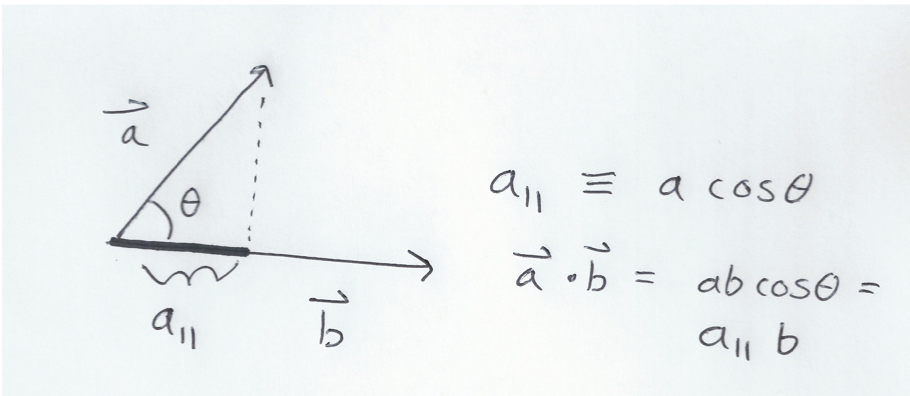

Figure 3: A geometric representation of the dot product.

For our coordinate vectors, since we know the lengths and orientations amongst them, we can see that

$$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1, \tag{10}$$

with all other inner products being zero. Using the above facts, we see that the dot product of two vectors can easily be written in terms of their components,

$$\vec{a} \cdot \vec{b} = (a_x\hat{x} + a_y\hat{y} + a_z\hat{z}) \cdot (b_x\hat{x} + b_y\hat{y} + b_z\hat{z}) = a_x b_x + a_y b_y + a_z b_z. \tag{11}$$

The above expression certainly seems like an intuitive guess for what the product between two vectors "should" be - we simply multiply all of the components, and then add. Two simple applications of the above formula immediately suggest themselves. First, if we take the two vectors to be the same, then we find

$$\vec{a} \cdot \vec{a} = a_x^2 + a_y^2 + a_z^2 = |\vec{a}|^2, \tag{12}$$

which relates the length of a vector to the inner product with itself. Also, since we can write the coordinate vectors as

$$\hat{x} = (1, 0, 0)\ ;\ \hat{y} = (0, 1, 0)\ ;\ \hat{z} = (0, 0, 1), \tag{13}$$

we find that

$$r_x = \hat{x} \cdot \vec{r}, \tag{14}$$

and likewise for the other two components.

The dot product between two vectors has an intuitive geometric interpretation, and will appear repeatedly throughout the course, especially when we consider the subjects of work and energy. Another way we can form the product between two vectors is known as the cross product, and while it may initially seem somewhat less intuitive, it is important nonetheless. The cross product, also known as the vector product or the outer product, takes two vectors as input, and returns a third vector as output. In terms of components, it can be expressed as

$$\vec{r} \times \vec{w} = (r_y w_z - r_z w_y)\,\hat{x} + (r_z w_x - r_x w_z)\,\hat{y} + (r_x w_y - r_y w_x)\,\hat{z}. \tag{15}$$

Alternatively, one can specify the cross product in terms of the determinant of a matrix,

$$\vec{r} \times \vec{w} = \det \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ w_x & w_y & w_z \end{bmatrix}. \tag{16}$$

Geometrically, the vector product can be expressed as

$$\vec{r} \times \vec{w} = rw \sin\theta\ \hat{n}, \tag{17}$$

where r and w are the lengths of the two vectors, θ is the angle between them, and $\hat{n}$ is the unit vector which is perpendicular to both $\vec{r}$ and $\vec{w}$. This still leaves an ambiguity as to which way $\hat{n}$ should point, which is resolved through the right-hand rule, shown in figure 4.

The vector product distributes in the same manner as the dot product,

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}. \tag{18}$$

Notice that while the dot product is symmetric,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}, \tag{19}$$

the cross product is anti-symmetric,

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}. \tag{20}$$

While the geometric interpretation of the vector product is less clear than that of the dot product, we will also see the vector product appear repeatedly, especially when we discuss torque and angular momentum. Later on when we discuss how to change between different coordinate systems, we'll get a better intuitive sense of *why* the dot product and cross product repeatedly show up in physics.

Kinematics

With the basic machinery of vectors in place, we want to be able to describe how material objects actually move through space. Emphasizing that the location of an object will generally change as time progresses, we have

$$\vec{r}(t) = r_x(t)\,\hat{x} + r_y(t)\,\hat{y} + r_z(t)\,\hat{z}. \tag{21}$$

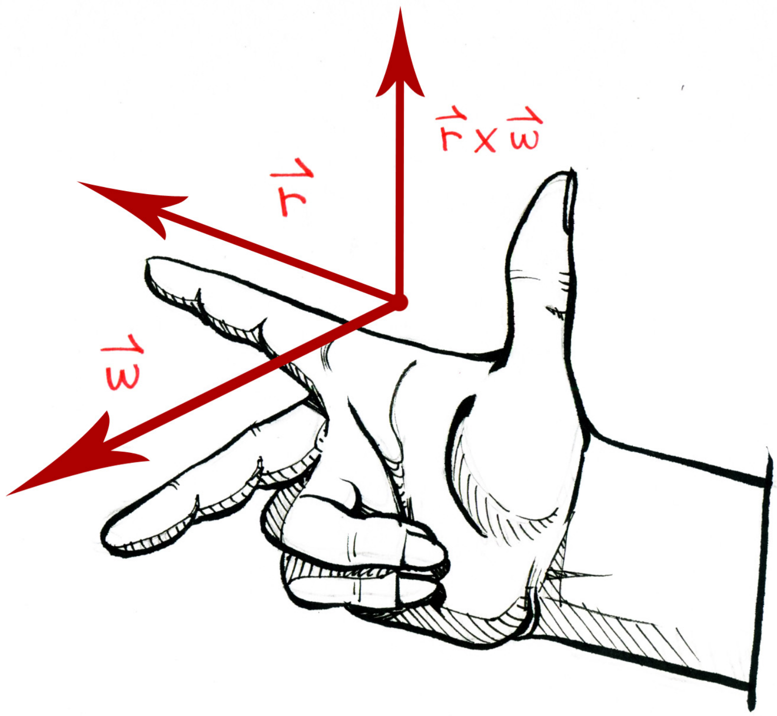

Figure 4: The right hand rule, explained graphically. Image credit: Kristen Moore

The three coordinate vectors themselves are fixed in space and do not change with time, although later when we use different coordinate systems, this will generally not be the case. Now, if my object is at a given point at some time t_1, and then it is at another point at time t_2, the displacement vector over that time interval is defined as

$$\Delta\vec{r} = \vec{r}\,(t_2) - \vec{r}\,(t_1)\,. \tag{22}$$

Remember: this is not the same thing as the total distance travelled between the two points! In general, even if the particle follows some sort of wiggly path in between these two points, the displacement is still defined as the above vector quantity.

Using the displacement, we can define the average velocity over the time

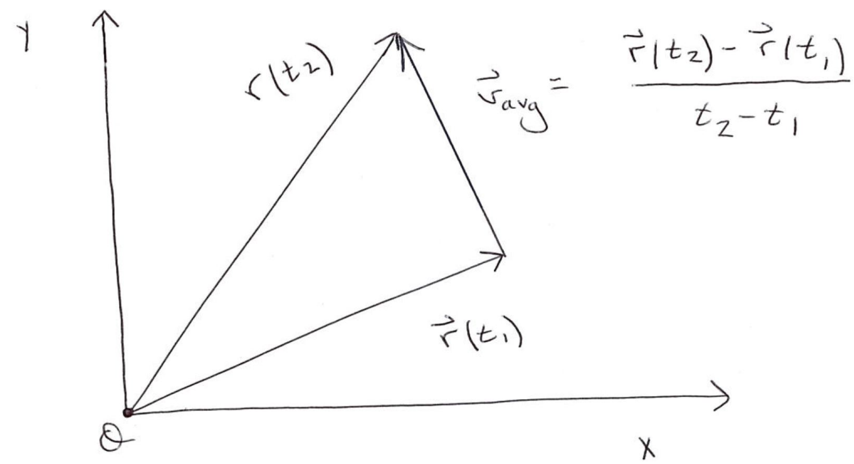

Figure 5: The definition of the average velocity over a time interval, defined in terms of the displacement vectors.

interval, which is given by

$$\vec{v}_{\rm avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}\,(t_2) - \vec{r}\,(t_1)}{t_2 - t_1}. \tag{23}$$

This agrees with our usual notion of velocity as being a notion of distance traveled per time, although it is important to remember that this is a vector quantity. This is shown in Figure 5. In the limit that the time interval goes to zero, we recover the instantaneous velocity,

$$\vec{v}\,(t) = \lim_{t_1 \to t_2} \frac{\vec{r}\,(t_2) - \vec{r}\,(t_1)}{t_2 - t_1} \equiv \frac{d\vec{r}}{dt}. \tag{24}$$

This defines the (instantaneous) velocity of an object as the derivative of the position with respect to time. This idea is shown in Figure 6. If we write out the position and velocity vectors in terms of coordinates, what we find is that

$$\vec{v}\,(t) = \lim_{t_1 \to t_2} \begin{pmatrix} \frac{r_x(t_2) - r_x(t_1)}{t_2 - t_1} \\ \frac{r_y(t_2) - r_y(t_1)}{t_2 - t_1} \\ \frac{r_z(t_2) - r_z(t_1)}{t_2 - t_1} \end{pmatrix} = \begin{pmatrix} \frac{dr_x}{dt} \\ \frac{dr_y}{dt} \\ \frac{dr_z}{dt} \end{pmatrix} \tag{25}$$

and so we have

$$v_x\,(t) = \frac{dr_x}{dt}, \tag{26}$$

and similarly for the y and z components. The magnitude of the velocity is referred to as the *speed*.

Part of the reason for being so pedantic in deriving the above results is to emphasize the fact that the simple relation

$$\vec{v} = \frac{dr_x}{dt}\hat{x} + \frac{dr_y}{dt}\hat{y} + \frac{dr_z}{dt}\hat{z} \tag{27}$$

relies on the fact that the coordinate vectors do not depend on time. For more general coordinate systems, which we will use later in the course, the expression for the velocity in terms of components does **not** look this simple!

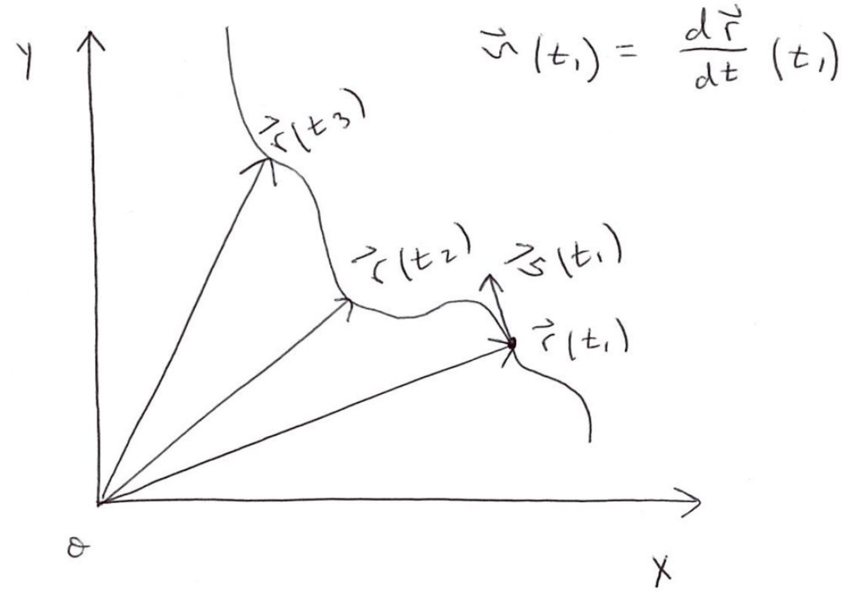

Figure 6: The definition of the instantaneous velocity as the time derivative of the displacement. The velocity at the first time is given, while several other position vectors along the motion of the particle are also shown.

The velocity vector is different from the displacement vector in an important way. The displacement vector is an oriented line from the origin to the location of the particle, and has physical units of length. The velocity vector, however, has units of length divided by time, and it is NOT a displacement vector that extends in space between two points, even though this is how we've drawn the average velocity. As shown in Figure 6, the tail of the instantaneous velocity vector is usually placed at the location of the particle at that time, but it is important to remember that drawing the vector as something that extends in space is really just a way for us to graphically represent the fact that it has a magnitude and direction. In reality, velocity doesn't have a length that extends through space, so in some sense it's not really correct to draw it on the same set of axes as the displacement vectors, although we usually still do this anyways.

Another important fact about the velocity vector is that if we shift the origin of our coordinates, while maintaining the orientation of the axes, the components of the velocity vector don't change, whereas the components of the displacement vector do change. We'll discuss this in more detail when we cover Galilean relativity.

We can find the displacement in terms of the velocity by integrating,

$$\Delta\vec{r} = \int_{t_1}^{t_2} \hat{v}\left(t\right) dt = \left[\int_{t_1}^{t_2} v_x\left(t\right) dt\right] \hat{x} + \left[\int_{t_1}^{t_2} v_y\left(t\right) dt\right] \hat{y} + \left[\int_{t_1}^{t_2} v_z\left(t\right) dt\right] \hat{z}. \quad (28)$$

One can check that this is correct using the fundamental theorem of calculus for each component of the position and velocity. The total distance travelled by the object, often referred to as the arc length of its trajectory, is given by the integral of its speed,

$$s = \int_{t_1}^{t_2} \left|\vec{v}\left(t\right)\right| dt \quad (29)$$

where s of course stands for "arc," since no other important terms I just mentioned start with the letter s.

After defining the velocity, we can also discuss higher order derivatives. The second derivative of the position, or the first derivative of the velocity, is of course the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}. \quad (30)$$

The average acceleration over some time is given by

$$\vec{a}_{\text{avg}} = \frac{\vec{v}\left(t_2\right) - \vec{v}\left(t_1\right)}{t_2 - t_1}. \quad (31)$$

Again, the change in velocity can be obtained from the integral of the acceleration,

$$\vec{v}\left(t_2\right) - \vec{v}\left(t_1\right) = \int_{t_1}^{t_2} \vec{a}\left(t\right) dt \quad (32)$$

Usually, acceleration is the highest order derivative that we consider. This is because Newton's laws tell us what the acceleration of an object is in terms of the forces acting on it. However, higher order derivatives occasionally come up. The third derivative of position, or first derivative of acceleration, is usually referred to as the "jerk." The fourth derivative is often referred to as the "jounce," or sometimes the "snap," while some sources cite the fifth and sixth derivatives as the "crackle" and "pop," respectively.

It is absolutely imperative to remember that these are all VECTOR quantities! They must be manipulated as such, making sure to correctly add the different components together.

While the mathematical language of kinematics that we've developed so far allows us to describe arbitrary motion, especially simple results can be derived when the motion that we are studying is the result of constant acceleration. These equations are probably known to you from your freshman mechanics course as the so-called "kinematic equations," which is not a very good name because it somehow suggests that they describe all of kinematics quite generally, which is of course not true. But let's derive them anyways, to show that they are a special case of the general formalism described here.

For simplicity, we'll assume that the motion begins at time zero, and denote the **final** time as simply t, with **intermediate** times (being integrated over) referred to as t'. The notation we'll use for any quantity at time zero will be

$$\vec{v}\,(0) \equiv \vec{v}_0. \tag{33}$$

Now, we know that we can find the final velocity according to

$$\vec{v}\,(t) = \vec{v}_0 + \int_0^t \vec{a}\ dt'. \tag{34}$$

In terms of components, we have

$$\vec{v}\,(t) = \vec{v}_0 + \left[\int_0^t a_x dt'\right]\hat{x} + \left[\int_0^t a_y dt'\right]\hat{y} + \left[\int_0^t a_z dt'\right]\hat{z}. \tag{35}$$

Because all of the components of the acceleration are constant, this simply becomes

$$\vec{v}\,(t) = \vec{v}_0 + a_x t\hat{x} + a_y t\hat{y} + a_z t\hat{z} = \vec{v}_0 + \vec{a}t. \tag{36}$$

Remember that the acceleration is constant in the sense that it does not change in time. It is, however, still a vector quantity, with a magnitude and a direction.

Now, similarly we can find the position according to

$$\vec{r}\,(t) = \vec{r}_0 + \int_0^t \vec{v}\,(t')\,dt' = \vec{r}_0 + \int_0^t (\vec{v}_0 + \vec{a}t')\,dt'. \tag{37}$$

Because $\vec{v}_0$ and $\vec{a}$ are constant, the result is

$$\vec{r}\,(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2. \tag{38}$$

As promised, we have arrived at one of the "kinematic equations." While it is obvious that this equation is only valid for motion under constant acceleration, it does demonstrate one important feature which will always be true for **any** mechanical problem - in order to specify the motion of an object, we need to supply the initial starting position ($\vec{r}_0$), the initial velocity, ($\vec{v}_0$), and some physical principle which determines the acceleration as a function of time (in this case the assumption that the acceleration is the constant vector $\vec{a}$).

I want to pause for a moment to point out that up until now we haven't actually done any *physics* yet. Everything we've talked about so far has involved how to develop a good language for describing motion, and is really just *math.* So far I haven't told you anything about what sort of motion actually occurs in nature as a result of some sort of physical principles. I'm going to change that now by introducing the subject of projectile motion.

Projectile Motion

Projectile motion is the motion that occurs for a material body under the influence of gravity alone. An example of this would be the motion of a bullet after it has been fired from a gun (assuming that we are neglecting the effects of air resistance). As we know from freshman mechanics, for objects moving near the surface of the Earth, it is usually a very good approximation to say that gravity causes the objects to move with a constant acceleration, with a value that is completely independent of the body in question. We generally refer to this value as g, and on Earth the numerical value for its magnitude is approximately 9.8 meters per second squared. Its orientation is directed towards the ground.

Since the motion of a body under the influence of gravity is described by a constant acceleration, we can use the equation we derived in the previous section to describe its motion. Let's imagine that we have a gun which fires a bullet with some initial velocity, held at some initial starting point. This is shown in Figure 7. What we want to do is find the position of the bullet as a function of time, after it is fired.

In solving this problem, we'll set up a coordinate system whose x axis is parallel with the ground and whose y axis points vertically upwards away from the ground. We'll place the origin so that it is on the ground, directly below the muzzle of the gun. This is shown in Figure 7. In our coordinates, we can see that the initial location of the bullet is specified by

$$\vec{r}_0 = \begin{pmatrix} 0 \\ h \end{pmatrix}, \tag{39}$$

where h is the height of the muzzle of the gun off of the ground. If the gun makes an angle of θ with respect to the horizontal, our initial velocity will be

$$\vec{v}_0 = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta \end{pmatrix}, \tag{40}$$

where v_0 is the initial speed of the bullet. Lastly, the acceleration is given by

$$\vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}, \tag{41}$$

where g is taken to be the (positive!) magnitude of the acceleration, in this case 9.8 meters per second squared (Notice that we are working with a two-dimensional coordinate system, even though space is three-dimensional - why are we allowed to do this?).

With my coordinates properly set up, finding the motion of the projectile is just a trivial application of the kinematic equation. The x component of the motion is described by the equation

$$r_x(t) = r_{x0} + v_{x0}t + \frac{1}{2}a_x t^2, \tag{42}$$

or,

$$r_x(t) = v_0 \cos\theta t. \tag{43}$$

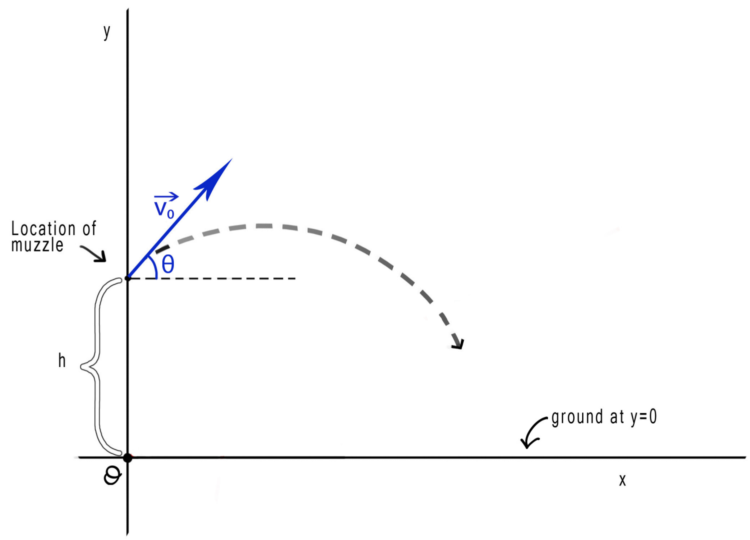

Figure 7: The set up of our projectile motion problem, complete with coordinate axes. The dotted line is a path we suspect the particle might take. Image credit: Kristen Moore

The y coordinate of the motion is described by

$$r_y(t) = r_{y0} + v_{y0}t + \frac{1}{2}a_y t^2, \tag{44}$$

or,

$$r_y(t) = h + v_0 \sin\theta t - \frac{1}{2}gt^2. \tag{45}$$

Combining these together, we can write the position vector of the bullet as a function of time as

$$\vec{r}(t) = \begin{pmatrix} v_0 \cos\theta t \\ h + v_0 \sin\theta t - \frac{1}{2}gt^2 \end{pmatrix} \tag{46}$$

So now we have an expression for the position of the bullet as a function of time. However, we know that this expression has an obvious limit to its validity. Eventually, one of two things will happen. One possibility is that the bullet will shortly fall back to the ground, at which point it will no longer be under the influence of gravity, and projectile motion will no longer be an adequate description of its trajectory. The second possibility is that we have an unimaginably powerful, rocket-powered rifle, and we've fired the bullet with such a large initial speed that it is able to travel very far from the surface of the Earth, and the approximation of constant acceleration is no longer valid. If the speed is large enough, it is possible for the bullet to completely escape the Earth's gravity, and never return.

Assuming that the first situation is more accurate, one thing we would like to do is figure out the time at which the bullet hits the ground, and then proceeds to do something else which is not correctly described by projectile motion (bounces off the ground, gets stuck in the dirt, or whatever). Mathematically, the bullet will hit the ground when

$$r_y(t) = h + v_0 \sin\theta t - \frac{1}{2}gt^2 = 0, \tag{47}$$

which is a quadratic equation with the solutions

$$t = \frac{-v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta + 2gh}}{-g}. \tag{48}$$

Choosing the positive solution, we see that the time that the bullet hits the ground must be

$$t_g = \frac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}}{g}. \tag{49}$$

An easy thing to immediately notice about this expression is that it is inversely proportional to g, so that when the acceleration due to gravity is larger, it takes a shorter amount of time for the bullet to hit the ground. This makes intuitive sense - if the acceleration due to gravity is not very strong, we expect it to not influence the motion of the bullet as much, and so it will take longer for the motion of the bullet to deviate from its initial velocity, which is pointing up and away from the ground. The final horizontal position of the bullet will be

$$x_{\max} = r_x(t_g) = \frac{v_0 \cos\theta \left(v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}\right)}{g}. \tag{50}$$

Differentiating the position as a function of time, we can find the y-component of the velocity to be

$$v_y(t) = v_0 \sin\theta - gt. \tag{51}$$

If we set this equal to zero, the time at which the maximum height is obtained is

$$t_{\max} = \frac{v_0 \sin\theta}{g}. \tag{52}$$

The maximum height is found by evaluating the y coordinate at this time,

$$y_{\max} = r_y\left(\frac{v_0 \sin\theta}{g}\right) = h + v_0 \sin\theta \frac{v_0 \sin\theta}{g} - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2, \tag{53}$$

or

$$y_{\max} = h + \frac{v_0^2 \sin^2\theta}{2g}. \tag{54}$$

Notice that the maximum height obtained depends on the starting height, but the time it takes to get there does not. Also, notice that in the case that $h = 0$, which is the case that the bullet is fired from the ground, the amount of time it takes to reach the maximum height is half the amount of time it takes to reach the ground. This says that **the amount of time it takes the bullet to rise to its maximum height is the same as the amount of time it takes to fall back down to the original starting height**, which is a nice feature of projectile motion to remember. In fact, when we study the subject of potential energy, we'll see that this type of behaviour is not specific to projectile motion, and is actually quite general.

Another nice feature of projectile motion is that it's relatively easy to write down a relation between the x and y coordinates of the motion at any given time. Because the x coordinate increases linearly with time, there is a one to one relationship between the x coordinate of the bullet, and the time that has elapsed. If we invert this relationship, we find

$$t = \frac{r_x}{v_0 \cos\theta}. \tag{55}$$

Using this expression, I can write the y coordinate as a function of the x coordinate, to find

$$r_y\left(r_x\right) = h + v_0 \sin\theta \left(\frac{r_x}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{r_x}{v_0 \cos\theta}\right)^2, \tag{56}$$

or

$$r_y\left(r_x\right) = h + \tan\theta r_x - \frac{g}{2 v_0^2 \cos^2\theta} r_x^2. \tag{57}$$

Thus, if we plot the y coordinate against the x coordinate, which indicates the bullet's trajectory, we will find a parabola, which is a generic feature of projectile motion. Specifically, this means that in Figure 7, the shape of the dotted line is parabolic.

This concludes our brief review of kinematics, before moving on to discuss Newton's equations. If any of this material seems rusty to you, review it now!