"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

-Hermann Minkowski
Address to the 80th Assembly of German Natural Scientists and Physicians
Sep. 21, 1908

Physics is an Experimental Science

The ideas we're going to discuss in the next few days will be severely difficult to reconcile with our everyday intuition about the world. For most of you, Special Relativity is the first topic you will encounter in physics which requires you to abandon your "common sense" ideas about the way the universe behaves. While other subjects, such as General Relativity and Quantum Mechanics, will also challenge your intuition about the physical world, personally, Special Relativity still strikes me as the strangest idea in modern physics.

However, all of the claims which I am going to present to you here have been verified experimentally. Thousands of physicists, working on hundreds of experiments, over the course of several decades, have all come to the same conclusions, all of which are in agreement on the subject of relativity. If tomorrow you wake up and suddenly have an epiphany that you no longer want to study physics, and never take another physics course in your life, if there is only one thing you take away from your physics education, I hope it is this: the ultimate validity of a physical theory is determined by experiment. If you want to understand how the universe behaves, you have to go out and actually do an experiment. No matter how "beautiful," "natural," or "'sensible" a theory may be, if it disagrees with experiment, it is wrong. Today, we are going to discover that our familiar laws of Newtonian mechanics are, in fact, wrong, and must be replaced by the Special Theory of Relativity.

The Constant Speed of Light

In the late 1800s, James Clerk Maxwell earned a place among the greatest minds in physics for his work in understanding electromagnetic phenomenon, in particular, the understanding that light itself is an electromagnetic wave. The equations which bare his name, Maxwell's equations,

$$\begin{array}{cc} \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} & \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \end{array}, \tag{1}$$

adequately describe all of classical electromagnetism. In particular, in the absence of any charge density, Maxwell's equations reduce to

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \quad ; \quad \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0, \tag{2}$$

which describes the propagation of an electromagnetic wave, with speed

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \text{ m s}^{-1}. \tag{3}$$

While Maxwell's predictions were ultimately verified by Heinrich Hertz in 1887, they presented a seeming contradiction to Newtonian physics. While it was widely believed that the ideas of Galilean Relativity were correct, Maxwell's equations make reference to one specific velocity, which is supposed to be a constant of nature. If there were a fundamental velocity of the universe, this would seem to preclude the possibility that all inertial frames are equally valid in their description of nature, since we previously argued that any reference frame moving at a constant velocity with respect to an inertial reference frame is also an inertial one. Since a Galilean transformation changes the velocities of material objects, any notion of the "correct" velocity of an object becomes meaningless.

However, to physicists at the time, the resolution to this problem was obvious. All previously known waves travelled in some sort of medium, and so electromagnetic waves should be no different. Water waves may have some characteristic speed at which they propagate through a stream, but no one would consider the stream to be a particularly special frame of reference. Likewise, electromagnetic waves must move through some sort of electromagnetic medium, which came to be known as the "ether," due to the fact that it had previously been unseen, and thus must possess some "ethereal" properties. This ether was supposed to extend throughout space, a provide a natural frame of reference for describing electromagnetic waves.

Not only was the idea of the ether a natural way to reconcile electromagnetism with Galilean Relativity, it also presented an opportunity to conduct an interesting experiment. Much as a boat travelling on a stream can "catch up" with a water wave by travelling through the water around it, if the Earth were moving through the ether, it may be able to "catch up" with a beam of light. In the late 1800s, Michelson and Morley devised an ingenious experiment to measure the speed of light, and to look for this behaviour. Since the Earth is orbiting around the Sun, and also rotating every 24 hours while it does so, the motion of the Earth with respect to the ether should change on a regular basis (assuming the ether is not being "dragged along" with the Earth). Thus, over the course of a day, and over the course of several months, a noticeable change in the speed of light should be detected in an experiment on an Earth-bound laboratory. Michelson and Morley performed a set of highly accurate measurements, in an effort to observe just such a behaviour. The idea behind such an experiment is sketched in Figure 1.

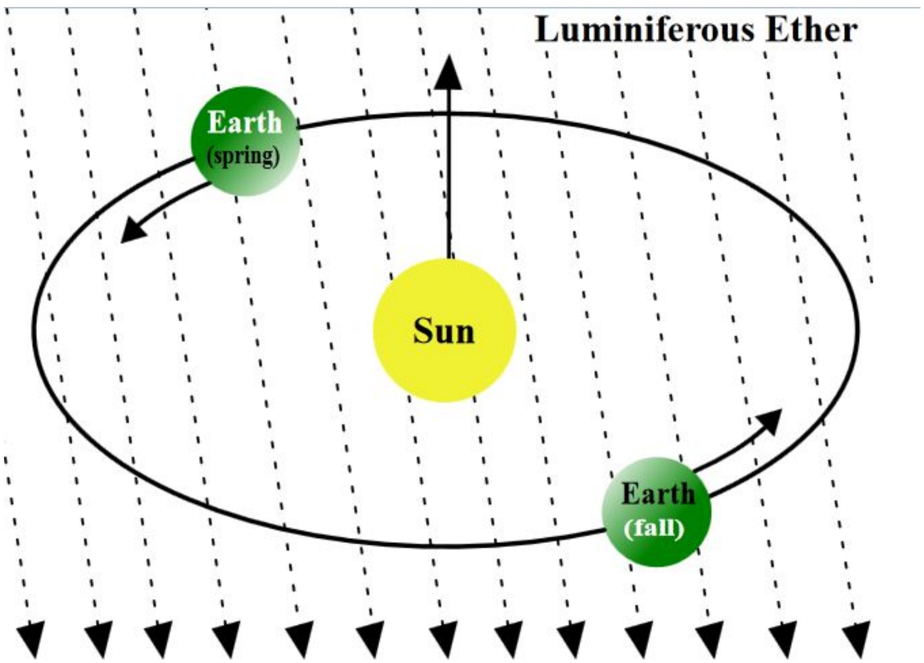

Figure 1: The motion of the Earth, with respect to the "Luminiferous Ether." The Michelson-Morley experiment of the late 1800s set out to measure this motion.

However, much to their surprise, no such variation was detected. While further refinements were made to their apparatus, and additional experiments were conducted, no variation in the speed of light was ever detected. Additional theories regarding either drag were devised, which purported to explain the lack of variation, though none of them were entirely consistent with all of the observational data. For several decades, the results of these experiments were not taken seriously, since the idea of an absolutely constant speed of light (regardless of one's orientation with respect to the light source) seemed to be utter nonsense. However, as we know, one of the few physicists who initially embraced this idea was Albert Einstein, who in 1905 published his theory of Special Relativity, which took the constant speed of light, along with the equivalence of all inertial reference frames, as its two basic postulates. Understanding the miraculous ramifications of these two simple postulates will be the focus of the last week of our course.

Time Dilation

To understand perhaps the most startling, and most famous, implications of the postulates of special relativity, we will again visit the example of two inertial observers, each of which has set up their own set of coordinate axes. For simplicity, we'll assume that they have aligned their coordinate axes in such a way that their origins coincide at time $t = 0$. Keeping with tradition, we'll assume that one of the observers is standing on the ground, while the other observer is in a passing train, moving with respect to the ground-based observer at a speed v. Additionally, we'll assume that as the two coordinate axes coincide with each other, the observer on the train shines a laser pointer from the floor of the train car, towards the ceiling, which is at a height L above the floor. It then reflects from a mirror on the ceiling, and travels back to the floor of the train car. This idea is illustrated in Figure 2. We now ask the seemingly innocuous question, how much time elapses between these two events, according to each observer?

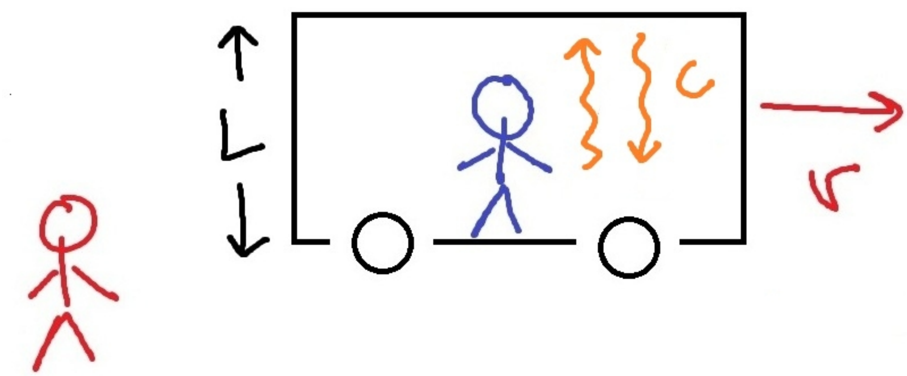

Figure 2: The set-up of the train problem.

For the observer on the train, the answer is straight-forward. Since the light travels a total distance of $2L$, and the speed of the beam of light is c, we find

$$\Delta t = \frac{2L}{c}. \tag{4}$$

For the observer on the ground, the calculation is slightly more involved, yet still straight-forward. Since the observer on the ground witnesses the train moving, he or she will see the light beam travel towards the right before hitting the floor of the train car. If the ground-based observer measures a time $\Delta t'$ between the two events in question, the horizontal distance travelled by the train will be

$$h = v\Delta t'. \tag{5}$$

For this reason, the total distance travelled by the light beam, according to the ground-based observer, is

$$2D = 2\sqrt{L^2 + \left(\frac{1}{2}v\Delta t'\right)^2}, \tag{6}$$

which can be seen by applying the Pythagorean theorem to the set-up in Figure 3. Thus, the amount of time elapse between the two events, according to the observer on the ground, satisfies

$$\Delta t' = \frac{2D}{c'} = \frac{2}{c'}\sqrt{L^2 + \left(\frac{1}{2}v\Delta t'\right)^2}. \tag{7}$$

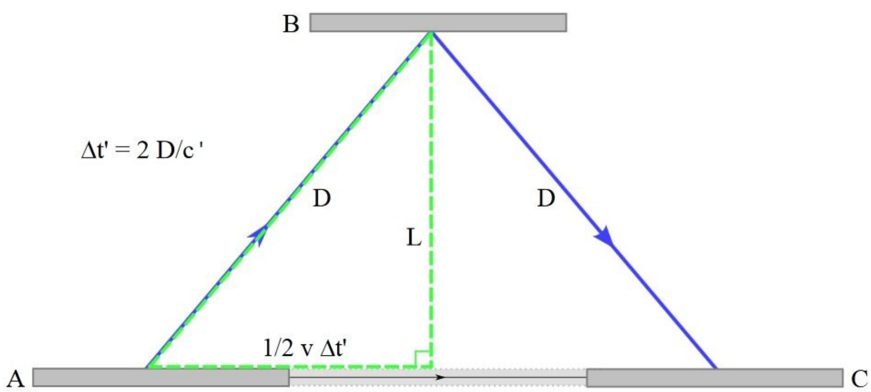

Figure 3: The distance travelled by the light beam, according to the ground-based observer.

Now, if we were describing the motion of some everyday object, such as a ball moving at relatively small velocities, we would find that c', the speed of the ball measured by the ground observer, is some value other than c, the value measured by the observer on the train. We would need to make use of our Galilean velocity addition formula, to find that c' was larger than c, since according to the ground-based observer, the train carries the ball along with it, increasing its velocity. But the postulates of relativity tell us that in fact, for the case of a light beam,

$$c' = c \;\Rightarrow\; \Delta t' = \frac{2D}{c} = \frac{2}{c}\sqrt{L^2 + \left(\frac{1}{2}v\Delta t'\right)^2}. \tag{8}$$

Solving for the time difference, we find

$$\Delta t' = \frac{(2L/c)}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} \equiv \gamma\Delta t. \tag{9}$$

Thus, we see that the time durations measured by the two observers differ by the **Lorentz factor**

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{10}$$

This effect that we have discovered is known as **time dilation**. It tells us that the amount of time observed between two events, as measured by the ground-based observer, is **longer** than that observed by the observer on the train (since the Lorentz factor is always greater than or equal to one). The ground-based observer might claim that the observer on the train has a clock which is running **too slow**. Make sure to understand, however, that this result is not a special property of light beams. While we have used an example regarding light beams to motivate this result, it is true for any two events. To understand why, notice that it is always possible for the observers to perform some action in unison with the motion of the light beam. For example, the observer on the train might snap his fingers as the light leaves the floor, and then stomp his foot as the light reaches the floor again. Since these two events occurs in the same time and same place as the two events we considered previously, our conclusions must be the same. While you may not be totally convinced by this argument, as we will see shortly, you won't need to be in order to believe that time dilation applies to all of events - the experimental status of relativity says that it is so.

Additionally, please make sure to understand that this result is **not** somehow an artefact of some sort of "optical illusion" due to the fact that light has a finite speed of propagation. The result we have derived here made no reference to how long it might have taken for the observers to witness these events after they happened. Optical effects which have to do with the finite propagation speed of light do indeed have interesting consequences, and they are discussed, for example, in section 10.3 of Griffiths' textbook on electrodynamics. But what we are discussing here is an entirely different matter.

Proper Time and The Relativity of Time Dilation

Notice that while the ground-based observer may believe he is measuring times correctly, since both observers are assumed to be inertial observers, the train-based observer should be equally justified in making any claim that the ground-based observer makes, in accordance with the postulates of special relativity. In particular, we could consider a situation in which the ground-based observer had set up a similar experiment, shining a beam of light between two surfaces. This is shown in Figure 4. Were we to perform the same considerations as before, we would now find in this case,

$$\gamma \Delta t' = \Delta t. \tag{11}$$

The time duration between the two events is now shortest for the ground-based observer, and not the train-based observer.

While this result seems to contradict what we found previously, it is in fact entirely consistent, and motivates the notion of **proper time**. Notice that when

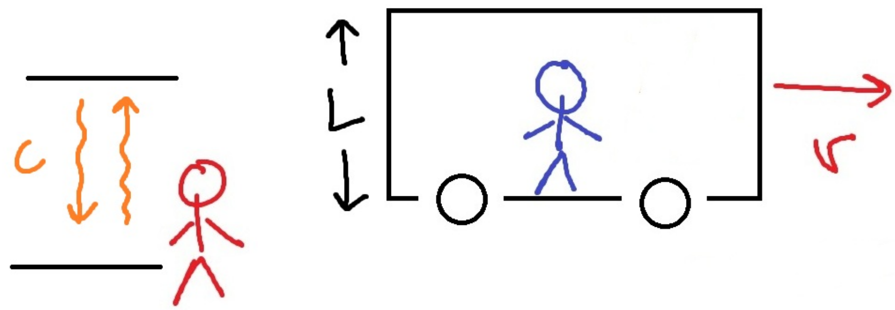

Figure 4: The light beam experiment, conducted on the ground instead of the moving train.

the train-based observer fired the laser beam, the two events we were interested in, light leaving the floor and then hitting it again, occurred in the same physical location, *according to the observer on the train.* This was not true according to the ground-based observer - the two events occurred in two different locations, since he believed the train-based observer was moving with some velocity. In this case, the shorter time duration was measured by the observer on the train. However, when the ground-based observer performed the same experiment on the ground, such that the two events did occur in the same place according to him, then he measured the shorter time duration. Thus, we have found an example of something which is in fact quite general - *the shortest possible time duration between two events is the time duration measured by an observer who witnesses those two events occur in the same place.* This time duration is known as the proper time.

Make sure to understand the the proper time between two events is a property of those two events. There is no such thing as the "proper time reference frame" in which all measured time durations are "correct". Which observer measures the proper time between two events will depend on which observer is stationary with respect to those events. As we saw previously, for some experiments this was the train observer, while for other experiments it was the ground observer.

However, while there is no such thing as the "proper time reference frame," we can define something known as the **proper time of an observer**. Notice that the light beam experiment we have performed is, in principle, an example of a time-keeping device - the two observers could use such an object to measure time in their own reference frame. Similarly, if I am wearing a watch, then (to a good approximation) each tick of the second hand is an "event" which occurs in the same location, according to me. Thus, the time being read off by my watch is the proper time between each tick of the second hand. Since this reading is the one which I would naturally use to measure time durations, it is sensible to refer to this as my own personal proper time. In other words, an observer's proper time is the amount of time duration he or she personally experiences.

The Twin Paradox

Our discussion above has led us to the conclusion that any observer is justified in making the claim that anyone whom he observes to be moving has a clock which is running too slow. But if this is true, it would seem that we can construct a paradox, often known as the **twin paradox**. The usual statement of the paradox is as follows: Two twins, each of course the same age, are on the Earth. One of them decides to embark on an interstellar journey travelling at speeds close to the speed of light, while the other remains on Earth. Each of the two twins is justified in believing that the other has a clock which is running too slow. If the travelling twin returns after many years, which of the two twins is the younger one? If they were to compare clocks after the journey, it would seem as though somehow each one would have a clock with a shorter time duration. But how can two people, both looking at the same clocks, disagree as to what the readings on those clocks are? This general state of affairs is illustrated in Figure 5.

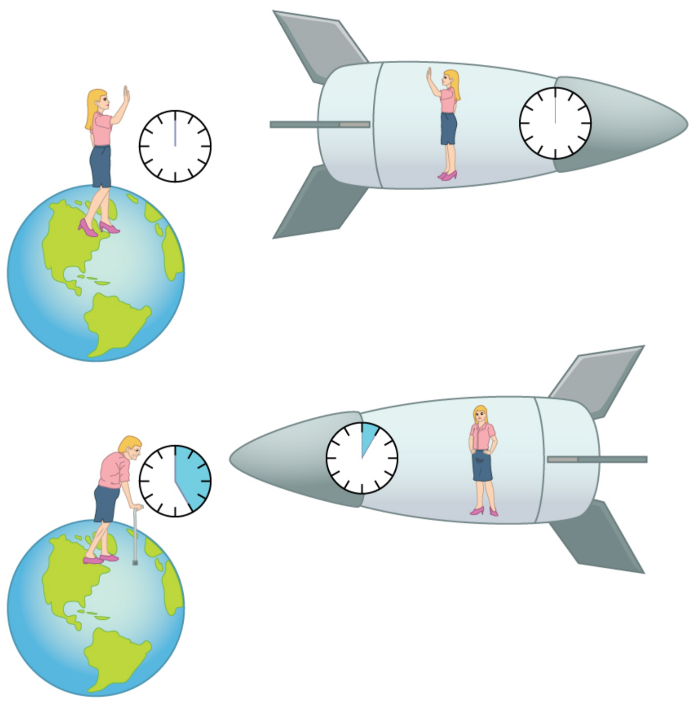

Figure 5: The twin paradox, in its usual formulation.

In fact, however, there is no paradox here - the travelling twin will return as the younger one. The resolution to the paradox comes from remembering that the postulates of special relativity are assumed to hold only for **inertial observers**. Assuming that the observer on Earth is in an (approximately) inertial frame, he is justified in his assumption that the twin in the rocket is experiencing the passage of time at a rate which is "too slow." The rocket observer, during portions of the journey in which she is moving at a constant velocity with respect to the Earth, is also justified in saying the same thing about the Earth-based observer. However, so long as the rocket-based observer wishes to return to Earth, she must accelerate at some point, as she turns around and changes her velocity. At this point, she is no longer an inertial observer, and is no longer justified in making the same claims about time dilation as the Earth-based observer is.

It is often mistakenly said that for this reason, special relativity is not capable of describing accelerated motion. This is false. It is similarly true that in Newtonian Mechanics, Newton's Laws are only valid in an inertial reference frame. However, the primary goal of Newtonian mechanics is to find the acceleration of material bodies. Likewise, while the postulates of special relativity hold only in inertial frames, there is no intrinsic limitation in special relativity to considering only the motion of non-accelerated bodies.

To demonstrate quantitatively that special relativity is in fact capable of making unambiguous statements about accelerated motion, let's consider a more specific example of the twin paradox, shown in Figure 6. An inertial observer is at rest with respect to an inertial set of coordinates. A second observer, indicated in red, then proceeds to travel around in a circle of radius R, with some constant speed v, according to the inertial observer. Because the second observer moves in a circle, he experiences an accelerated motion with respect to the inertial frame. As this motion occurs, according to our time dilation formula, the relation between the two time durations will be

$$\Delta t' = \gamma \Delta t, \tag{12}$$

where $\Delta t'$ is the duration measured by the inertial observer, and Δt is the duration measured by the accelerated observer. According to the inertial observer, the total amount of time elapsed as the accelerated observer moves around in the circle is

$$T' = \frac{2\pi R}{v}. \tag{13}$$

By the time dilation formula, this means the total time the accelerated observer experiences is

$$T = \frac{2\pi R}{\gamma v} = \sqrt{1 - v^2/c^2}\frac{2\pi R}{v}. \tag{14}$$

Thus, as the two observers meet up to compare their clocks, they will find that the accelerated observer will have aged less, and his clock will read less time. There is in fact nothing wrong or paradoxical about this result. We simply must accept the fact, as we did in the case of Newtonian Mechanics, that there are some frames of reference which are inertial, in which a particle that experiences no forces acting on it will feel no acceleration. Any frame of reference moving at constant velocity with respect to such a frame will also be an inertial one. However, any frames of reference which are accelerating with respect to an inertial frame are **not** themselves inertial. While in the Newtonian case the rectifications of this were not particularly profound, we have found here, in the relativistic case, that the difference between an inertial and accelerated frame can affect the passage of time itself.

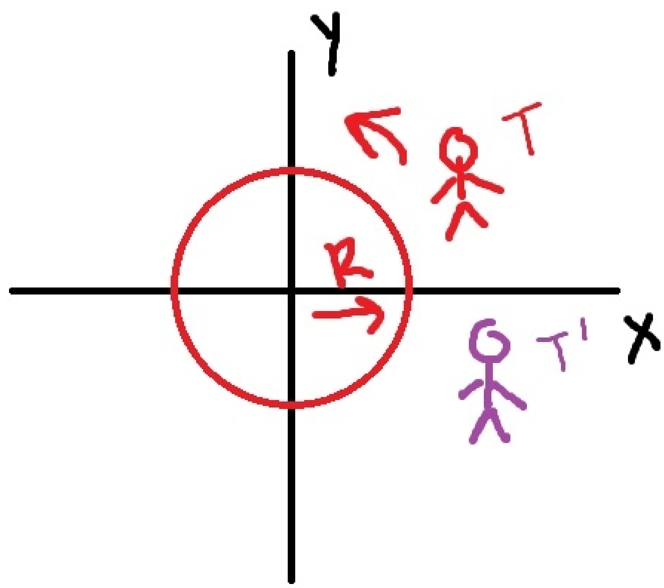

Figure 6: The twin paradox, examined more quantitatively.

In general, since any time duration can be viewed as a sequence of infinitesimal time steps, our formula for time dilation can be generalized to describe accelerated observers which move at varying speeds. At any given moment, our time dilations formula reads

$$dt' = \frac{1}{\sqrt{1 - v\left(t'\right)^2/c^2}} dt. \tag{15}$$

Thus, assuming that our observers have synchronized their clocks so that $T = T' = 0$, if we wish to find the relation between T and T' at some later point, we simply integrate, to find

$$\int_0^{T'} \sqrt{1 - v\left(t'\right)^2/c^2}\; dt' = \int_0^{T} dt = T \tag{16}$$

This gives the amount of time T experienced by the accelerated observer, in terms of the duration T' measured by the inertial observer, assuming that the inertial observer measures a velocity $v\left(t'\right)$ for the accelerated observer.

Experimental Verifications of Time Dilation

At this point, I would like to be able to give some sort of "explanation" for time dilation, or provide some intuition as to why the result we have found "makes sense." But I cannot do this. Even today, our most sophisticated theories of physics simply take special relativity as a starting assumption - no one knows "why" it is true. As far as we know, it is simply a part of the laws of physics which we must accept. Special Relativity disagrees with our intuition about the world, but our intuition about the world around us is typically not based upon experiences with objects that move at speeds close to the speed of light.

To perhaps understand better why our everyday intuition does not inform us about the laws of special relativity, we can examine the behaviour of the Lorentz factor for velocities which are small in comparison with the speed of light. In this case, we can perform a Taylor series expansion, in order to find

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2. \tag{17}$$

The deviation from unity is given by a term which is quadratically small in the velocity ratio. For example, for a car moving at 30 meters per second, the Lorentz factor would be

$$\gamma \approx 1 + \frac{1}{2}\left(10^{-7}\right)^2 = 1 + 5 \times 10^{-15}. \tag{18}$$

Thus, during one second of elapsed time, the difference between special relativity and Newtonian mechanics amounts to a mere five femtoseconds! This is the time scale over which most molecular interactions occur, and the time it takes light to travel a distance roughly equal to the diameter of a virus. It is no surprise then that it took until the 20$^{\text{th}}$ century for physicists to notice any discrepancy between the two theories.

However, the predictions of time dilation were, in fact, eventually verified. The first direct evidence for time dilation came from the experiments of Rossi and Hall in 1941. The two experimenters were interested in the behaviour of **cosmic muons**. Highly energetic charged particles (whose origin is still somewhat of a debate) from elsewhere in the galaxy are constantly striking the upper atmosphere of the Earth. The resulting collisions possess enough energy that they can in fact create new subatomic particles, in a process similar to the one which occurs in particle accelerators here on Earth. A large fraction of the particles produced are muons, a subatomic particle which is similar in almost every way to the electron, except with a mass which is about 206.7 times larger. These particles have a half-life of about $\tau = 1.5\ \mu$s.

After these muons are created in the upper atmosphere, they travel towards the surface of the Earth, typically at very large velocities with respect to the ground. For this reason, their motion is typically within the regime which would be considered relativistic. Rossi and Hall set up a detector at the top of a mountain in Colorado, and measured the number of muons travelling towards the surface of the Earth. If the Earth were some distance d below, then the amount of time an Earth-based observer would measure for the transit of the muons would be

$$t' = d/v, \tag{19}$$

where v is the velocity of the muons. Based on the usual formula for radioactive decay in terms of half-life, this implies that if the experimenters measured a total of N_0 muons at the top of the mountain, then they should observe

$$N = N_0 e^{-t'/\tau} = N_0 e^{-d/v\tau} \tag{20}$$

muons at the bottom of the mountain.

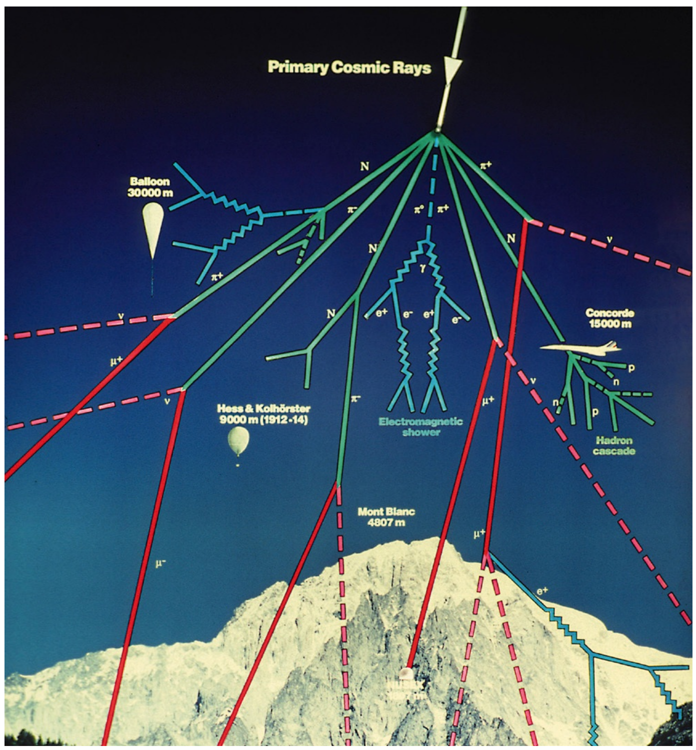

Figure 7: Rossi and Hall were the first to measure the effects of time dilation, by observing the survival probability of cosmic muons.

However, this is in fact **not** the number of muons they observed. The reason that our prediction was incorrect is that we have not properly taken into account the effects of time dilation. Because the muons are moving relativistically, we must take into account the fact that time elapses more slowly in their frame of reference. The time duration in the frame of the muons is in fact

$$t = t'/\gamma = d/v\gamma, \tag{21}$$

so that the correct prediction for the number of muons is

$$N = N_0 e^{-t/\tau} = N_0 e^{-d/v\gamma\tau}. \tag{22}$$

This does indeed agree with the number of muons measured by Rossi and Hall.

Since the experiments of Rossi and Hall, many more experiments have verified the predictions of time dilation to an incredible level of accuracy. In 1971, Hafele and Keating flew a set of two highly sensitive atomic clocks in two airplanes, travelling around the Earth. After the journey, the clocks were compared with a set of clocks that had been left on the surface of the Earth, and the predictions were perfectly in line with the effects of special relativity. As recently as 2010, highly sensitive optical clocks were used to verify the presence of time dilation for objects moving as slowly as ten meters per second. In fact, the global positioning system is constantly verifying the predictions of special relativity, since its satellites must take into account the effects of time dilation in order to provide accurate results.

The fact that all of these experiments result in precisely the same conclusions is further evidence that time dilation is in fact a fundamental property of the universe, and not simply some weird effect that is just a more mundane consequence of bodies experiencing motion. Some students first learning about special relativity wonder if perhaps the effects of time dilation are just a result of material bodies being affected by motion - perhaps, for example, a watch being accelerated reads a slower time because its motion causes its internal components to be "shaken around," or something of that manner. But the results of Rossi and Hall show that this cannot be true. As far as we know, muons are fundamental point particles with no internal structure - there is nothing to "move around," and no weird optical illusions which might lead a muon (which is of course not a sentient being) to somehow "see things funny." In fact, the formula for the Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{23}$$

makes no reference to any kind of "material composition" or "internal structure" - it only depends on the velocity of the object, and the speed of light. The fact that the time dilation formula remains the same for muons, atomic clocks, optical clocks, and even trains, implies that it truly is a fundamental property of time itself.

Length Contraction

Our relativistic prediction for the correct number of muons was based on the assumption that the muons had "clocks" which were "running slow." However, according to the postulates of relativity, an observer moving with the muons would be equally justified in claiming that his clock was running correctly. Why, then, was our prediction for the number of muons correct? The only possible resolution is that according to the muons, as they travel from the top of the mountain to the bottom, the distance they believe they travel is not d, but rather

$$d' = d/\gamma. \tag{24}$$

In other words, the muons, which see the ground moving up towards them, believe that the height of the mountain is in fact less than d. This effect, in which an observer believes that moving objects are shorter than when they are at rest, is known as **length contraction**. We have discovered this effect based upon the results of our muon experiment, but it could be equally well derived using the same sorts of considerations we made when working out the predictions of time dilation (as Taylor does in section 15.5). This idea is illustrated further in Figure 8.

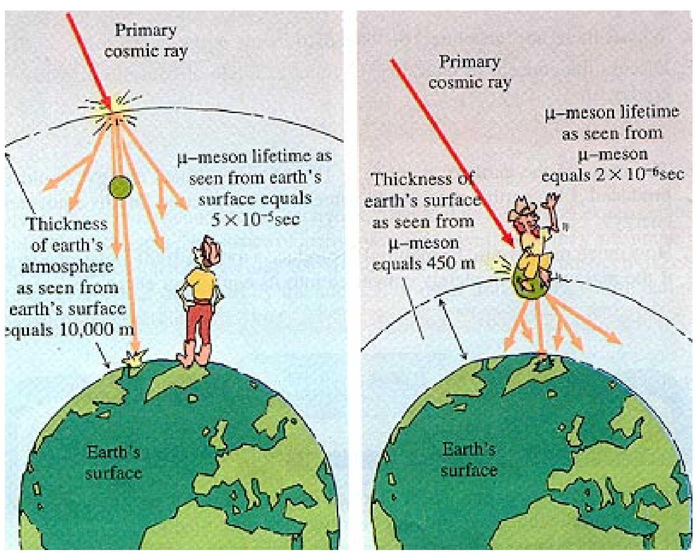

Figure 8: The results of the Rossi and Hall experiment, as seen from two different, yet equally valid, frames of reference.

Similar to the definition of proper time, we can define the **proper length** of an object to be its length as measured by an observer who is at rest with respect to the object. The proper length of an object is always the longest possible length measured by any observer - all other observers will measure a length which is shorter.

To better understand the ramifications of length contraction, let's consider a famous thought experiment known as the **barn and ladder paradox**, shown in Figure 9. In this "paradox," we consider a barn, sitting on the ground, which has a length of four meters, as measured by someone sitting on the ground, observing the barn to be stationary. This is then the proper length of the barn. A particularly skilled runner then runs towards the open door of the barn at a speed which is 3/5 of the speed of light, carrying a ladder with a rest length of five meters. In particular, since the runner is at rest with respect to the ladder, the proper length is the length he measures for the ladder. The question we then ask is this: as the runner passes through the barn, is the ladder ever fully contained within the barn?

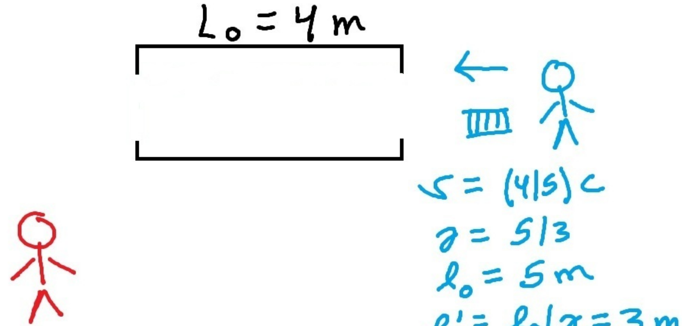

Figure 9: The interplay between length contraction and time dilation, as demonstrated by the barn and ladder paradox.

According to the observer on the ground, the answer is a clear and resounding yes. As the runner travels towards the barn, length contraction effects reduce the length of the ladder to a mere three meters. Since this is a full meter less than the length of the barn, there is certainly a point in time in which the ladder is fully contained within the barn - after the back end of the ladder passes through the front door, but before the front end of the ladder exits through the back door. However, the runner comes to a different conclusion. The barn is moving towards him, and as far as he is concerned, it is the object which contracts, not the ladder. Thus, the barn has a length less than four meters, and can never fully contain the ladder, with a length of five meters. How can two observers disagree on what is seemingly a very simple question?

The resolution to this paradox lies in the fact that in our fully relativistic universe, we are forced to accept the fact that questions regarding the simultaneity of two events may no longer be meaningful. Having abandoned any notion of one universal time in the universe, we must also accept the fact that whether or not the front end of the ladder left the barn before the back end entered, or whether the ladder was ever simultaneously contained within the barn, is a question which has no correct answer.

Causality

The barn and ladder paradox can also help us understand one of the most important, and most profound, implications of special relativity. To see how, we imagine a slightly modified version of the original set-up. This time, the back door of the barn is closed, while the front door remains open. Additionally, there is now a sensor positioned at the location of the front door, which detects whether or not the back end of the ladder has passed through the front door. Once the back end of the ladder has passed through the front door, the sensor sends a signal to the back door, telling it to open. This is indicated in Figure 10. The question we now ask is the following: does the door open in time for the ladder to pass through the barn, or does the ladder crash into the back door?

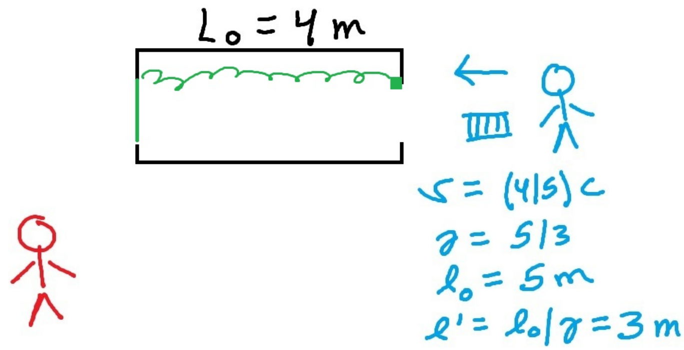

Figure 10: A modified version of the barn and ladder paradox, which demonstrates the role of causality in our universe.

According to the runner, the answer is that the door does **not** open in time - the ladder is longer than the barn (according to him), and thus he can never fit the entire ladder inside of the barn. As a result, the sensor does not trigger in time, and the ladder crashes into the back door. However, according to the ground-based observer, the ladder is capable of fitting within the barn - the back end should pass through the front door, setting off the sensor, and opening the back door in time. In this case, the disagreement between the two observers is even more severe - they disagree about whether the back door will be smashed to pieces or not! If we are to accept the ramifications of this paradox, then we would be led to believe that after the dust settles, and the two observers come to examine the barn, they will somehow disagree as to whether the door which they are both looking at is either broken or intact! This is certainly nonsensical, and no such type of behaviour has ever been observed in our universe. Similarly, any sort of idea that the door might magically "fix itself" as the runner slows down, so that his observations agree with those of the ground observer, seems equally ridiculous (and would certainly constitute an enormous violation of the second law of thermodynamics).

However, we have so far been neglecting to consider the fact that the signal from the sensor needs to travel to the back door, and we should expect that this will take some finite amount of time. This may alter our conclusions about what the ground-based observer witnesses, since this delay in propagation may cause a failure of the back door to open. Indeed, if the signal takes too long to reach the back door, the ladder will still crash into the closed door, even though it would be valid for the ground-based observer to conclude that the ladder was fully contained within the barn. In fact, it turns out that so long as the signal cannot travel faster than the speed of light, the door will not open in time. While I will leave the verification of this fact as an exercise for those who are interested, the corresponding math is just some simple algebra. In any event, the important conclusion we have come to is this: in order to avoid severe physical paradoxes, we must assume that **no causal influence or signal can ever move faster than the speed of light, with respect to any observer**. This is known as the principle of **causality**. Without it, the effects of length contraction and time dilation would cause our universe to be rife with absurd physical paradoxes.

A few years ago, physicists from the Gran Sasso national laboratory in Italy made national headlines for supposedly observing particles moving faster than the speed of light. In their experimental set-up, neutrinos, a type of fundamental particle, were fired from CERN in Switzerland, and travelled through the Earth to the OPERA experiment (which is based in the exact same experimental hall I worked in as an undergraduate). Since neutrinos interact very weakly with other forms of matter, this beam passes through the Earth mostly unaffected, and the detector in the OPERA experiment occasionally detects a neutrino. Initial experimental data indicated that these neutrinos which were detected had somehow travelled **faster** than the speed of light! While the results were all over the news for several days, most physicists (including myself) were quite skeptical, and the reason was because of causality. While it was tempting to believe the results, and while it would have been incredibly exciting were they true, time dilation and length contraction are not compatible with faster-than-light travel, unless we accept some very absurd physical paradoxes. Since time dilation and length contraction are an experimentally verified property of the universe, and since no one has yet witnessed a paradox as absurd as the one demonstrated by the barn and ladder paradox, the likely conclusion was that the OPERA results were in error. For days, many potential explanations were proposed, including some relatively outlandish ones (it was even supposed that if the physicists transporting the synchronized clocks between the two laboratories had stopped for lunch for too long at the top of the Swiss Alps, that that effects of gravitational time dilation, an effect which we have not yet discussed, may have caused the error). However, after several weeks of conjecture, the ultimate explanation proved to be much more mundane - a loose cable in the OPERA experiment had caused a faulty reading. It was a sad result for those of us looking to find new and exciting physics, but a result which was expected nonetheless.

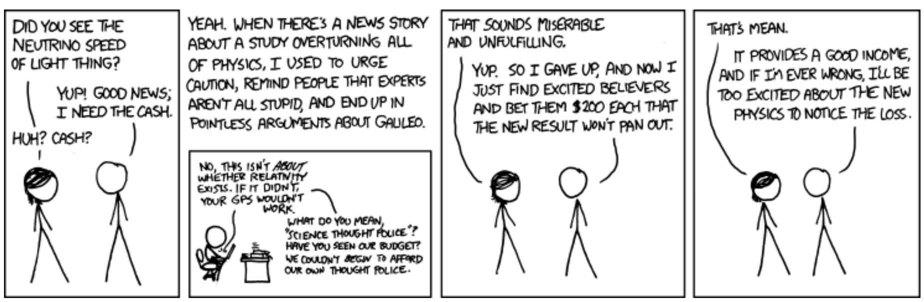

Figure 11: XKCD # 955