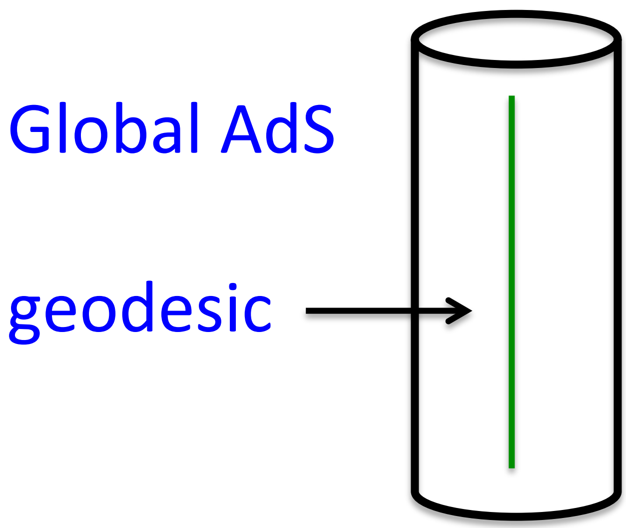
Hovering Black Holes

Gary Horowitz
UC Santa Barbara
(with Iqbal, Santos, and Way,
1412.1830)

Geometry of AdS_4

Globally, AdS can be conformally rescaled to fit inside a cylinder.

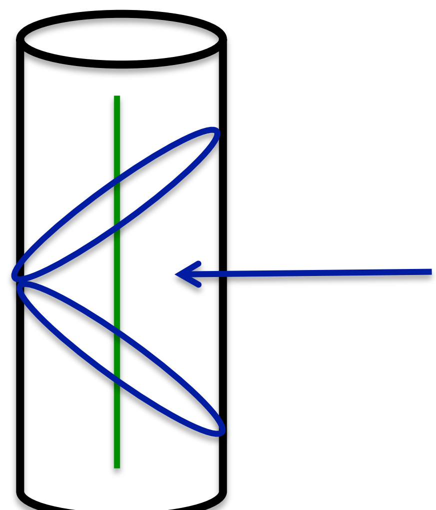


Geometry of AdS_4

Globally, AdS can be conformally rescaled to fit inside a cylinder. Poincare coordinates cover only one part of this cylinder:

$$AdS_4 : \quad ds^2 = \frac{1}{z^2}[-dt^2 + dr^2 + r^2 d\phi^2 + dz^2]$$

Global AdS



Poincare patch

The problem

Consider D = 4 Einstein-Maxwell with $\Lambda < 0$.

Fix the metric and vector potential on the AdS boundary to be

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2, \quad A = \mu(r) dt$$

Find the zero temperature solutions. Simple solutions known for $\mu = 0$ (AdS) and $\mu = \text{const}$ (planar RN AdS). We will demand $\mu \to 0$ as $r \to \infty$.

(Related work by Blake, Donos, and Tong, 1412.2003.)

Gauge/gravity duality

String theory on spacetimes that asymptotically approach anti-de Sitter (AdS) x compact space, is equivalent to an ordinary gauge theory.

The asymptotic (small z) region is dual to UV physics in the gauge theory, while large z (near the Poincare horizon) is dual to IR physics.

Motivation

In applications of holography to condensed matter, μ represents the chemical potential. Our gravity solution describes the effects of a single charged defect at a quantum critical point.

Questions:

1) What is the induced charge density?
2) Is the IR behavior modified?
3) Are there universal quantities which are independent of the shape of the impurity?

Scaling argument

Adding a chemical potential corresponds to adding to the CFT action: $\int d^3x\, \mu(r)\rho(r)$

↑ charge density

If $\mu(r) = a/r^{\beta}$ asymptotically, then dimension of a is $1 - \beta$. So

$\beta > 1$ is irrelevant ← Start with this

$\beta = 1$ is marginal

$\beta < 1$ is relevant

Linear example

Start with pure AdS and μ = 0:

$$ds^2 = \frac{1}{z^2}[-dt^2 + dr^2 + r^2 d\phi^2 + dz^2]$$

Solve Maxwell's equation with $\mu(r) = \frac{a}{(r^2+\ell^2)^{3/2}}$

Solution is $A_t(r,z) = \frac{a(z+\ell)}{\ell[r^2+(z+\ell)^2]^{3/2}}$

The induced charge density is found by expanding the solution in z: $A_t = \mu(r) - z\,\rho(r) + O(z^2)$

Expanding our linearized solution we find:

$$\rho(r) = -\frac{a\left(r^2 - 2\ell^2\right)}{4\pi\ell(r^2+\ell^2)^{5/2}}$$

Note:

1) $\rho > 0$ for small r but $\rho < 0$ for large r. In fact, the total charge vanishes!
2) ρ falls off like $1/r^3$

Claim: These properties hold for all $\mu(r) \to a/r^\beta$ with $\beta > 2$ both for linearized and exact solutions.

$\rho \sim 1/r^3$ even when μ falls off faster:

$$\text{If} \quad \tilde{\mu} = d\mu/d\ell = \frac{3a\ell}{(r^2+\ell^2)^{5/2}}$$

$$\text{Then} \quad \tilde{\rho} = d\rho/d\ell \quad \text{and} \quad \rho \sim -\frac{a}{4\pi\ell r^3}$$

$$\text{So} \quad \tilde{\rho} \sim \frac{a}{4\pi\ell^2 r^3}$$

$$Q = 0 \text{ for all } \ell \;\Rightarrow\; \tilde{Q} = 0$$

Can continue to take more derivatives with same result.

The (numerical) solutions

We are looking for static, axisymmetric Einstein-Maxwell solutions. Have to solve coupled nonlinear PDE's for 6 functions of two variables.

Ansatz: $A = A_t\, dt$ and

$$ds^2 = -G_1 dt^2 + G_2(dz + G_3 dr)^2 + G_4 dr^2 + G_5 d\phi^2$$

Boundary conditions are: smooth extremal horizon, asymptotically AdS (with flat boundary metric), and $A_t = \mu(r)$ at infinity.

(Solutions found by J. Santos and B. Way)

We considered 4 different profiles for $\mu(r)$ (with $\beta > 2$)

$$\mu_{I_1}(r) = \frac{a}{\left(\frac{r^2}{\ell^2} + 1\right)^{3/2}}$$

$$\mu_{I_2}(r) = \frac{a}{\left(\frac{r^2}{\ell^2} + 1\right)^{4}}$$

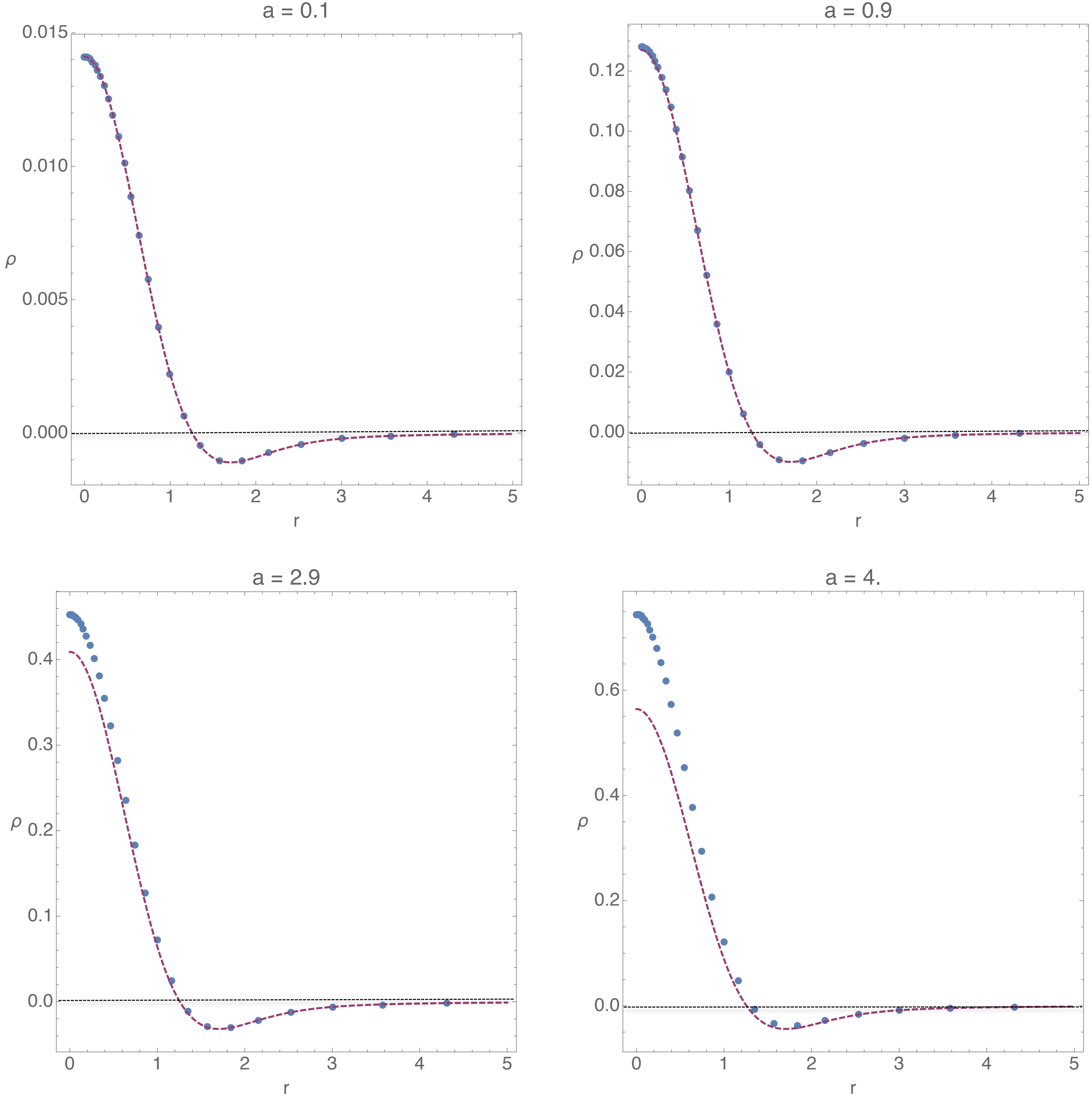
$$\mu_{I_3}(r) = a\, e^{-\frac{r^2}{\ell^2}}$$

$$\mu_{I_4}(r) = \frac{a\, r^2}{\ell^2 \left(\frac{r^2}{\ell^2} + 1\right)^{4}}$$

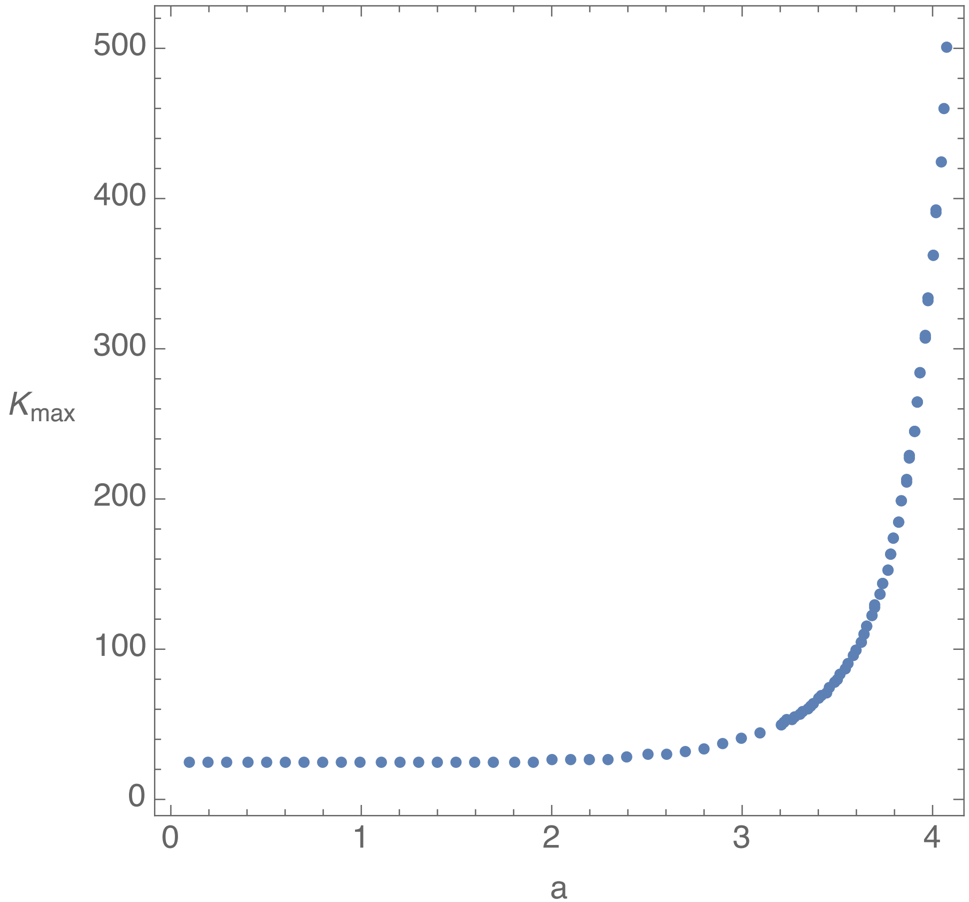
1^{st} Set of Results (for $\beta > 2$)

- The solutions all have a standard Poincare horizon (cf Hickling, Lucietti, and Wiseman, 1408.3417)
- Charge density $\rho \sim 1/r^3$ (in all cases)
- Total charge $Q = 0$ (in all cases)
- Solutions only exist for $a < a_{max}$

ρ vs r



Maximum value of $K = R_{abcd} R^{abcd}$ as a function of the amplitude of chemical potential



Simple argument for $\rho \sim 1/r^3$

$$\text{Suppose} \quad \mu = a\delta(\vec{x})$$

δ has dimension 2 so a has dimension -1.
Linear response: ρ is proportional to a,
But ρ has dimension 2 so $\rho \sim a/r^3$.

Simple argument for Q = 0

Suppose we start with $\mu = 0$ and slowly increase it: $\mu = a(t)/r^{\beta}$. Current conservation implies

$$D_i j^i = -\partial_t \rho \sim k \frac{\dot{a}(t)}{r^3}$$

So total charge satisfies

$$\frac{dQ}{dt} = \lim_{r\to\infty} r \oint d\phi j^r \sim \lim_{r\to\infty} k \frac{\dot{a}(t)}{r} = 0$$

Since Q = 0 initially, it stays zero.

What about a > a_{max} ?

Solutions do exist if you allow for a static, spherical, extremal BH hovering above the Poincare horizon.

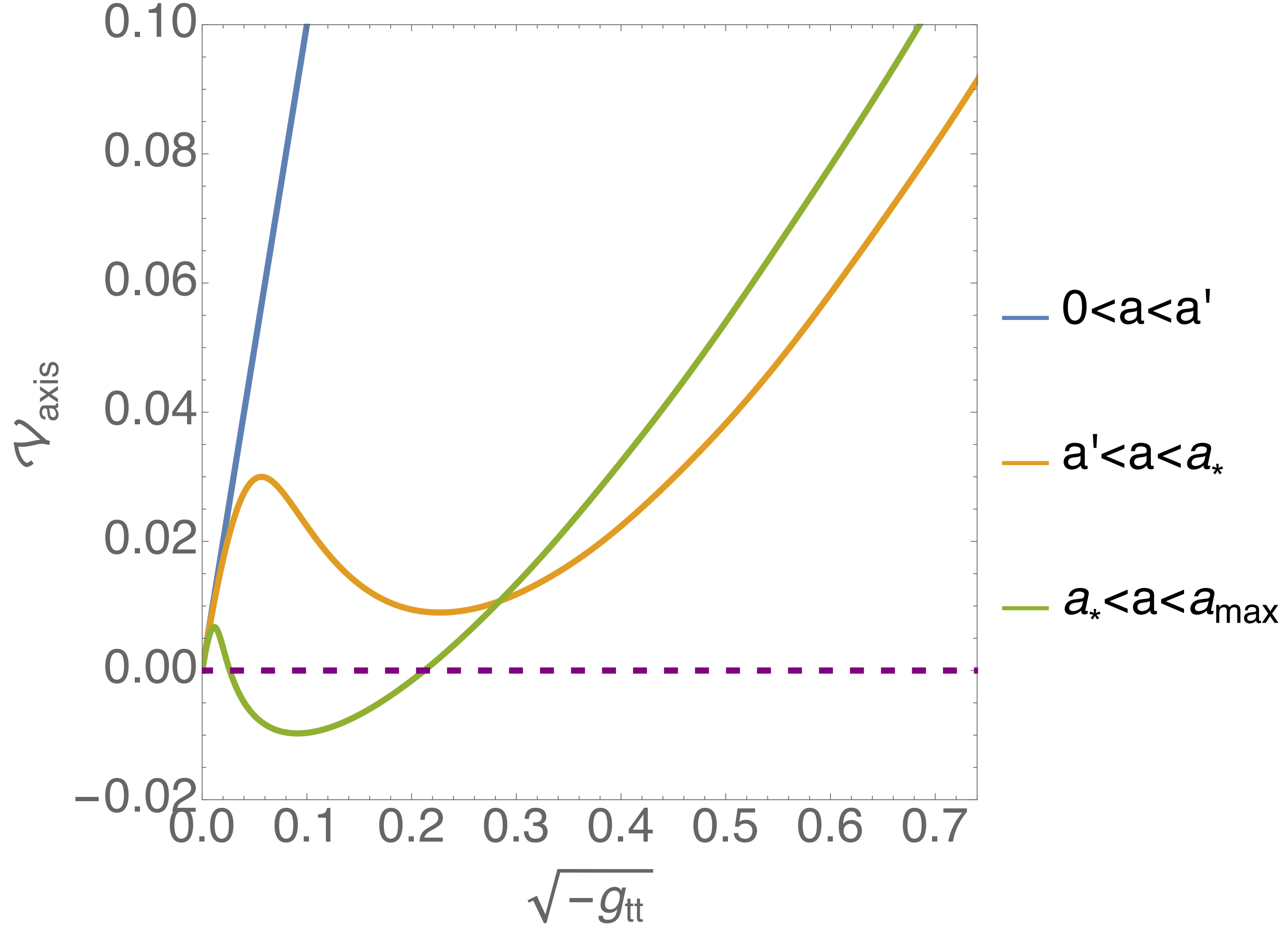
To explore this possibility, look for static orbits of q = m test particles (cf Anninos et al 1309.0146). Extremize

$$S = \int \left[\sqrt{-g_{ab}\dot{X}^a \dot{X}^b} - A_a \dot{X}^a \right] d\tau$$

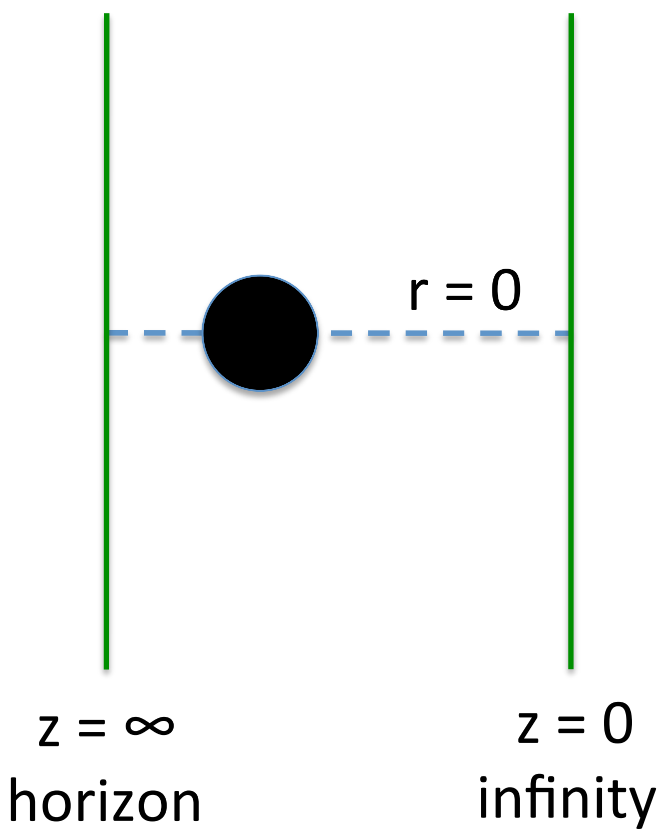
Static orbits correspond to local extrema of

$$\mathcal{V} = \sqrt{-g_{tt}} - A_t$$

The local minimum must occur along the axis of rotational symmetry. We find: (V_{min} = 0 at a = a_*)



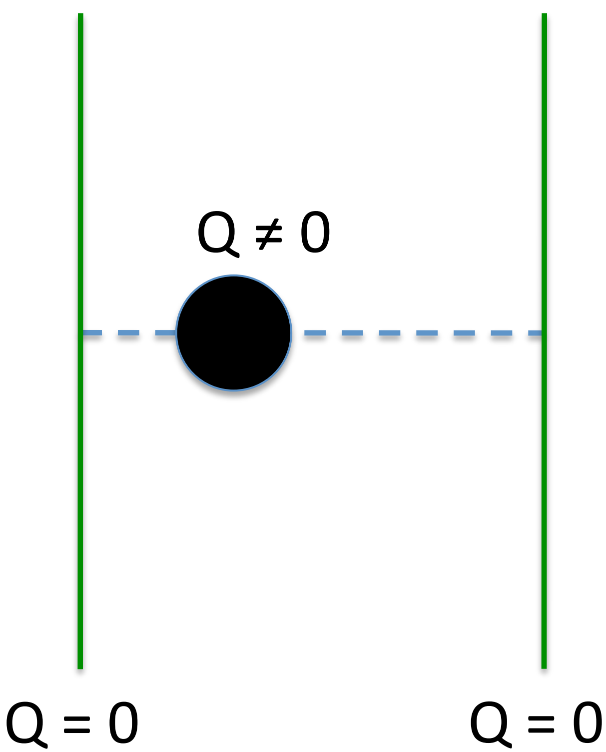
Hovering black hole solutions have been constructed numerically



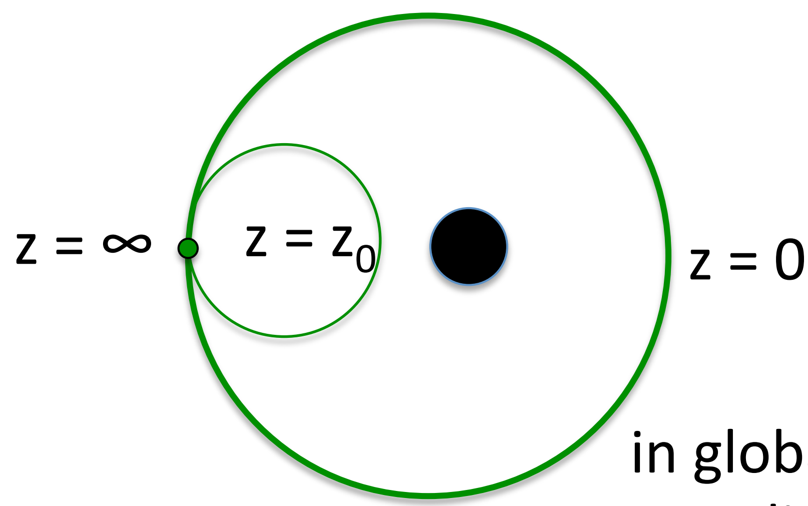
Properties

- Still have standard Poincare horizon in IR
- Near horizon geometry is exactly RN AdS
- A_{BH} -> 0 as a -> a_*, and grows monotonically as amplitude increases
- BH bigger than the AdS radius have been found

Where does the flux go?

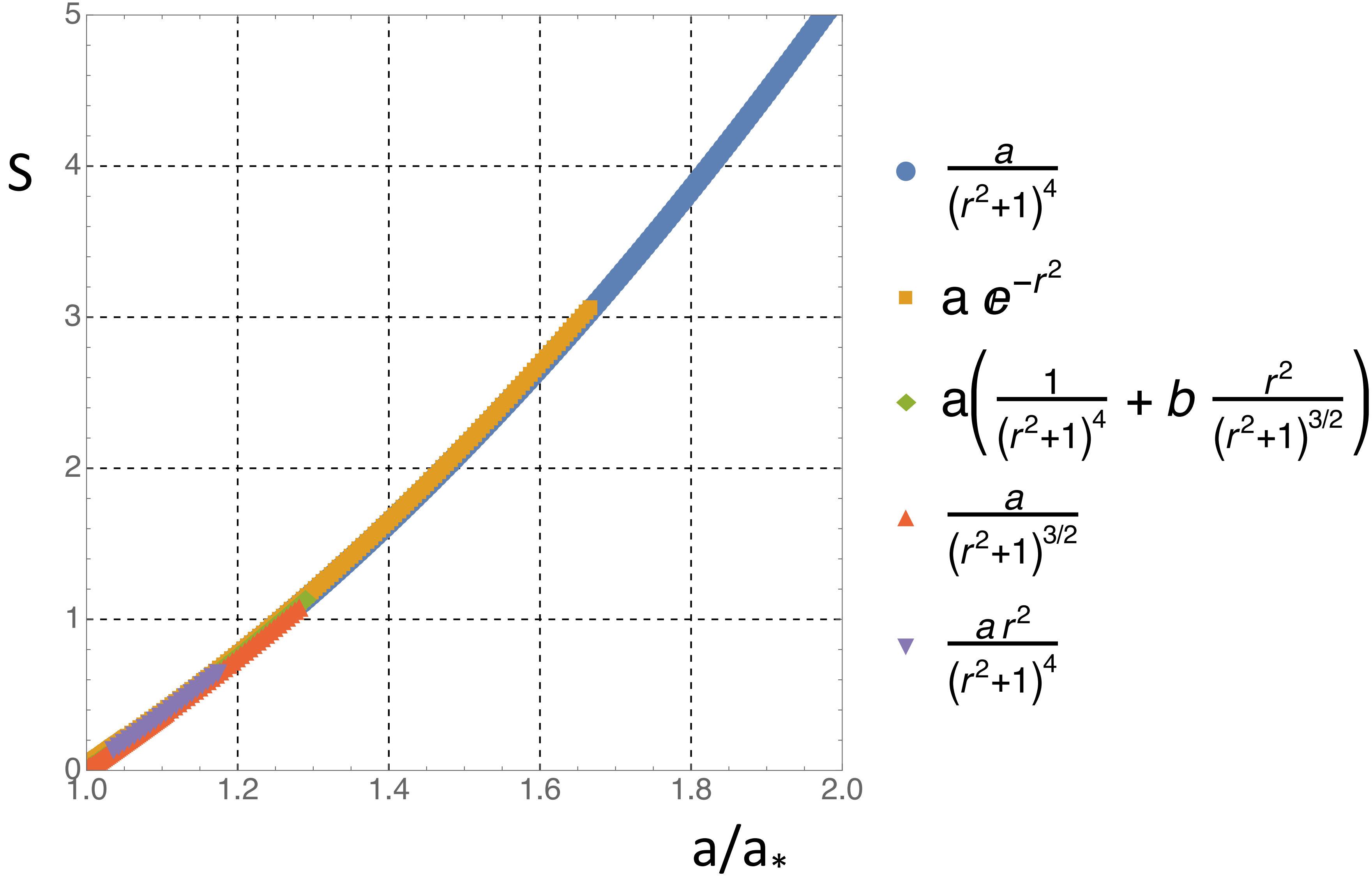


It can't go to the Poincare horizon or infinity. It must leak out the sides.

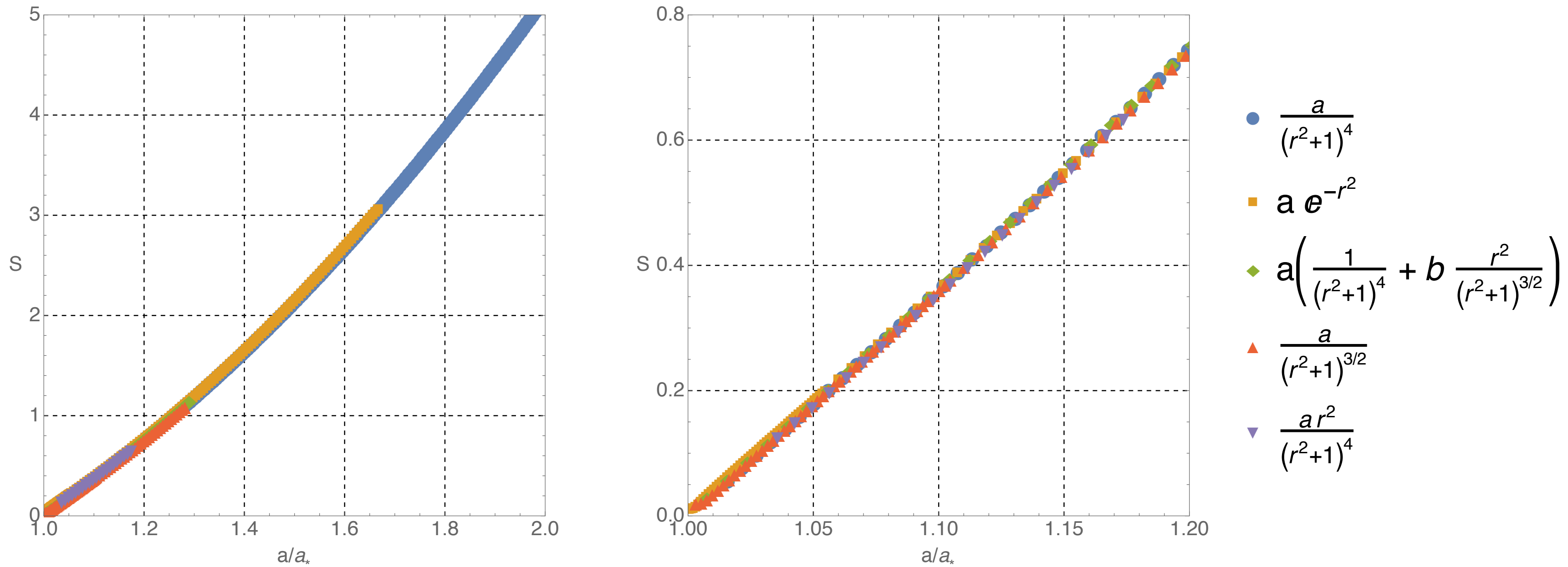


in global coordinates

Growth of BH with amplitude is universal!



The behavior near a_* is linear: $S = c\,(a - a_*) + \ldots$



This is similar to extreme RN AdS: $S = \pi(\mu^2 - 1)/3$
but the slope is different.

The special case: $\mu \sim 1/r$

Suppose $\mu(r) = a/r$ everywhere. The boundary condition $A = \mu(r)\, dt$ is invariant under scaling symmetry $(r, t) \to \lambda\, (r, t)$.

In fact, SO(2,1) x SO(2) subgroup of full SO(3,2) conformal symmetry is preserved.

To make this manifest, rescale the boundary metric:

$$-dt^2 + dr^2 + r^2 d\phi^2 = r^2 \left(\frac{-dt^2 + dr^2}{r^2} + d\phi^2 \right)$$

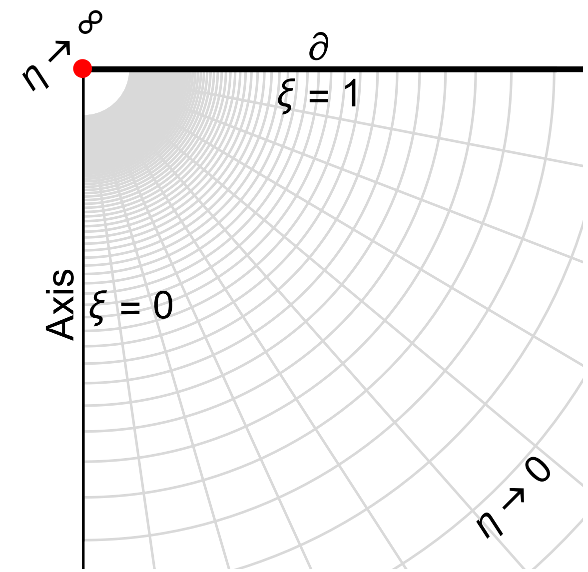
First write AdS_4 with SO(2,1) x SO(2) symmetry:

$$ds^2 = \frac{1}{z^2}[-dt^2 + dr^2 + r^2 d\phi^2 + dz^2]$$

Introduce polar coordinates for the r, z plane, with $\xi = \sin\theta$ and inverse radius η. Then $r = \xi/\eta$ and

$$\mathrm{d}s^2 = \frac{1}{(1-\xi^2)}\left[-\eta^2\mathrm{d}t^2 + \frac{\mathrm{d}\eta^2}{\eta^2} + \frac{\mathrm{d}\xi^2}{1-\xi^2} + \xi^2\mathrm{d}\phi^2\right]$$

$z^2 = (1-\xi^2)/\eta^2$ so the AdS_4 Poincare horizon is the same as the AdS_2 horizon.



Next consider the magnetically charged hyperbolic black hole:

$$ds^2 = \rho^2 \left[-f(\rho) dt^2 + \frac{d\rho^2}{\rho^4 f(\rho)} + \eta^2 dx^2 + \frac{d\eta^2}{\eta^2} \right]$$

where

$$f(\rho) = 1 - 1/\rho^2 - 2M/\rho^3 + q^2/\rho^4, \quad A_x = q\eta$$

Analytically continue t = i φ, x = i t, q = ia.

Fix M so period of φ is 2π. Then $\rho_{min} < \rho$ and

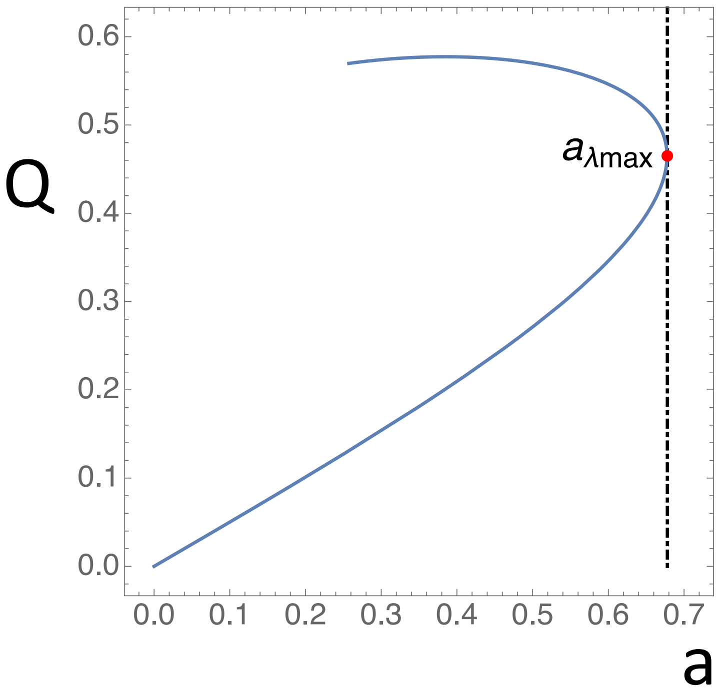
$$ds^2 = \rho^2 \left[-\eta^2 dt^2 + \frac{d\eta^2}{\eta^2} + \frac{d\rho^2}{\rho^4 f(\rho)} + f(\rho) d\phi^2 \right], \quad A_t = a\eta$$

This is the exact bulk solution for μ(r) = a/r.

What is the charge density?

The charge density vanishes over most of the boundary. However there is a contribution from the asymptotic region of AdS_2. In the original Poincare coordinates, this is concentrated at r = 0.

The solution describes a point charge. (Total charge is now nonzero.)

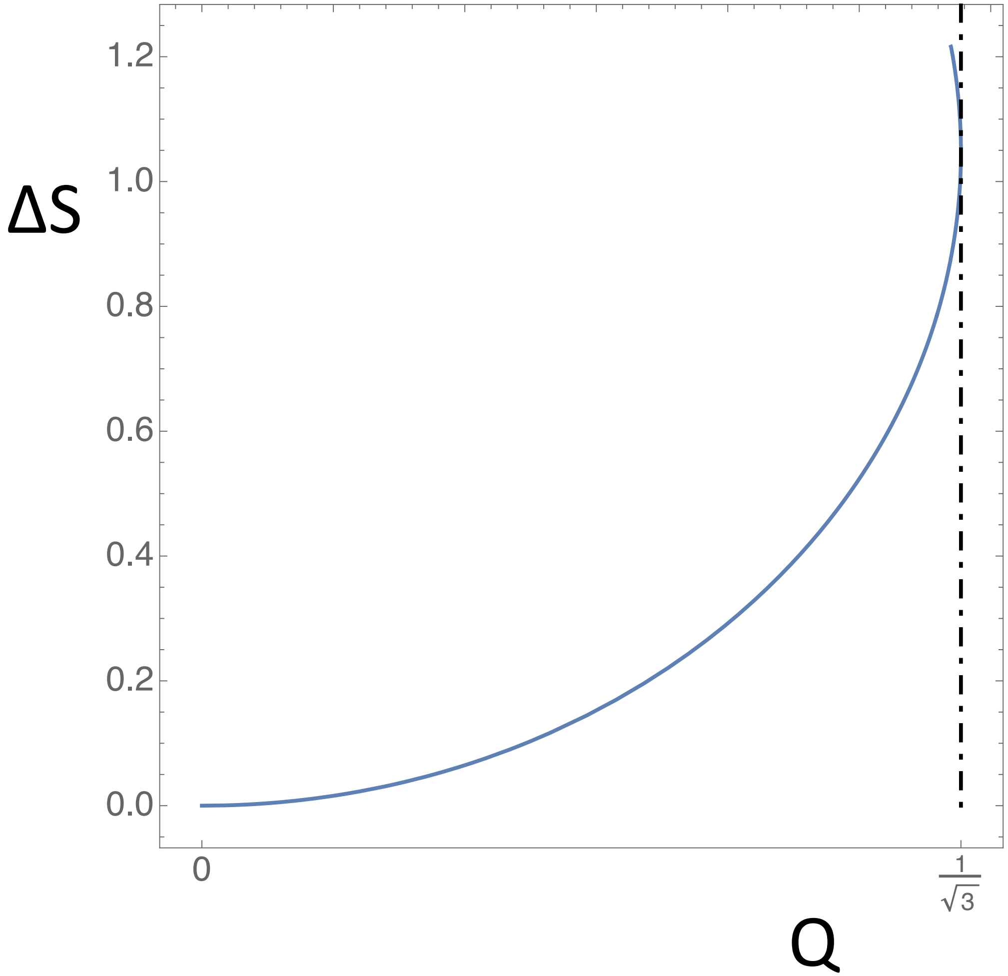


A new extremal horizon

The Poincare horizon of AdS_2 ($\eta = 0$) defines a new extremal horizon, extending out to infinity.

Its area is infinite, but one can define a regulated area by subtracting the area of the Poincare horizon without the defect.

Entropy of the defect



More general marginal μ(r)

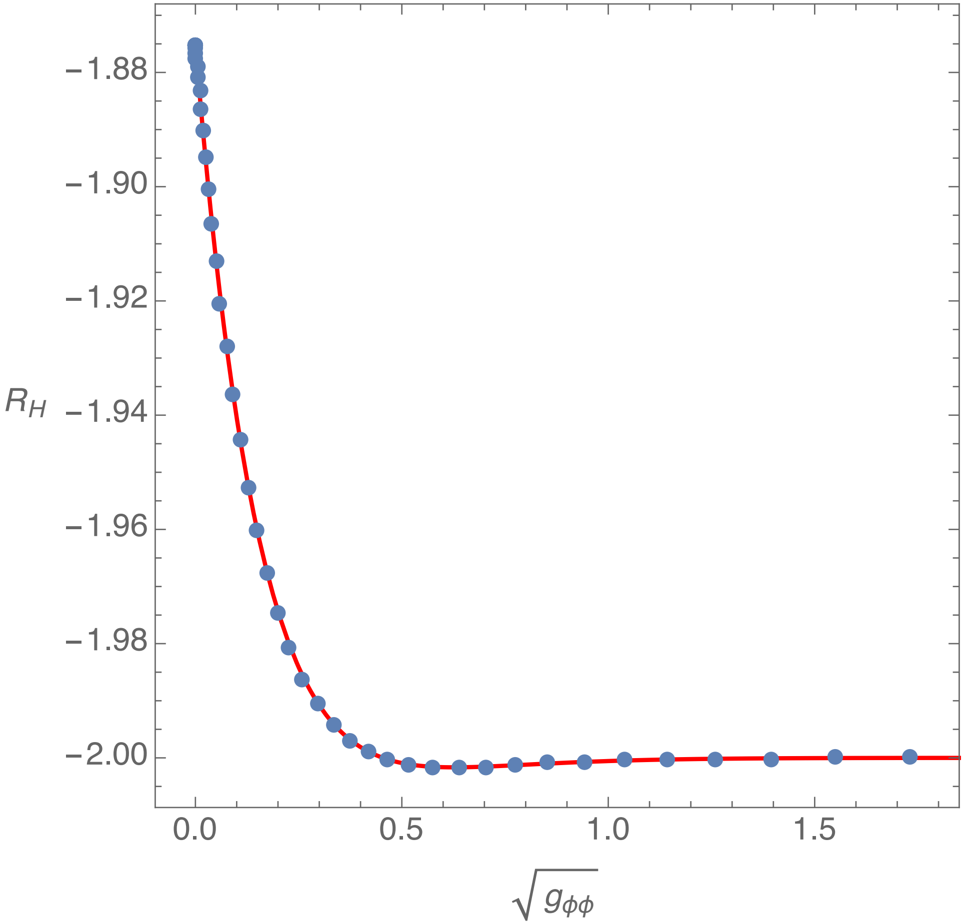
We constructed solutions with

$$\mu_{M_1}(r) = \frac{a}{\left(\frac{r^2}{\ell^2}+1\right)^{1/2}}$$

$$\mu_{M_2}(r) = a\left[\frac{1}{\left(\frac{r^2}{\ell^2}+1\right)^4} + \frac{b\,r^2}{\ell^2(\frac{r^2}{\ell^2}+1)^{3/2}}\right]$$

both with and without black holes. In all cases, the horizon geometry agrees with the point charge.

Scalar curvature of horizon geometry



Red line is analytic solution from point charge. Blue dots are numerical data for M1 with a = .2

The charge density is now a smooth function, and again falls off like $1/r^3$.

The total charge is determined by the coefficient of the $1/r$ fall off of μ. It agrees with the point charge solution with the same fall-off.

This is required since the total charge on the IR horizon must agree with the charge at infinity.

Discussion

What are the consequences of hovering black holes in the dual theory?

There are a large number of approximately degenerate states localized around the defect. Analogy: QM in 2+1 dim

A signal sent toward the defect will be largely unaffected if its energy is very high or low. But at intermediate energies it will thermalize with the degenerate states.

The universal growth of hovering black holes is reminiscent of Choptuik scaling but:

1) We are considering static T = 0 ground states not dynamical collapse.
2) Our universality extends to large BH not just small ones.

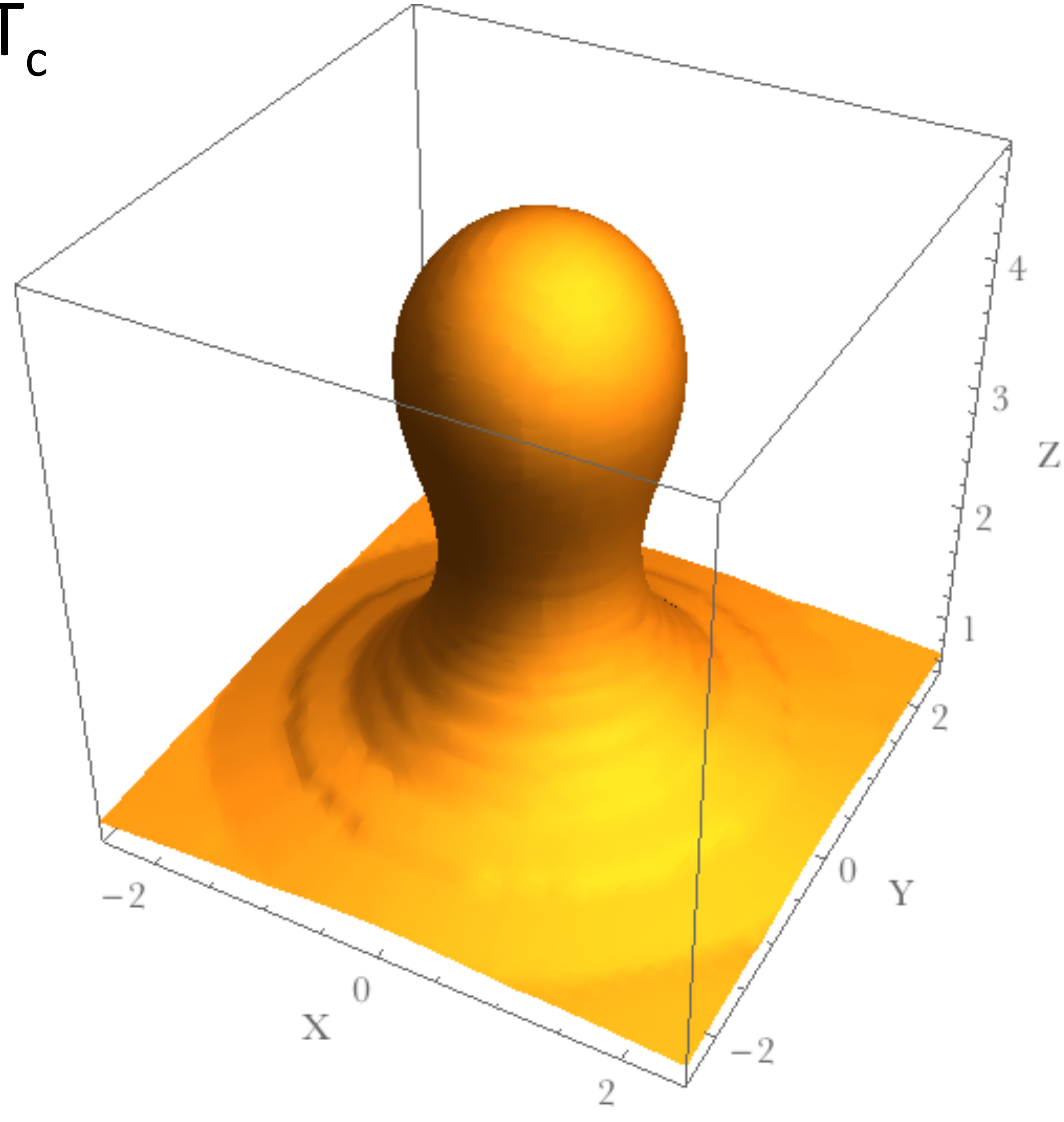
We have focused on $T = 0$ solutions, but $T > 0$ solutions with nonextremal hovering black holes should exist as well.

There should exist a finite T phase transition in which the topology of the horizon changes:

At high T, the horizon should be connected. As you lower T, a bubble appears around $r = 0$. At a critical T_c, the horizon should pinch off and below T_c there will be a hovering black hole.

(Santos, Way, GH, in progress)

Embedding diagram of the horizon at T near T_c



Form a "black mushroom"

Another extension of this work:

Rather than a single defect, we could consider an array of defects. As you increase the amplitude, one expects to form an array of hovering black holes. (This might be related to many body localization in the dual theory.)

Possible violation of cosmic censorship in 4D?

Suppose we fix the radial profile of $\mu(r)$ and slowly increase the amplitude. Bulk solution can't form hovering black hole since there is no charged matter.

At the critical amplitude, static solution becomes singular. Can one get arbitrarily close to this singular solution? (Santos, Way, GH, in progress)

Summary

- If $\mu(r) = a/r^{\beta}$ asymptotically with $\beta > 2$,
  $\rho \sim 1/r^3$ (for $1 < \beta < 2$, one has $\rho \sim \mu(r)/r$).
  $Q = 0$ in both cases.
- If $\beta = 1$, the total charge is nonzero and there is a new extremal horizon.
- In both cases, there are hovering black holes for large enough $\mu(r)$.
- These black holes grow in a universal way independent of the details of $\mu(r)$.