

UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

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Physics 229A

Winter 2007

Gauge Theories

ASSIGNMENT #3

Due Thursday, February 1, 2007

1. $SU(2) \times U(1)$ Chiral Lagrangians
 - a) Derive the gauge boson mass from \mathcal{L}_Σ in unitary gauge.
 - b) Derive the deviation in ρ from 1 in terms of γ_1 .
 - c) Check the invariance of \mathcal{L}_Σ under custodial $SU(2)$. Show that the term proportional to γ_1 is not invariant.
2. a) Assuming a lagrangian $\mathcal{L} = \mathcal{L}_{\text{gauge}} - M_w^2 W^+ W^- - \frac{M_z^2}{2} Z^2$, compute the high-energy behavior of the amplitude $M(W_L W_L \rightarrow Z_L Z_L)$ for scattering of longitudinal states.
 - b) Now consider the standard model lagrangian, with a minimal Higgs. Compute the extra contribution to $M(W_L W_L \rightarrow Z_L Z_L)$ from Higgs exchange and show that it improves the high energy behavior. (Hint: you may wish to use unitary gauge.)

Extra Credit Anomalies in the standard model

As you know from field theory, symmetries can be anomalous, and an anomaly in a gauge symmetry in particular spoils the symmetry structure of the theory. Such anomalies, which in four dimensions arise from triangle diagrams, are proportional to the quantity $\text{Tr} \left[\{T^a(R), T^b(R)\} T^c(R) \right]$ where $T^a(R)$ is the symmetry generator in representation R , and by convention the trace is over the left-handed fermions. (So right-handed fermions contribute through the complex conjugate representation.)

- a) Show that all anomalies cancel for standard model gauge symmetries.
- b) Compute the anomalies in the baryon and lepton number symmetries, B and L . Find the combination of these which is non-anomalous.