UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Prof. S.B. Giddings **Physics 229A** Winter 2007

Gauge Theories

ASSIGNMENT #6
Due Thursday, February 22, 2007

Suggested Reading: Wess and Bagger, I – VII

1. Check that the supersymmetry algebra given in class follows from the definitions

$$\delta\phi = \sqrt{2}\,\epsilon^{\alpha}\psi_{\alpha}$$

$$\delta\psi_{\alpha} = \sqrt{2}\,i\sigma^{\mu}_{\dot{\alpha}\beta}\partial_{\mu}\phi\bar{\epsilon}^{\dot{\beta}}$$

2. Show that $\bar{D}_{\dot{\alpha}}F = D_{\alpha}F = 0$, for an arbitrary superfield F, implies $F = a =$ constant. Demonstrate that $\bar{D}_{\dot{\alpha}}F = 0$ and $D^{\alpha}D_{\alpha}F = 4mF^{+}$ yield massive field equations for the components of F.

3. Construct the superfield whose lowest component is F, rather than φ. Compare this to the superfield $DD\Phi$.

4. Define the components of a chiral superfield ($\bar{D}_{\dot{\alpha}}\Phi = 0$) as follows:

$$\mathcal{A} = \Phi|_{\theta=\bar{\theta}=0}$$

$$\Psi_{\alpha} = D_{\alpha}\Phi|_{\theta=\bar{\theta}=0}$$

$$\mathcal{F} = DD\Phi|_{\theta=\bar{\theta}=0}\ .$$

Express these components in terms of the component fields φ, ψ, F of the chiral superfield given in class. Compute the transformation laws for $\mathcal{A}$, Ψ, and $\mathcal{F}$ using Q and $\bar{Q}$ in the following form:

$$Q_\alpha = D_\alpha - 2i\sigma_{\alpha\dot{\alpha}}{}^{\mu}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}$$

$$\bar{Q}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} + 2i\theta^{\alpha}\sigma_{\alpha\dot{\alpha}}{}^{\mu}\frac{\partial}{\partial x^\mu} \ .$$

5. Show that $\Phi = \bar{D}\bar{D}U$ is chiral for any superfield U. Relate the components of U to those of Φ.