UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

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Gauge Theories

ASSIGNMENT #8
Due Thursday, March 8, 2007

1. Consider a real superfield V. One may add a supersymmetry invariant (but not gauge invariant) mass term of the form $\int d^4\theta m^2 V^2$ to its lagrangian. Find the resulting component-field lagrangian, and derive its Euler-Lagrange equations.

2. The $U(1)_R$ symmetry of a system Φ^i of chiral multiplets is given by the infinitesimal transformation.

$$\delta\varphi^i = i\rho r_i \varphi^i$$
$$\delta\psi^i_\alpha = i\rho(r_i - 1)\psi^i_\alpha \quad \text{(no sum on } i\text{)}$$
$$\delta F^i = i\rho(r_i - 2)F^i$$

where ρ is a real constant parameter and r_i is the R charge of the superfield Φ^i. ($r_i \epsilon \mathcal{R}$)

   (a) It should be obvious that the R-transformation does not commute with SUSY transformations. Define the infinitesimal generator R by $\delta\Phi = i[R, \Phi]$ where Φ is any field of the system, and calculate $[R, Q_\alpha]$ where Q_α is the 4-component supercharge. Interpret this result.

   (b) Show that the action of the multicomponent chiral SUSY theory with superpotential $W(\Phi)$ is R invariant if and only if

$$\sum_i r_i \Phi^i W_{,i}(\Phi) = 2W \ .$$

3. Compute the gauge anomalies in the MSSM and show they cancel. Show that eliminating the field $\bar{H}$ spoils this.

4. Begin with the WZ model lagrangian

$$\mathcal{L} = \int d^4\theta Z\phi^+\phi \ + \int d^2\theta \Big(\frac{m\phi^2}{2} + \frac{\lambda}{3}\phi^3\Big) + h.c.$$

Replace the parameters Z, m, λ by the "spurion" vevs,

$$m \to m + \theta^2 F_m$$
$$\lambda \to \lambda + \theta^2 F_\lambda$$
$$Z \to Z + (\theta^2 B + h.c.) + \theta^2\bar{\theta}^2 C \ .$$

Compute the resulting scalar potential, determining its parameters in terms of the original parameters m, λ, Z and F_m, F_λ, B and C.