ASSIGNMENT #8
Due Thursday, March 8, 2007

1. Consider a real superfield $V$. One may add a supersymmetry invariant (but not gauge invariant) mass term of the form $\int d^4\theta m^2 V^2$ to its lagrangian. Find the resulting component-field lagrangian, and derive its Euler-Lagrange equations.

2. The $U(1)_R$ symmetry of a system $\Phi^i$ of chiral multiplets is given by the infinitesimal transformation.

$$
\delta \varphi^i = i\rho r_i \varphi^i \\
\delta \psi^i_\alpha = i\rho (r_i - 1) \psi^i_\alpha \quad \text{(no sum on } i) \\
\delta F^i = i\rho (r_i - 2) F^i
$$

where $\rho$ is a real constant parameter and $r_i$ is the $R$ charge of the superfield $\Phi^i$. ($r_i \epsilon R$)

(a) It should be obvious that the R-transformation does not commute with SUSY transformations. Define the infinitesimal generator $R$ by $\delta \Phi = i[R, \Phi]$ where $\Phi$ is any field of the system, and calculate $[R, Q^K]$ where $Q^K$ is the 4-component supercharge. Interpret this result.

(b) Show that the action of the multicomponent chiral SUSY theory with superpotential $W(\Phi)$ is $R$ invariant if and only if

$$
\sum_i r_i \Phi^i W^i(\Phi) = 2W .
$$

3. Compute the gauge anomalies in the MSSM and show they cancel. Show that eliminating the field $H$ spoils this.
4. Begin with the WZ model lagrangian

\[ \mathcal{L} = \int d^4 \theta Z \phi^+ \phi + \int d^2 \theta \left( \frac{m \phi^2}{2} + \frac{\lambda}{3} \phi^3 \right) + h.c. \]

Replace the parameters \( Z, m, \lambda \) by the “spurion” vevs,

\[
\begin{align*}
    m &\rightarrow m + \theta^2 F_m \\
    \lambda &\rightarrow \lambda + \theta^2 F_\lambda \\
    Z &\rightarrow Z + (\theta^2 B + h.c.) + \theta^2 \bar{\theta}^2 C.
\end{align*}
\]

Compute the resulting scalar potential, determining its parameters in terms of the original parameters \( m, \lambda, Z \) and \( F_m, F_\lambda, B \) and \( C \).