

UNIVERSITY OF CALIFORNIA, SANTA BARBARA  
Department of Physics

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**Physics 229A**

Winter 2007

Gauge Theories

**ASSIGNMENT #9**

Due Thursday, March 15, 2007

1. Verify the cancellation of anomalies between the  $\bar{5}$  and 10 representations of  $SU(5)$ .
2. Perform the calculation of coupling unification in the  $SU(5)$  model, verifying that you have the correct formulas for the  $SU(3)$ ,  $SU(2)$  and  $U(1)$  beta functions. Start with the measured values of the  $SU(2)$  and  $U(1)$  couplings, being careful about the differing normalizations in the Standard Model and in  $SU(5)$ . Compute the value of the unification scale (the point where these two couplings are equal); then determine the value of  $\alpha_3$  at  $M_Z$ . Compare with the value given by the Particle Data Group. You need only study the equations to one-loop order. In practice, two-loop corrections, as well as threshold effects and higher-order corrections to the beta function, are often included.
3. In SUSY  $SU(5)$ , find the vacua of the superpotential

$$W(\Sigma) = m \operatorname{tr} \Sigma^2 + \frac{\lambda}{3} \operatorname{tr} \Sigma^3 ,$$

and characterize their symmetry breaking patterns.

4. Beginning with the known beta-function coefficients for QCD, verify our formulas for the beta function coefficients above the SUSY breaking scale in the MSSM.