VI Supersymmetry

Hierarchy : why is $M_{EW} \ll M_p$. (also $\Lambda_c \lll M_p^4$?)

Options
1) No fundamental scalars - compositeness
2) New symmetry
3) M_{EW} is fundamental cutoff, M_p is derived.
   (TeV gravity)

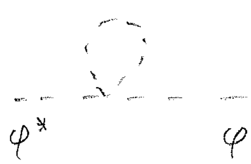
Others??

New symmetry:

Recall the problem. E.g. "massless" scalar

$$\mathcal{L}_\phi = -|\partial_\mu \phi|^2 - \lambda |\phi|^4$$

but at one loop



$$\Rightarrow \quad m_\phi^2 = \frac{4\lambda}{i} \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2}$$

$$\sim \Lambda^2$$

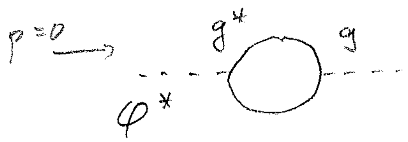
but, massless (Weyl) ferm = ok

$$\mathcal{L}_\psi = i\bar{\psi}\not{\partial}\psi - (g\psi\psi\phi + hc)$$

why?

protected : $\psi \to e^{i\alpha}\psi \quad \phi \to e^{-2i\alpha}\phi$ forbids $m\bar{\psi}\psi$.

but this also $\leadsto$ scalar mass



$$\propto \quad |g|^2 \int \frac{d^4 q}{(2\pi)^4} \, \mathrm{Tr}\left[\frac{1}{\not{q}}\,\frac{1}{\not{q}}\right]$$

$$= \; - \frac{4|g|^2}{i} \int \frac{d^4 q}{(2\pi)^4} \, \frac{1}{q^2}$$

Ferm loop

<u>cancels</u> above, if $\lambda = |g|^2$. Also $\delta\Lambda_c = 0$! (ex)

Reason: $\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\psi$, w/ $\lambda = |g|^2$, has an

<u>extra symmetry</u> that prevents mass corrections.
(~ extension of chiral symm to scalars)

<u>supersymmetry</u>

Fermions $\longleftrightarrow$ Bosons

Try to infer structure directly, from this fact plus Lorentz invariance.

Compare $U(1)$ symm:

$U(1)$

$\varphi = \varphi_1 + i\varphi_2$

$\delta\varphi_1 = \alpha\varphi_2$

$\delta\varphi_2 = -\alpha\varphi_1$

SUSY

Now

only way to make scalar

$$\delta\phi = \sqrt{2}\ \varepsilon^\alpha \psi_\alpha$$

dim $-1/2$, odd

dim $3/2$

$$\delta\psi_\alpha = \sqrt{2}\, i\, \sigma^\mu_{\alpha\dot\beta}\, \underbrace{\partial_\mu\phi\ \bar\varepsilon^{\dot\beta}}_{\text{spinor}}$$

convention

dim 2

dim $-1/2$

(note: henceforth Wess & Bagger sign + spinor conventions)
(e.g. $\sigma^0_{\text{Srednicki}} = -\sigma^0_{WB}$)

generators

$$\delta\varphi_i = \alpha Q \varphi_i$$

$$\delta_\epsilon\phi = (\varepsilon^\alpha Q_\alpha + \bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha})\,\phi$$

$$\delta_\varepsilon\psi = (\text{—— '' ——})\,\psi$$

$$[Q, Q] = 0$$

SUSY algebra

$$[\delta_\epsilon, \delta_\eta]\,\phi = [(\epsilon Q + h.c), (\eta Q + h.c)]\,\phi$$

$$= [-\epsilon^\alpha \eta^\beta \{Q_\alpha, Q_\beta\} - \cdots \quad]\,\phi$$

compare to combined transforms, from above (ex)

$$\leadsto \quad \boxed{\begin{aligned} &\{Q_\alpha, Q_\beta\} = 0 = \{\bar{Q}^{\dot\alpha}, \bar{Q}^{\dot\beta}\} \\ &\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu = 2P_\mu \sigma^\mu_{\alpha\dot\alpha} \\ &[Q_\alpha, P_\mu] = 0 = [\bar{Q}_{\dot\alpha}, P_\mu] \\ &N=1 \text{ SUSY algebra} \end{aligned}}$$

– 2/13/07

Essentially unique extension of Poincare group that is not a direct product w/ internal symms. (Coleman-Mandula). + extended SUSYs.

U(1)

Could work in real basis, but more convenient to invent i, consider

$$\phi = \phi_1 + i\phi_2$$

$$\delta\phi = -i\alpha\phi$$

SUSY

Could work with ordinary fields, but more convenient to introduce

$\theta_\alpha, \bar{\theta}_{\dot\alpha}$

constant (in x) Grassmann-odd "coordinates."

spin-statistics

$$\underbrace{x^\mu, \ \theta^\alpha, \ \bar{\theta}^{\dot\alpha}}$$

superspace coordinates

4 real $\left\{ \begin{matrix} \text{even} \\ \text{odd} \end{matrix} \right\}$ components (for $D=4, N=1$)

Fields on superspace: (richer than fields on $(1,i)$ space, since $i^2=-1$)

$\Phi(x^\mu, \theta_\alpha, \bar{\theta}_{\dot\alpha})$ power series:

$$= \phi(x) + \sqrt{2}\,\theta\psi + \sqrt{2}\,\bar{\theta}\bar{\psi} + \cdots + \theta^2\bar{\theta}^2 \rho(x) \qquad (*)$$

convention

note $\theta^3 = \bar{\theta}^3 = 0$

can represent our field transform using

$$\boxed{\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_\mu \\ \bar{Q}_{\dot\alpha} &= -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \end{aligned}}$$

"superchargees"

usual Grassman differentiation

$$\rightsquigarrow \quad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu + i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\{Q_\alpha, Q_\beta\} = hc = 0 .$$

$\delta\Phi = (\epsilon Q + \bar\epsilon \bar Q)\Phi$ (more explicit shortly)

But (*) has too many components!

$\varphi, \psi, \bar\psi,$ + coeffs of $\theta^2, \bar\theta^2, \theta\bar\theta, \theta^2\bar\theta,$ etc. ??

Reason: general Φ gives a <u>reducible</u> rep of SUSY.

to reduce: extra condition

... commuting w/ SUSY transform.

One way: use <u>superderivative</u>

$$\boxed{\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \\ \bar D_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \end{aligned}}$$

note relative sign from Q: $\partial_\mu \to -\partial_\mu$

Note $\{D_\alpha, \bar D_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$, so $D \sim \sqrt{\partial}$

Also, easily check: $\{D_\alpha, Q_\alpha\} = \{D_\alpha, \bar Q_{\dot\alpha}\} = 0$

$\bar D$ —— " ——

(exs.)

Why useful? Suppose $\bar D_{\dot\alpha}\Phi = 0$. (**)

then $\delta_\epsilon \bar D_{\dot\alpha}\Phi = 0$

i.e. condition <u>inv't.</u>

$(**) \rightsquigarrow \Phi$ irreducible. "Chiral superfield"

To solve $(**)$:

1) Note $\bar{D}_{\dot\alpha}\, \theta^\beta = 0$

2) Define $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$

$$\bar{D}_{\dot\alpha} = i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha} = 0$$

So $\Phi = \Phi(y,\theta)$ (no explicit $\bar\theta$) is chiral!

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

(note: $\theta^\alpha\theta^\beta = \frac{1}{2}\epsilon^{\alpha\beta}\epsilon_{\gamma\delta}\theta^\gamma\theta^\delta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta^2$)

			# DOF
φ	...	cplx, 2 bosons	2
ψ	...	ferm, 4 real components	2 (EOM lin. in ∂_μ)
F	...	boson, 2	0 ?

Likewise $\Phi^\dagger = \varphi^*(y^+) + \sqrt{2}\,\bar\theta\bar\psi(y^+) + \bar\theta^2 F^*(y^+)$

$D_\alpha \Phi^\dagger = 0$ antichiral

But: What is F ?

SUSY transforms:

$$\delta\Phi = (\epsilon Q + \underbrace{\bar\epsilon \bar Q}_{-\bar\epsilon^{\dot\alpha}\bar Q_{\dot\alpha}})\,\Phi$$

$Q^\alpha y^\mu = 0$ (like $\bar D$) $\qquad Q_\alpha \theta^\beta = \delta_\alpha{}^\beta$

$\bar Q_{\dot\alpha} y^\mu = 2i\,\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \qquad \bar Q_{\dot\alpha}\theta^\beta = 0$

$$\Rightarrow \quad \delta\Phi = \epsilon^\alpha \sqrt{2}\,\psi_\alpha + 2\epsilon^\alpha \theta_\alpha F(y) + 2i\,\theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\left(\partial_\mu\phi + \sqrt{2}\,\theta\partial_\mu\psi\right)\bar\epsilon^{\dot\alpha}$$

So:

$$\delta\varphi = \sqrt{2}\,\epsilon^\alpha\psi_\alpha$$

$$\delta\psi_\alpha = \sqrt{2}\,i\,\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\phi\,\bar\epsilon^{\dot\alpha} + 2\epsilon^\alpha F$$

$$\delta F = \underbrace{\qquad\qquad}_{\text{as before}} \qquad \underset{\uparrow\ ??\ (=0 \text{ on shell})}{(\#)\,\bar\epsilon\,\bar{\partial\!\!\!/}\psi}$$

<u>Actions</u>

space: $\displaystyle\int d^4x\,\mathcal{L}$

$\uparrow$ translation, etc invt.

superspace: $\displaystyle\int d^4x\; d^4\theta\; K(\Phi, \Phi^\dagger)$

... "D terms"

($K \sim$ Kahler potential)

Berezin integration $\quad \int d\theta\, \theta = 1 \qquad \int d\theta\, 1 = 0$

$$\int d\theta\, \partial_\theta f = 0$$

($\equiv$ differentiation!)

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

Normalize: $\int d^4\theta\, \theta^2\bar\theta^2 = 1$

SUSY inv?

$$\delta K = (\underbrace{\epsilon Q}_{\partial_\theta + \theta\partial_x} + \bar\epsilon\bar Q)K = (\text{total deriv})\, D$$

$$\Rightarrow \int d^4x\, d^4\theta\, \delta K = 0$$

Another invariant! $\quad W(\Phi(y,\theta))$

"only 2 θ's"

$$\Rightarrow \int d^4x\, d^2\theta\, W(\Phi)$$

"F-terms"

W = superpotential

invariance:

chiral

$$\delta W = (\epsilon Q + \bar\epsilon\bar Q)W = \underbrace{\epsilon^\alpha \partial_\alpha W - \bar\epsilon^{\dot\alpha}\cdot 2i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\partial_\mu W}_{\text{total derivs}}$$

just $\partial/\partial\theta^\alpha$, no $\partial/\partial\bar\theta^{\dot\alpha}$