$$\Rightarrow \int d^4x\, d^2\theta\, \delta W = 0. \quad \checkmark$$

— 2/15/07

SUSY EFTs:

$$\Phi = \varphi + \cdots \quad \Rightarrow \quad [\Phi] = 1$$

$$\mathcal{L} = \int d^4\theta\, \Phi^\dagger \Phi + \left(\int d^2\theta\, W[\Phi] + hc \right)$$

dim 2 2 1 (Φ, Φ^2, Φ^3: 1 2 3)

Direct calc: $\phi(x^\mu + i\theta\sigma^\mu\bar\theta, \theta)$; int over $\theta, \bar\theta$

(WB) exercise – go through

$$\rightsquigarrow \quad \mathcal{L}_0 = \underbrace{-|\partial_\mu\phi|^2 + i\,\partial_\mu\bar\Psi\,\bar\sigma^\mu\psi}_{\text{expected}} + |F|^2 + t.d.$$

if $W=0$, $F=0$
auxilliary field.

F terms:

$$W(\Phi)\big|_{\theta^2} = \theta^2 F \frac{\partial W}{\partial\phi} + \theta^\alpha\psi_\alpha\,\theta^\beta\psi_\beta\,\frac{\partial^2 W}{\partial\phi^2}$$

$$= \theta^2\left(F\frac{\partial W}{\partial\phi} - \frac{1}{2}\psi^2\frac{\partial^2 W}{\partial\phi^2}\right)$$

$\Rightarrow$

$$\mathcal{L}_F = F \frac{\partial W}{\partial \varphi} - \frac{1}{2} \psi^2 \frac{\partial^2 W}{\partial \varphi^2} + hc$$

F: no derivs; can trivially "integrate out"
(= elim. by equ of motion)

$$\delta F \Rightarrow F^* = -\frac{\partial W}{\partial \varphi}, \quad \text{so}$$

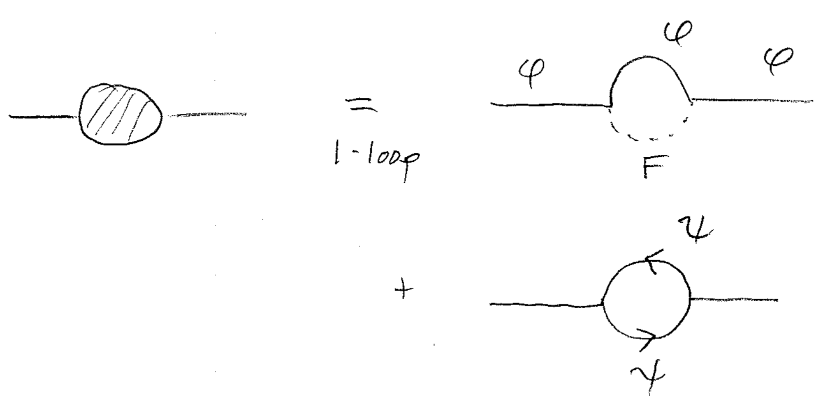
$$\mathcal{L} = -|\partial_\mu \phi|^2 + i \bar{\psi} \partial\!\!\!/ \psi - \left|\frac{\partial W}{\partial \phi}\right|^2 - \left(\frac{1}{2} \psi^2 \frac{\partial^2 W}{\partial \varphi^2} + hc\right)$$

In particular $W = \frac{g}{3} \phi^3 \Rightarrow$

$$\mathcal{L} = -|\partial_\mu \phi|^2 + i \bar{\psi} \partial\!\!\!/ \psi - |g|^2 |\phi|^4 - (g \psi^2 \phi + h.c.)$$

as before!

Cancellation



extends to higher loops ...

More fields: ϕ^i, ψ^i:

$$\boxed{\mathcal{L} = -\partial_\mu \phi^{i*} \partial^\mu \phi^i + i \bar{\psi}^i \bar{\partial\!\!\!/} \psi^i - |\partial_i W|^2 - \frac{1}{2}\left(\psi^i \psi^j \partial_i \partial_j W + h.c.\right)}$$

... good for Higgs, fund fermions. Gauge fields?

Approaches: 1) Define gauge X_m, cov. deriv (West 15.3)
2) Another invariant condition:

$$\Phi^\dagger = \Phi$$

Arb. superfield:

$$V = \underbrace{C(y,\theta)}_{\text{chiral: } 1,\theta,\theta^2} + \underbrace{\bar{C}(y^\dagger,\bar{\theta})}_{\text{antichiral } 1,\bar{\theta},\bar{\theta}^2} - \theta\sigma^\mu\bar{\theta} A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

$V^\dagger = V \Rightarrow$

1. $\bar{C} = C^\dagger$
2. $A_\mu = A_\mu^*$
3. $\lambda^\dagger = \bar{\lambda}$
4. $D = D^*$

... Real or vector superfield.

But: still too many components? $\supset$ chiral

$\leftrightarrow$ gauge invariance !

clues: 1. eliminate C
2. $A_\mu \to A_\mu + \partial_\mu \alpha$

Candidate:

$$V \to V + \Phi + \Phi^\dagger \qquad \Phi = \text{chiral}$$

if choose $\Phi = -C$, 1 ✓

alternately, let $\Phi = -C + \frac{i}{2}\alpha(y)$ ↖ $\times\, i\theta\sigma^\mu\bar\theta$

$$V \to V - C - C^\dagger + \underbrace{i\; i\theta\sigma^\mu\bar\theta\,\partial_\mu\alpha}_{-\theta\sigma^\mu\bar\theta\,\partial_\mu\alpha}$$

ie $A_\mu \to A_\mu + \partial_\mu\alpha$! 2 ✓

WZ gauge $\quad C = 0 \quad$ (breaks SUSY, but useful)

note $V_{WZ}^3 = 0$

Field strength ? clue: λ = invt; isolate as

$$W_\alpha = -\frac{1}{4}\bar D^2 D_\alpha V \qquad \bar W_{\dot\alpha} = -\frac{1}{4} D^2 \bar D_{\dot\alpha} V$$

1) gauge invt: $\delta W_\alpha = -\frac{1}{4}\bar D^2 D_\alpha \Phi$

but $[\bar D^2, D_\alpha] \propto \bar D$ (ex)

and $\bar D \Phi = 0 \implies \delta W_\alpha = 0$.

2. Chiral : $\bar{D}_{\dot\alpha} W_\alpha = 0$: $\bar{D}^3 = 0$.

3. Contains $F_{\mu\nu}$:

$$W_\alpha \sim \bar{D}^2 D_\alpha \left(\theta \sigma^\mu \bar\theta \, A_\mu \right)$$

$$\left(-\frac{\partial}{\partial \bar\theta} - i \theta \sigma^\nu \partial_\nu \right)^2$$

$$\theta \sigma \sigma \partial_\nu A_\mu$$

indeed (ex) :

$$W_\alpha = -i \lambda_\alpha(y) + D(y)\, \theta_\alpha - \frac{i}{2} (\sigma^\mu \bar\sigma^\nu)_\alpha{}^\beta F_{\mu\nu} \theta_\beta$$
$$+ \theta^2 \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\lambda^{\dot\alpha}$$

note $\sim$ analog field strength

Lagrangian ? W_α = chiral, $W^2 \sim F^2$

suggest :

$$\mathcal{L}_{gauge} = \frac{1}{4g^2} \int d^2\theta \; W^\alpha W_\alpha + h.c.$$

clearly of form (coeffs = exercise)

$$\int d^4x \, \mathcal{L}_{gauge} = \frac{1}{g^2} \int d^4x \left(\frac{D^2}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \lambda \sigma^\mu \partial_\mu \bar\lambda \right)$$

$\therefore$ D = auxil. λ = "gaugino"

here "photino" (U(1))

Interactions ? $\sim$ minimal coupling.

Had $\int d^4\theta\, \Phi^\dagger \Phi$

clearly invt. under $\Phi \to e^{-iq\Lambda}\Phi$ for $\Lambda =$ const, $\in \mathbb{R}$.

let $\Lambda =$ general chiral field

$$\Phi^\dagger \Phi \to \Phi^\dagger \Phi\, e^{-iq\underbrace{(\Lambda - \Lambda^\dagger)}_{\neq 0}}$$

But if $V \to V + \dfrac{i\Lambda - i\Lambda^\dagger}{2}$ gauge invc.

then

$$\Phi^\dagger e^{2qV}\Phi \longrightarrow \Phi^\dagger e^{2qV}\Phi$$ <u>min coupling</u>

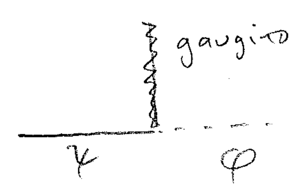
$\left(\text{note } V_{here} = \dfrac{V_{WB}}{2}\right)$

$\leadsto$ $\mathcal{L}_{matter} = -\int d^4\theta\, \Phi^\dagger e^{2qV}\Phi$

$$= -|D_\mu \phi|^2 + i\bar\psi\bar\sigma^\mu D_\mu\psi + F^*F$$

$$+ qD\phi^*\phi + i\sqrt{2}q\left(\lambda\psi\phi - \bar\lambda\bar\psi\phi^*\right)$$

supersym. :



$\mathcal{L}_{superpot} = \int d^2\theta\, W(\Phi_i)$ ... only gauge invt terms allowed

– 2/20/07

Note $\frac{\delta}{\delta D} \rightsquigarrow \frac{1}{g^2} D = -q\,\phi^* \phi$

more generally $= -\sum_i q_i \phi_i^* \phi_i$

$$\Rightarrow \quad V_D = -\frac{g^2}{2}\Big(\sum_i q_i \phi_i^* \phi_i\Big)^2 \qquad \text{D-term pot.}$$

E.g. SQED

$$\phi_1 = E \qquad q = 1$$

$$\phi_2 = \bar{E} \qquad q = -1$$

$$\mathcal{L}_{SQED} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + m \int d^2\theta \, E \bar{E}$$

↑ gauge invt.

Nonabelian G? <u>Generalize</u>

$$\Phi \to \underbrace{e^{-ig\Lambda}}_{\text{rep } T(G)} \Phi \qquad \text{(put coupling in transform)}$$

$V \in \mathrm{Adj}(G)$

$$e^V \to e^{-i\Lambda^\dagger/2} e^V e^{i\Lambda/2}$$

$$W_\alpha = -\frac{1}{4} \bar{D}^2 e^{-V} D_\alpha e^V$$

$$\to e^{-i\Lambda^\dagger/2} W_\alpha e^{i\Lambda/2} \qquad \text{(ex - easy)}$$

Action:

$$\mathcal{L} = \frac{1}{2}\int d^2\theta \, \mathrm{Tr}(W^\alpha W_\alpha) + h.c. - \int d^4\theta \sum_i \Phi_i^\dagger e^{2gV} \Phi_i$$

$$+ \int d^2\theta \; \underbrace{W(\Phi_i)}_{\text{holomorphic, gauge invt.}} + h.c.$$

(Non) renormalization - introduction.

promised miraculous properties, e.g. no mass renorm.

(saw - 1 loop)

Can prove

1. Supergraph techniques (... tedious)
2. Holomorphy + symmetry.

WZ model: $\mathcal{L} = \int d^4\theta \, \phi^\dagger \phi + \int d^2\theta \left(m \cdot \frac{\phi^2}{2} + \frac{\lambda}{3!}\phi^3 \right) + h.c. \quad (*)$

Idea #1 Think of m, λ as <u>vevs</u> <u>of</u> <u>fields</u>

e.g. $\mathcal{L} \to \mathcal{L} + \mathcal{L}_m + \mathcal{L}_\lambda$ with

$$\mathcal{L}_m = \int d^4\theta \, m^\dagger m + \int d^2\theta \, \frac{M}{2}(m - m_0)^2 + h.c.$$

Take $M \to \infty$ $\rightsquigarrow$ m fixed at classical value:

$$0 = \frac{\partial W}{\partial m} = M(m - m_0)$$

$\rightsquigarrow$ Ingredient # 1: $W(\phi, m, \lambda)$ <u>no</u> $\phi^\dagger, m^\dagger, \lambda^\dagger$

... <u>holomorphy</u>.

Ingredient # 2: Symmetry

R symmetries

$\int d^4\theta \, \phi^\dagger \phi$ has a symmetry we haven't noted

suppose
$$\theta \to e^{i\lambda}\theta = \theta'$$
$$\phi \to e^{ir\lambda}\phi = \phi'$$

then
$$d^4\theta \to d^4\theta$$
$$\phi^\dagger \phi \to \phi^\dagger \phi$$

in components:
$$\phi'(\theta') = e^{ir\lambda}\phi(\underset{\theta' e^{-i\lambda}}{\underset{\shortparallel}{\theta}})$$

$$= \underbrace{e^{ir\lambda}\varphi}_{\varphi'} + \sqrt{2}\,\theta' \underbrace{e^{i(r-1)\lambda}\psi}_{\psi'} + \theta'^2 \underbrace{e^{i(r-2)\lambda}F}_{F'}$$

r = arbitrary $\Rightarrow$ really 2 symmetries.

broken by W:

$$\int d^2\theta\, \Phi^2(\theta) \to \int d^2\theta\, e^{2i(r-1)\lambda}\, \Phi^2$$

recall, $\sim \left(\frac{\partial}{\partial\theta}\right)^2$

$\therefore$ $r = 1 \leftrightarrow$ symmetry. (note $\sim$ chiral, but not)

$\leftrightarrow$ ϕ^2 : R charge two

in general, unbroken R symm $\Leftrightarrow$ $W(\phi_i)$ has R charge $= 2$

Now, $(*)$ has two symmetries :

plain old global symm

	$U(1)_R$	$U(1)$
Φ	1	1
m	0	-2
λ	-1	-3

namely: int out modes above some scale Λ.

S_{eff} ... Wilsonian effective action ($\sim S_{1PI}$, not quite, but good enough for now) : must preserve

$$\Rightarrow W_{eff} = m\,\phi^2\, f\left(\frac{\lambda\phi}{m}\right)$$

$U(1)_R$	2	0
$U(1)$	0	0

need to find

Ingredient # 3 weak coupling:

let $\lambda \to 0$ (suppress loops) w/ $\frac{\lambda}{m} = t$ = fixed
$m \to 0$

then $W_{eff} \to W_{tree}$, $\Rightarrow$ $f(t\phi) = \frac{1}{2} + \frac{t\phi}{3!}$

true for arb t!

$\therefore$ $W_{eff} = \frac{m\phi^2}{2} + \frac{\lambda\phi^3}{3}$ Q.E.D. (Note: not even perturbative!)

Generalizes (ex!) $\underbrace{\int d^4\theta\, K(\phi_i, \phi_i^\dagger)}_{\text{renormalized}} + \underbrace{\int d^2\theta\, W(\phi_i) + h.c.}_{\text{not renormalized.}}$

(in particular no $W \rightsquigarrow$ SUSY + $\Lambda_c \neq 0$)

Gauge theory?

$$-\frac{i}{8\pi}\int d^2\theta\, \tau\, \mathrm{Tr}(\underbrace{W^\alpha W_\alpha}_{\in \text{ adjoint}}) + h.c.$$

in general $\underline{\text{cplx}}$ $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

g certainly renormalized (runs!)

But : $N=1$ SUSY $\Rightarrow$ one-loop renorm. only

$\updownarrow$

holomorphy
anomalies
Adler-Bardeen

$\therefore$ Can calculate exact β-fcn.

E.g. SQCD : $SU(N_c)$
N_f flavors

$$\beta(g) = \frac{-g^3}{16\pi^2}\left[3N_c - N_f\right]$$

$$\Leftrightarrow \qquad e^{-\frac{8\pi^2}{g^2(\mu)}} = \left(\frac{\Lambda}{\mu}\right)^{3N_c - N_f}$$

asymp free : $N_f < 3N_c$

nice explicit verification of QCD discussion.

... Small piece of very beautiful story involving SUSY gauge dynamics, instatons, etc.; also related to SUSY breaking

— 2/22/07

But now : apply SUSY to particle physics