VII Unification

Saw EW "unification." But $g_1 \neq g_2 \neq g_3$, 3 groups.

Other puzzles:

1. Charge quantization $\underbrace{Q_u = \frac{2}{3},\ Q_d = -\frac{1}{3}\ \ Q_e = -1}_{\text{different kinds of matter}}$

$(\updownarrow)$

2. Anomaly cancellation
3. Weird multiplet structure

$$\underset{Q}{(3,2)_{1/6}} \quad \underset{\bar{u}}{(\bar{3},1)_{-2/3}} \quad \underset{\bar{d}}{(\bar{3},1)_{1/3}} \quad \underset{\ell}{(1,2)_{-1/2}} \quad \underset{\bar{e}}{(1,1)_1} \quad \underset{\bar{\nu}}{\left[(1,1)_0\right]}$$

Idea: unify into one group G, above structure from SSB.

How to realize?

simplest

$$\left(\begin{array}{c|c} SU(3) & \\ \hline & SU(2) \end{array}\right) \rightsquigarrow SU(5)?$$

Y ? $\quad [Y, T_{su(2), su(3)}] = 0$

$$\Rightarrow \quad Y = \left(\begin{array}{ccc|cc} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ \hline & & & -1/2 & \\ & & & & -1/2 \end{array}\right)$$

relative coeffs fixed by $\mathrm{Tr}\, Y = 0$.

Reps of $su(5)$? $\quad$ 5 $\quad V^i \to U^i{}_j V^j$

$\bar{5}$

10 $\quad A^{ij}$

$\vdots$

Ferms? $\quad \bar{5}$ made to order: (see Y!) $\quad \begin{pmatrix} \bar{d} \\ l \end{pmatrix}$

remaining states: $\quad Q \; \bar{u} \; \bar{e}$

$6 + 3 + 1 = 10$!

gives $(3, 2)$

$$\left[\begin{array}{ccc|cc} 0 & \bar{u} & \bar{u} & u & d \\ -\bar{u} & 0 & \bar{u} & u & d \\ -\bar{u} & -\bar{u} & 0 & u & d \\ \hline & (-) & & 0 & \bar{e} \\ & & & -\bar{e} & 0 \end{array}\right]$$

$3 \times 3 = \bar{3} + 6$

$\bar{3}$: AS

$2 \times 2 = 1 + 3$

1: AS

ex - check Y's

SM fits - <u>perfectly</u>. ($\bar{\nu}$ = extra singlet. singlets are cheap.)

but $g_3 = g_2 = g_1'$

not usual normalization

Canonicaly normalized generators:

$$\mathrm{Tr}(T^a T^b) = \frac{\delta^{ab}}{2}$$

$$Y' = cY \quad \rightsquigarrow \quad c^2 \Big(\underbrace{3\times\left(\tfrac{1}{3}\right)^2 + 2\times\left(\tfrac{1}{2}\right)^2}_{5/6}\Big) = \frac{1}{2}$$

$$\Longrightarrow \quad c = \sqrt{\frac{3}{5}}$$

So:

$$\cdots \quad -\frac{1}{4} B_{\mu\nu}^2 + g_1' B \sqrt{\frac{3}{5}}\, Y$$

$$g_1$$

in particular, $\sin^2\theta_W = \dfrac{g_1^2}{g_1^2 + g_2^2} = \dfrac{3}{8} \simeq .37$ ✗

But: <u>couplings run</u>.

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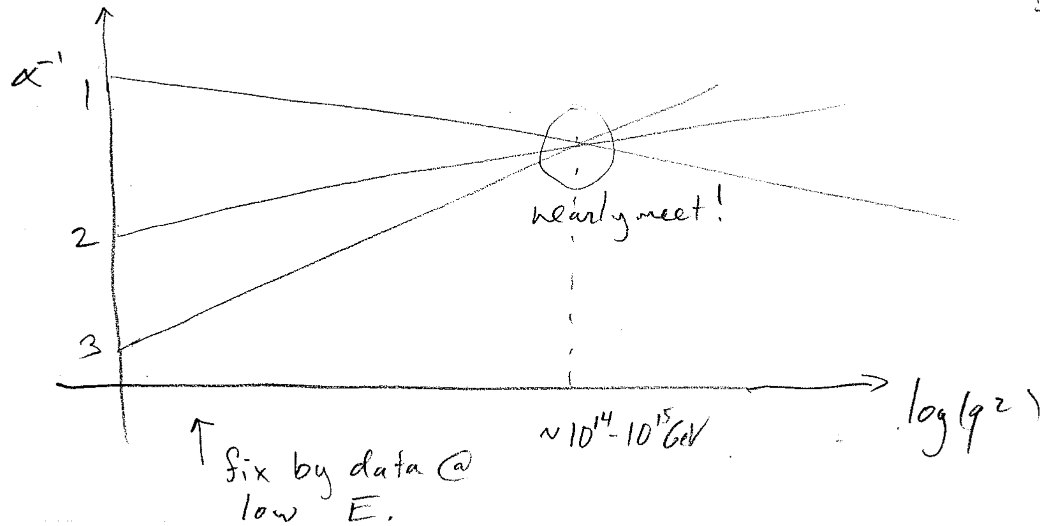
Recall

$$\frac{\alpha_3^{-1}(q^2)}{\alpha_3^{-1}(\Lambda^2)} = \frac{b_3}{4\pi} \log\left(q^2/\Lambda^2\right)$$

$$b_0 = \frac{11}{3}N - \frac{2}{3}n_F$$

$$\Rightarrow b_3 = 11 - \frac{2}{3}n_F > 0$$

$$(n_F = 6)$$



$$\frac{\alpha_2^{-1}(q^2)}{\alpha_2^{-1}(\Lambda^2)} = \frac{b_2}{4\pi} \log\left(q^2/\Lambda^2\right)$$

$$b_2 = \frac{22}{3} - \frac{2}{3}n_F - \frac{1}{6}n_H > 0$$

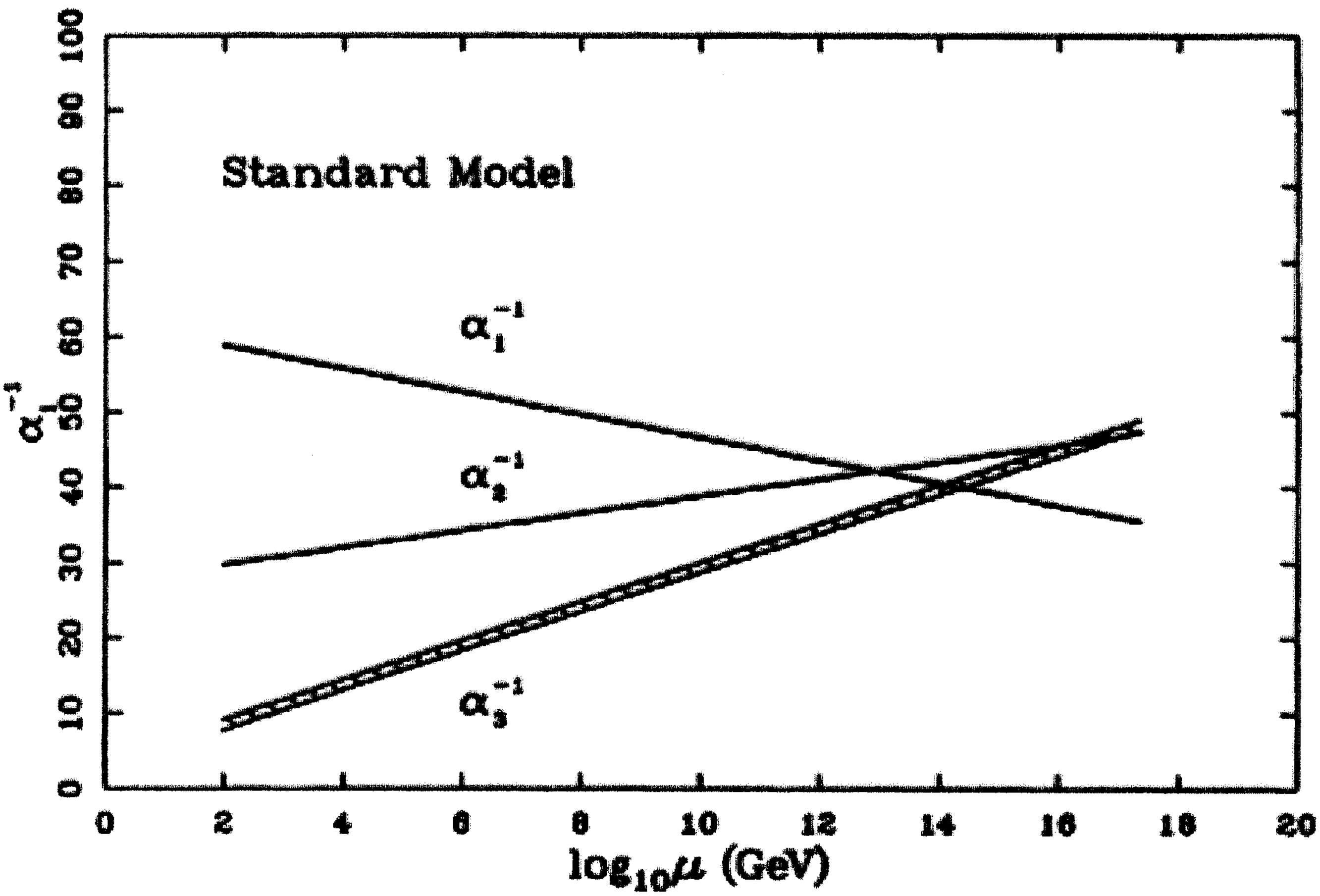
scalar contrib

$$\frac{\alpha_1^{-1}(q^2)}{\alpha_1^{-1}(\Lambda^2)} = \frac{b_1}{4\pi} \log\left(q^2/\Lambda^2\right)$$

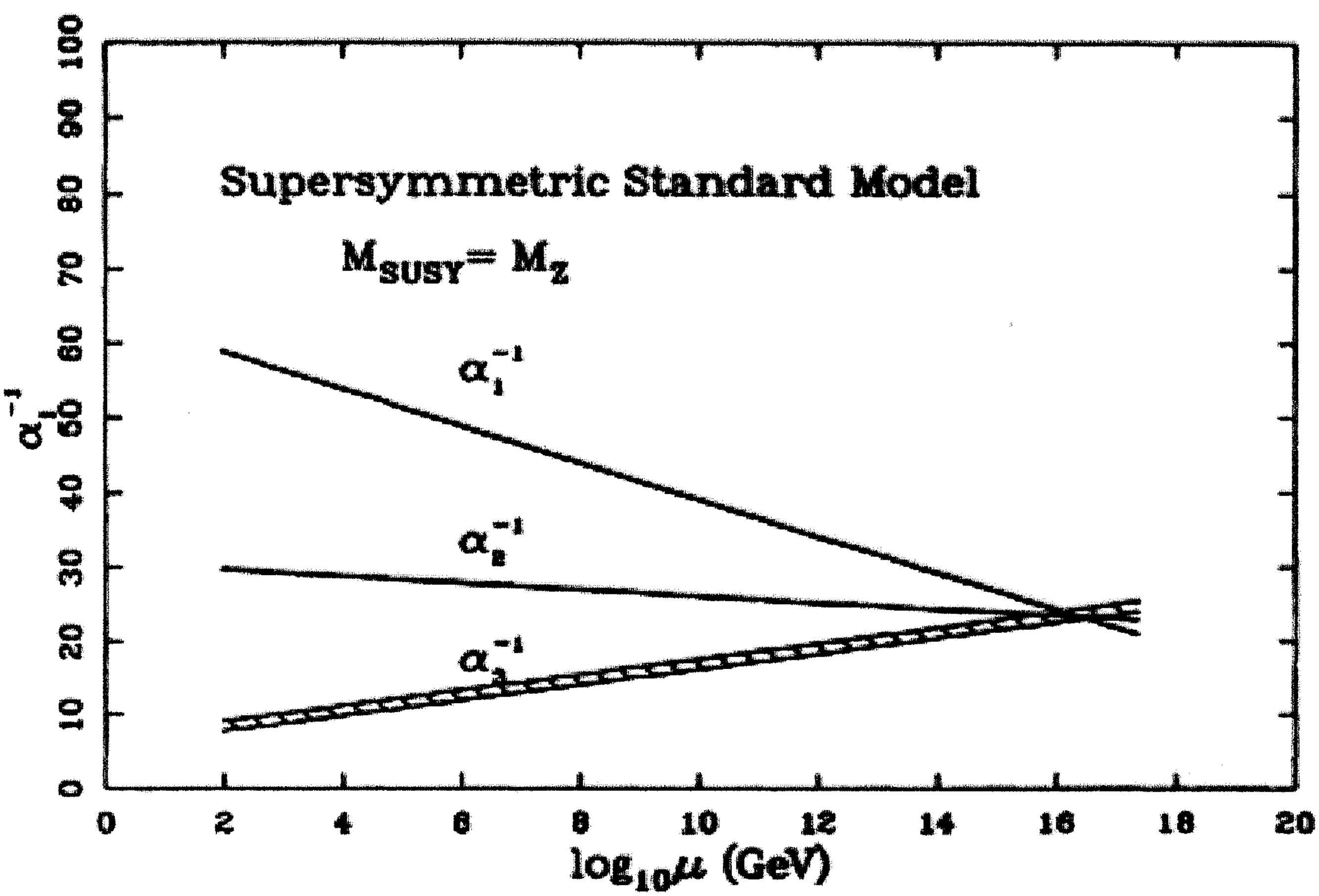
$$b_1 = -\frac{2}{3}n_F - \frac{1}{10}n_H < 0$$

So: idea is SSB at Λ_{GUT} then couplings run down to 'observed values.

but: LEP precise measurements: don't quite meet.



Standard Model
α_1^{-1}
α_2^{-1}
α_3^{-1}
α_i^{-1}
0 10 20 30 40 50 60 70 80 90 100
0 2 4 6 8 10 12 14 16 18 20
$\log_{10}\mu$ (GeV)



Supersymmetric Standard Model
$M_{SUSY} = M_Z$
α_1^{-1}
α_2^{-1}
α_3^{-1}
α_i^{-1}
0 10 20 30 40 50 60 70 80 90 100
0 2 4 6 8 10 12 14 16 18 20
$\log_{10}\mu$ (GeV)

But: $M_{susy} \sim 1$ TeV ? $\quad$ SM $\longrightarrow$ MSSM

$$b_i = 3N - N_F - N_H \qquad (ex)$$

E.g. QCD $\quad N=3 \quad N_F = 6 \quad N_H = 0 \quad b_3 = 3$

SU(2) $\quad N = 2 \quad N_F = \dfrac{3 \times (\overset{\text{color}}{3}+1)}{\underset{\text{chiral}}{2}} \quad N_H = \dfrac{2}{\underset{\text{chiral}}{2}} \quad b_2 = -1 \quad$ (note <0)

U(1) $\qquad b_1 = -\frac{33}{5} \quad (ex)$

Assume spartners @ ~ 1 TeV $\Longrightarrow$

They meet, within error bars ! (see fig)

$$M_{GUT} = 1.2 \times 10^{16}\ GeV, \quad \alpha_{GUT} \simeq \frac{1}{25}$$

(also 2-loop and threshhold corrections included)

<u>SUSY bonus # 2</u> .

Recall also: $M_{\bar{\nu}} \sim$ few $\times 10^{14}$ GeV $-$ related ??

SUSY/GUT bonus # 3 ?

Much more can be said about GUTs. (good ... and bad)

<u>Some highlights:</u>

- Anomaly cancellation. $SU(5)^3$ $\bar{5}, 10$. simpler (ex)

- Breaking

$$A^I T^I = \left(\begin{array}{c|cc} G\text{'s}, B & X & Y \\ & X & Y \\ & X & Y \\ \hline h.c. & & W\text{'s}, B \end{array}\right)$$

broken @ M_{GUT}?

Simplest: $\Sigma \in \mathrm{adj}(SU(5))$ $\underline{24}$

e.g. SUSY $W(\Sigma) = m \, \mathrm{tr}\, \Sigma^2 + \frac{\lambda}{3} \mathrm{tr}\, \Sigma^3$ (renorm)

$\rightsquigarrow$ 3 possible vacua (ex)

one: $\Sigma = V \, \mathrm{diag}(2,2,2,-3,-3)$ w/ $V = \frac{m}{\lambda}$

$\longrightarrow SU(3) \times SU(2) \times U(1)$ ✓

$\mathcal{L} = \cdots - |D_\mu \Sigma|^2 \quad \rightsquigarrow \quad M_X \sim M_Y \sim gV$

- The Higgs(es) $\quad H \quad (1,2)_{+1/2} \longrightarrow 5$

  $\bar{H} \quad (1,2)_{-1/2} \longrightarrow \bar{5}$ (like leptons)

ferm. masses:

$$W = \underset{3\times 3}{G_D}\, \bar{5}_i\, \bar{H}_j\, 10^{ij}$$

$\rightsquigarrow \quad m_d = m_e = G_D V$ at unification

"Yukawa unification"

off by ~3, but RG $\rightsquigarrow$ close for m_b, m_τ

$$+ \underset{3\times 3}{G_u}\, H^i\, 10^{jk}\, 10^{lm}\, \epsilon_{ijklm}$$

$\rightsquigarrow \quad m_u = G_u V$

Potential issue: $\quad H = \begin{pmatrix} H_3 \\ H_2 \end{pmatrix} \leftarrow$ color triplet

also $\bar{H}$

$H_3, \bar{H}_3 \rightsquigarrow B, L$ violation, $\therefore$ want <u>massive</u>

achieve by coupling to Σ

$$W_H = \mu H \bar{H} + y \bar{H} \Sigma H$$

want $H_2, \bar{H}_2 \sim 10^3$ GeV

$H_3, \bar{H}_3 \sim 10^{16}$ GeV

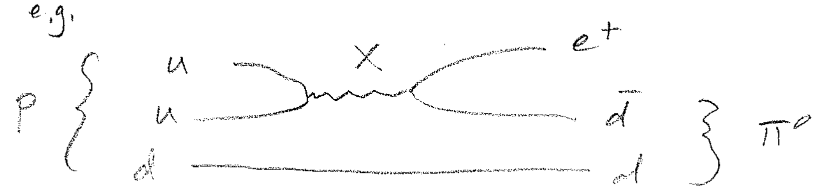
$\rightsquigarrow$ $\mathcal{O}(10^{-13})$ fine tuning ...

if no SUSY : radiatively unstable (hierarchy)

SUSY : tree level only, stable.

• p decay X, Y mediate

e.g.



$p \to e^+ \pi^0$

non SUSY $\Gamma \sim \frac{\alpha^2}{M_X^4} \times m_p^5 \sim (10^{31} \text{ years})^{-1}$

not seen yet.

Super K : $\tau(p \to e^+ \pi^0) > 5 \times 10^{33}$ yr.

SUSY : ~~dim 4~~ R parity

dim 5 $W = \frac{1}{M_1} \bar{u}\bar{u}\bar{d}e^+ + \frac{1}{M_2} QQQL$ (*)

Int out squarks, gluinos $\rightsquigarrow$ dim 6 in SM fields (e.g. $\underbrace{\bar{u}\bar{u}\bar{d}e}_{\text{Ferms}}$ = dim 6)

coeff (est.) $\sim \frac{\alpha}{4\pi} \frac{1}{M_{GUT} M_{SUSY}}$

need 10^{-9} suppression

plausible! 1) (*) typically from intermediate Higgs trips $\Rightarrow \propto (\text{Yuk})^2$

2) $Q_i Q_j Q_k L_\ell$ = SU(3) invt

$\Rightarrow$ antisym ijk

$\Rightarrow$ 2^{nd} or 3^{rd} gen. involved for $\neq 0$

$\Rightarrow$ CKM

$\rightsquigarrow 10^{-9} - 10^{-11}$ suppression.

claim: Super K rules out minimal SUSY SU(5)

but

- non-min Higgs
- other groups

— 3/6/07

- SO(10) : $> SU(5)$

to see: $z^j = x^{2j-1} + i x^{2j}$ $j = 1, \dots, 5$

$\rightarrow U^j{}_k z^k$

preserving $z^j z^*_j$

special case of $x^I \rightarrow \underset{\substack{\uparrow \\ 10\times 10}}{O^{IJ}} x^J$

SM ? $\qquad \bar{5} + 10 + 1 = 16$

SO(10) has spinor reps $\quad 2^{D/2} \times \underbrace{\tfrac{1}{2}}_{\text{Weyl}} = 16\,!$

$$\{\Gamma^I, \Gamma^J\} = 2\delta^{IJ}$$

let $\quad a^j = \frac{1}{2}\left(\Gamma^{2j-1} + i\,\Gamma^{2j}\right)$

$\rightsquigarrow \quad \{a^i, a^{\bar{j}}\} = \delta^{i\bar{j}} \qquad$ (ex)

$\qquad \{a^i, a^j\} = 0$

define $\qquad a^i|0\rangle = 0$

odd # $a^{\bar{i}}$ $\leftrightarrow$ irrep.

$$\left\{\begin{array}{ll} a^{\bar{i}}|0\rangle = \bar{5} & \text{(ex - ck Y)} \\ a^{\bar{i}}a^{\bar{j}}a^{\bar{k}}|0\rangle = 10 & \text{(use } \epsilon \text{)} \end{array}\right.$$

ex: construct $1 = \bar{\nu}$

$\therefore$ exactly accomodates <u>matter w/ $\bar{\nu}$</u>

also

$$\underbrace{E_8 > E_6}_{\text{strings!}} > SO(10)$$

a story for another day ...