

VIII SUSY breaking - brief intro.

- SUSY:
- desirable features
 - need to understand breaking (devil in details??)

Overview of some basic aspects
(more lectures needed to do justice)
→ Luty, TASI

Idea: SSB: $Q_\alpha |0\rangle \neq 0$

Note $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$

$$\times \sigma^0, \text{ trace} \quad \leadsto \quad \mathcal{L} \sum_{\alpha} (Q_\alpha \bar{Q}_{\dot{\alpha}} + \bar{Q}_{\dot{\alpha}} Q_\alpha) = \cancel{\mathcal{L}} \cdot 4i\partial_0 = 4E$$

$$\text{so } \langle 0|H|0\rangle = \sum_{\alpha} \|Q_\alpha |0\rangle\|^2 + \|\bar{Q}_{\dot{\alpha}} |0\rangle\|^2$$

$$\text{thus: } Q_\alpha |0\rangle = 0$$



$$H|0\rangle = 0 \quad (\text{again, no c.c.!!})$$

$$\therefore E_0 = \text{order param } \underline{\text{SUSY}} \quad (\text{also } E \geq 0)$$

$$\text{alternately } \langle S \rangle \neq 0 = \text{order param, if } Q_\alpha \langle S \rangle \neq 0 \text{ or } \bar{Q}_{\dot{\alpha}} \langle S \rangle \neq 0$$

$$\text{Lorentz inv} \Rightarrow \langle \bar{\Phi} \rangle = \theta^2 F \quad \text{F-breaking}$$

$$\text{or } \langle V \rangle = \theta^2 \bar{\theta}^2 D \quad \text{D-breaking}$$

as we've described!

Simple tree models: O'Raifeartaigh

$Q^a = N$ chiral fields

$$\text{let } \mathcal{L}_{\text{O'R}} = \int d^4\theta Q_a^\dagger Q^a + \left(\int d^2\theta W(Q) + \text{h.c.} \right)$$

$$\text{SUSY vac} \iff 0 = \langle F_a^\dagger \rangle = \left\langle \frac{\partial W}{\partial Q^a} \right\rangle$$

... N conditions

but not always possible

e.g. $W = \kappa Q$!

but trivial, no mass splittings

better

$$W = \frac{1}{2} \lambda_1 S_1 X^2 + \frac{1}{2} \lambda_2 S_2 (X^2 - v^2)$$

$$\frac{\partial W}{\partial S_1} = \frac{1}{2} \lambda_1 X^2 \quad \frac{\partial W}{\partial S_2} = \frac{1}{2} \lambda_2 (X^2 - v^2)$$

can't both vanish!

$$\frac{\partial W}{\partial X} = (\lambda_1 S_1 + \lambda_2 S_2) X \rightsquigarrow$$

$$V = \frac{1}{2} |\lambda_1|^2 |X|^4 + \frac{1}{2} |\lambda_2|^2 |X^2 - v^2|^2 + |\lambda_1 S_1 + \lambda_2 S_2|^2 |X|^2$$

$$\frac{\partial}{\partial S_1}, \frac{\partial}{\partial S_2} \rightsquigarrow \langle \lambda_1 S_1 + \lambda_2 S_2 \rangle = 0 \text{ or } \langle X \rangle = 0$$

$$\frac{\partial}{\partial X} \rightsquigarrow \langle X^2 \rangle = \frac{|\lambda_2|^2}{|\lambda_1|^2 + |\lambda_2|^2} v^2 \quad \text{or} \quad \langle X \rangle = 0$$

$$\langle V \rangle = \begin{cases} \frac{|\lambda_2|^2}{4} |v|^4 & \langle X \rangle = 0 \\ \frac{|\lambda_1|^2 |\lambda_2|^2}{2(|\lambda_1|^2 + |\lambda_2|^2)} |v|^4 & \langle X \rangle \neq 0 \end{cases} \quad (\text{note } \geq 0)$$

$$|\lambda_2| > |\lambda_1| \Rightarrow \langle X \rangle \neq 0 = \text{min.}$$

but flat dirⁿ: vacua satisfying $\langle \lambda_1 S_1 + \lambda_2 S_2 \rangle = 0$.

other masses lifted to $\mathcal{O}(\lambda v)$ (ex)

LEFT: X and $\lambda_1 S_1 + \lambda_2 S_2$ fixed
param flat dirⁿ by S_2 .

$$\rightsquigarrow S_1 = -\frac{\lambda_2}{\lambda_1} S_2$$

$$W_{\text{eff}} = -\frac{1}{2} \lambda_2 v^2 S_2 \quad (\text{ex})$$

$$\Rightarrow \mathcal{L}_{\text{eff}} = \int d^4\theta \left(1 + \frac{|\lambda_2|^2}{|\lambda_1|^2} \right) S_2^\dagger S_2 + \left(\int d^2\theta W_{\text{eff}} + \text{h.c.} \right)$$

note $\frac{\partial W_{\text{eff}}}{\partial S_2} \neq 0 \Leftrightarrow$ ~~SUSY~~

See in masses in higher-dim terms.

D-breaking

$$V = \theta^2 \bar{\theta}^2 D$$

one approach:
if $U(1)$,

$$\mathcal{L}_{FI} = \int d^4\theta \xi V$$

$$\delta \mathcal{L}_{FI} = \text{T.D. ex}$$

but not true if $\xi = \text{superfield}$
doesn't arise from SUSY

alt. ~~SUSY & gauge sym~~ $D = - \sum g_i \phi_i^\dagger \phi_i \neq 0$

(charged fields get vevs).

complicated, not clearly useful, no time...

More General features:

$$V = F^a F_a^* + \frac{1}{2} D_A D_A$$

$$\uparrow$$

$$\frac{\partial W}{\partial Q^a}$$

$$\uparrow$$

$$g_A Q_a^\dagger T_A^a Q^b$$

- goldstino can find directly from ferm mass matrix
or, current algebra.

$$\text{SUSY} \xrightarrow{\text{Noether's Thm}} \partial_\mu j_\alpha^\mu = 0; \quad Q_\alpha = \int d^3x j_\alpha^0$$

Suppose, e.g., F breaking (w/ D simple generalization.)

$$\text{Consider } \langle 0 | j_\alpha^\mu(q) \psi_\phi | 0 \rangle = \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \langle T j_\alpha^\mu(x) \psi_\phi(0) \rangle$$

$$q_\mu \cdot \left(\quad \right) = \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \underbrace{i \frac{\partial}{\partial x^\mu} \langle T j_\alpha^\mu(x) \psi_\phi(0) \rangle}_{\text{partner to } F_\phi \neq 0}$$

$$= i \langle [j_\alpha^0(x), \psi_\phi(0)] \rangle \delta(x^0)$$

take $q \rightarrow 0$

$$= i \langle [Q_\alpha, \psi_\phi] \rangle = i F$$

\Rightarrow pole in $\langle 0 | j_\alpha^\mu(q) \psi_\phi | 0 \rangle$ @ $q=0$: massless part.

Indeed (ex) $j_\alpha^\mu = i\sqrt{2} \sigma_{\alpha\dot{\alpha}}^\mu \psi^{*\dot{\alpha}} F$

$$\partial_\mu j_\alpha^\mu = 0 \Leftrightarrow \not{\partial} \psi^* = 0.$$

ex find the goldstino in O'Raifeartaigh.

local SUSY \Rightarrow eaten: super Higgs.

• another general statement about mass matrix w/ tree breaking:

$$0 = \text{tr} (-1)^F M^2$$

$$(-1)^F = \begin{array}{l} 1 \text{ bosons} \\ -1 \text{ fermions} \end{array}$$

can prove in general; e.g. w/ F breaking/chiral mults.

Recall $M_{Fab} = \partial_a \partial_b W$

Bosons from $V = |\partial_a W|^2$

$$M_{a\bar{b}}^2 = \partial_a \partial_{\bar{b}} V = \partial_a \partial_c W \partial_{\bar{b}} \partial_{\bar{c}} W^*$$

$$M_{ab}^2 = \partial_a \partial_b V = \partial_a \partial_b \partial_c W \partial_{\bar{c}} W^*$$

↑
off diag (recall cplx basis for scalar field)

$$\Rightarrow \text{Tr } M_B^2 = \sum_{a,c} \partial_a \partial_c W \partial_{\bar{a}} \partial_{\bar{c}} W^* = \text{Tr } M_F^2 \quad \checkmark$$

Trouble! light superpartners.

- Outs:
- large loop corr^s
 - non-renormalizable terms in K . (eg $K_{a\bar{b}} \neq \partial_a \partial_{\bar{b}}$)

these $\propto \frac{1}{M_P}$ \Rightarrow typically

SUSY at high scale

↓ messenger dynamics

SUSY at low scale.

maybe helps w/ flavor?
(if "blind")

E.g. gravity (or gauge.)

→ SUGRA

summarize effects in LEFT by higher dim ops. $\propto \frac{1}{M_P^{d}}$.

Let $X =$ field in "hidden sector", $\langle F_X \rangle \neq 0$, $\langle X \rangle = 0$

~

$$\Delta \mathcal{L} = \int d^4\theta \left\{ \frac{XX^\dagger Q_i Q_j^\dagger}{M_P^2} Z_{ij} + \frac{b}{M_P} X H \bar{H} + \frac{b'}{M_P^2} X^\dagger X H \bar{H} + \text{hc} + \dots \right\}$$

$$+ \int d^2\theta \left(\frac{s_i}{M_P} X W^\alpha W_\alpha + \text{hc} + \dots \right)$$

$$+ \int d^2\theta \left(\frac{a_{ij}}{M_P} X Q_i \bar{U}_j H + \dots \right)$$

$\langle F_X \rangle \sim$ soft SUSY described before!

$$\text{want } \text{TeV} \sim M_{\text{susy}} \sim \frac{F_X}{M_P} \Rightarrow F_X \sim (10^{11} \text{ GeV})^2$$

"intermediate scale"

difficulty: Z_{ij} , a_{ij} on visible flavor.

symmetry avoids?

but gravity isn't expected to respect global symmetries, gauge??

Minimal Ansatz: $Z_{Qij} = Z_{Li_j} = \dots = z_0 \delta_{ij}$
 etc. \leftrightarrow "minimal SUGRA" as described.

feature (ex): H mass runs negative from large λ_t
 \leftrightarrow radiative symmetry breaking.

Flavor issues suggest: gauge mediation ... another story

Final comment on SUSY breaking & LHC:

$$V_{\text{Higgs}} = M_{H_u}^2 |H_u|^2 + M_{H_d}^2 |H_d|^2 - M_{H_u H_d}^2 (H_u H_d + \text{hc}) \quad \leftarrow \mu + \text{soft terms}$$

$$+ \frac{1}{8} (g_1^2 + g_2^2) (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g_1^2 |H_u H_d|^4 \quad \leftarrow \text{quant terms from D-terms.}$$

requirements for $SU(2) \times U(1)$ breaking pot (ex)

$$M_{H_u}^2 + M_{H_d}^2 - 2|M_{H_u H_d}|^2 > 0$$

$$|M_{H_u H_d}|^4 > M_{H_u}^2 M_{H_d}^2$$

$$\leadsto \langle H_u \rangle \neq 0, \langle H_d \rangle \neq 0, \quad \tan \beta \equiv \frac{\langle H_u \rangle}{\langle H_d \rangle}$$

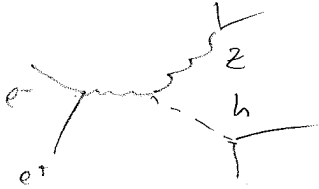
$$\text{define } m_A^2 = \frac{M_{H_u H_d}^2}{\sin \beta \cos \beta}$$

can show:

$$M_{H_{1,2}}^2 = \frac{1}{2} \left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos(2\beta)} \right)$$

note $m_{h_0} \leq m_Z$!

but LEP bd: ≥ 115 GeV.



but :
• loop corr^{ns} \Rightarrow MSSM not ruled out
• beyond MSSM ?

A very interesting story to be revealed.