SM$^-$ (SM - Higgs : what has been observed)

Known matter :

focus

e_i : e^-, μ^-, τ^-

ν_i

u_i : u, c, t

d_i : d, s, b

$s = 1/2$; fermions

Eff lagrangian $\longrightarrow$ interactions

EM $U(1)$, gauge.

$$\psi_I(x) \longrightarrow e^{iQ_I e\Gamma(x)}\psi_I(x)$$

local $U(1)$

Q_I = arbitrary $e \simeq -.30$ $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$

Effective lagrangian - gauge invc. (local symmetry)

$$D_\mu \psi_I = (\partial_\mu + iQ_I e A_\mu)\psi_I \longrightarrow e^{iQ_I e\Gamma} D_\mu \psi_I$$

gauge field $A_\mu \to A_\mu - \partial_\mu \Gamma$

spin 1

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \longrightarrow F_{\mu\nu}$

$$\mathcal{L} = i\bar{\Psi}_I \not{D} \Psi_I - m_I \bar{\Psi}_I \Psi_I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots$$

Dirac ferm

$\gamma^\mu D_\mu$

diagonalize

arb. (absorb into A)

dims:

$[\Psi] = \frac{3}{2}$ $\quad [A] = 1$

dim ≤ 4 (most general)

higher dim

F^4, $\bar{\Psi} F \Psi$, ...

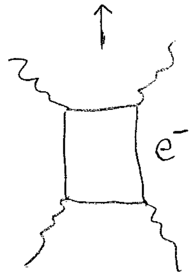
irrelevant

$Q_e = -1 \quad Q_\nu = 0 \quad Q_u = 2/3 \quad Q_d = -1/3$

E.g. of matching EFTs: (another e.g.: HW)

$E \ll m_e \; (\leq m_I)$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_4 \frac{F^4}{m_e^4} + \cdots$$



@ $E \sim m_e$, match to above.

A_μ ... spin 1, massless ($m^2 A^2$ forbidden - gauge sym.)

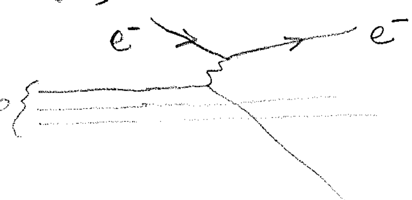
QCD

Hadrons $\begin{cases} \text{Mesons} & \pi, K, \dots \\ \text{Baryons} & p, n, \Sigma, \dots \end{cases}$ made of quarks

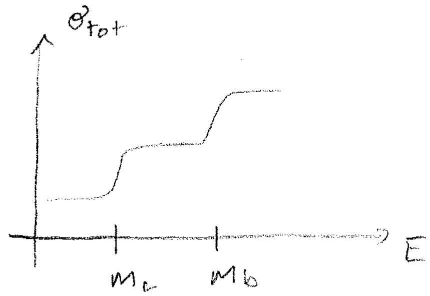
Evidence :

a) Spectra eg $(u,d,s) = 3_f$, $3 \times \bar{3} = 8 + 1$

b) deep inelastic scatt.
~ modern Rutherford



c) $e^+e^- \longrightarrow$ hadrons



...

Never seen free : tightly bound

sca~~lar~~ ? likes attract, etc (see Ross p 90)

gauge thy - non abelian G

$$\phi \longrightarrow U(h(x))\, \phi \qquad h \in G$$

ϕ: n; $U(h(x))$: $n \times n$

$$D_\mu \phi = (\partial_\mu - i g A_\mu)\phi$$

A_μ : $n\times n$, $\in$ adj(G)

i.e. $U(x) = e^{-i\gamma^a(x) T^a}$; $A_\mu = A_\mu^a T^a$
(T^a: $n\times n$)

$$A_\mu \to U(x) A_\mu U^\dagger(x) + \frac{i}{g} U(x) \partial_\mu U^\dagger(x)$$

$$\Longrightarrow \quad D_\mu \phi \to U(x) D_\mu \phi(x)$$

Also $$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$
$$= F^a_{\mu\nu} T^a$$

Lagrangian : quarks $q_{\alpha i}$ ← rep R of G
(α: u, d, ...)

$$\mathcal{L} = i \sum_\alpha \left(\bar{q}_\alpha \not{D} q_\alpha - m_\alpha \bar{q}_\alpha q_\alpha \right) - \underbrace{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})}$$

$+$ ~~higher dim.~~ → irrelevant, drop

$(+ \theta \epsilon^{\mu\nu\lambda\sigma} F^a_{\mu\nu} F^a_{\lambda\sigma})$
... later

w/ $\mathrm{Tr}(T^a T^b) = \frac{\delta^{ab}}{2}$

— 1/11/07

Experiment : $G = SU(3)$, $R = 3$

e.g.
- spectrum $qqq \sim 3\times 3\times 3 = 10+8+8+1$
- $\pi^0 \to 2\gamma$
- $e^+e^- \to q\bar{q}$

⋮