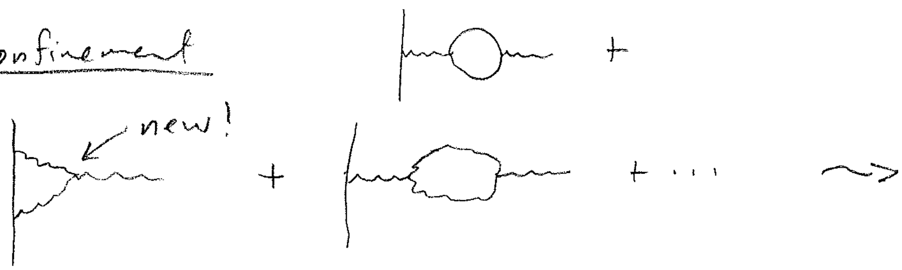
Confinement



$$\beta(g) = \mu \frac{d}{d\mu} g \underset{\text{1 loop}}{\simeq} -\frac{g^3}{16\pi^2}\left[\frac{11}{3}T(A) - \frac{4}{3}T(F)\right]$$

Here: $\quad T^a_{ij} T^a_{jk} = \underset{\text{rep}}{T(R)}\, \delta_{ik} \quad \ldots$ quadratic Casimir

$T(A) = N$ for $SU(N)$ $\quad$ (ex)

$T(F) = 1/2$ for each quark

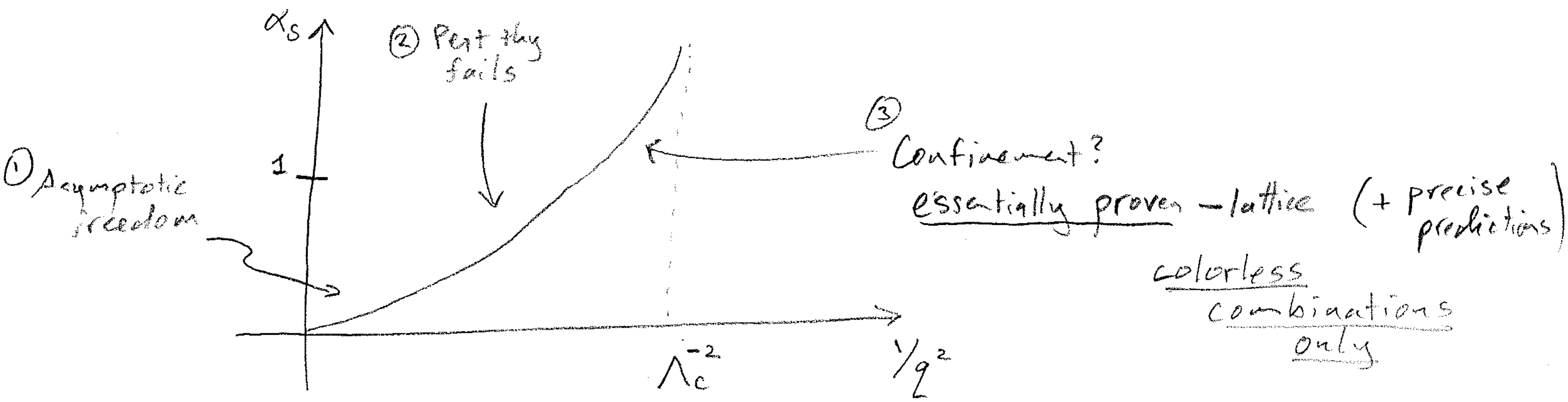
$$\Rightarrow \beta(g) \simeq -\frac{g^3}{16\pi^2} b_0 \qquad \text{w/} \quad b_0 = \frac{11}{3}N - \frac{2}{3}n_F$$

$$N = 3, \; n_F = 6 \Rightarrow \; > 0$$

$$\Rightarrow \alpha_s(q^2) \equiv \frac{g^2}{4\pi} \simeq \frac{1}{\frac{b_0}{4\pi}\log(q^2/\Lambda_c^2)}$$

int const.

Supports picture:



<u>dimensional transmutation</u> : parameter $\Lambda_c \simeq 200\,\text{MeV}$

<u>QCD-symmetries</u>

- P, C, T
- flavor $\quad u \to e^{i\alpha_u} u \qquad U(1)^{n_F}$
  $\quad d \to e^{i\alpha_d} d$
  $\quad \vdots$

Also, e.g. PD book:
$$\left.\begin{aligned} m_u &= 1.5-3\ \text{MeV} \\ m_d &= 3-7\ \text{MeV} \\ m_s &= 95 \pm 25\ \text{MeV} \end{aligned}\right\} \ll \Lambda_c$$
(though masses subtle business)

$\Longrightarrow$ 1. Most mass of $p, n, \cdots$ <u>not in quarks</u>!

2. Approx global chiral flavor symm

$$U(3)_L \times U(3)_R \qquad \left(\begin{array}{l}\text{don't confuse} \\ \text{w/ } SU(3)_c\end{array}\right)$$

$u_L \leftrightarrow d_L \leftrightarrow s_L$

exact as $m_{u,d,s} \to 0$

$$J^a_\mu = \sum_{\alpha = u,d,s} \bar{q}_\alpha \gamma_\mu \frac{T^a_{\alpha\beta}}{2} q_\beta \qquad U(3)$$

$$J^a_{A\mu} = \bar{q} \gamma_\mu \gamma_5 \frac{T^a}{2} \bar{q}$$

Low-E spectrum $SU(3)_V \times \cancel{SU(3)_A}$

$\uparrow$ don't see

eg $(\pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta) = 8$ ... no chiral partners

Explanation: $SU(3)_A$ <u>spontaneously broken</u>

Recall Spontaneous symm. breaking:

- non-singlet field gets vev
- preserve Lorentz invc.

$\bar{q}_\alpha q_\beta$ ... Lorentz invt
not invt under chiral symm.

$\Rightarrow$ expect $\langle 0 | \bar{q}_\alpha q_\beta | 0 \rangle \neq 0$, $\sim \Lambda_c^3$

($\leftrightarrow$ superconductivity)

$SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$ <u>χSB</u>

SSB $\rightarrow$ Goldstone bosons? $\underset{\substack{\uparrow \\ \dim SU(3)}}{8}$: <u>(π, K, η)</u>

if so, massless for $m_\alpha = 0$

π: 140 MeV "$\ll$" p 1 GeV
K: 500 MeV ρ 776 MeV
η: 549 MeV

pseudo
or world-be Goldstone bosons

<u>$U(1)_V$, $U(1)_A$</u>

$$J_\mu = \bar{q}\gamma_\mu q \qquad \text{baryon \#} \qquad \partial_\mu J^\mu = 0$$

$$J_{A\mu} = \bar{q}\gamma_\mu\gamma_5 q \qquad \text{axial " "} \qquad \partial_\mu J^\mu \neq 0$$

<u>Anomaly</u>

<u>Chiral lagrangian</u> : (another EFT example!)

- $SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$
- π = Goldstone bosos.

let $\Sigma(x) = e^{i\gamma^a(x)T^a} \in SU(3)$

$$\gamma^a = \frac{2\pi^a}{f} \leftarrow \text{dim } 1$$

~ chiral scale

$$\Sigma \to L\Sigma R^+$$

Correctly captures the SSB :

$\langle\Sigma\rangle = 1$ invt. if $L = R$

breaks $L \neq R$

Param $L = e^{ic}e^{iv}$

$R = e^{-ic}e^{iv}$

v: $\pi \to e^{iv}\pi e^{-iv}$ (ex)

c: $\pi \longrightarrow \pi + fc + \cdots$ ~ Goldstone boson.

Lagrangian:

Note $V(\Sigma) = \text{const}$ if inv't eg $tr(\Sigma^\dagger \Sigma) = tr(1)$

2 derivs: $$\frac{f^2}{4} tr(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) = \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a + \dots \qquad (ex)$$

interactions

4 derivs ...

Masses = pert : in QCD $\sim \bar{q}_L M q_R$ $\qquad M = \begin{pmatrix} m_u & & 0 \\ & m_d & \\ 0 & & m_s \end{pmatrix}$

Σ : linear, breaks $SU(3) \times SU(3)$:

$$v^3 \left[tr(\Sigma^\dagger M) + h.c. \right]$$

dim analysis.

$\rightsquigarrow$ $3m_\eta^2 + m_\pi^2 = 4m_K^2$ Gell-Mann-Okubo

etc.

more: Georgi.

Lattice QCD: many consistency checks of this chiral EFT