

A leading candidate:

The Higgs

ϕ scalar, 2 of $SU(2)$

$$\mathcal{L}_\phi = -|D_\mu \phi|^2 - \frac{\lambda}{4} \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

$$\Rightarrow |\langle \phi \rangle|^2 = \frac{v^2}{2} \quad SU(2) \text{ rot. } \rightsquigarrow \langle \phi \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\text{need } Q=0 = T_3 + Y \Rightarrow Y = -1/2$$

$$\therefore \text{breaks } SU(2) \times U(1) \rightarrow U(1)$$

relate to chiral lag.:

$$\text{let } \tilde{\phi} = \epsilon \phi^* \quad \dots \text{ also 2}$$

and

$$\hat{\Sigma} = (\phi, \tilde{\phi}).$$

Transforms as:

$$\hat{\Sigma} \rightarrow U_L \hat{\Sigma} e^{-i\alpha \tau_3/2}$$

\uparrow obvious \uparrow ex (easy)

$$\text{note } \text{Tr } \hat{\Sigma}^\dagger \hat{\Sigma} = 2|\phi|^2$$

$$\text{in general } \phi(x) = \underbrace{\Sigma(x)}_{SU(2)} \begin{pmatrix} (v+h(x))/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\Rightarrow \hat{\Sigma} = \frac{v+h(x)}{\sqrt{2}} \Sigma$$

$$- |D\phi|^2 = -\frac{1}{2} \text{Tr}(D\hat{\Sigma})^2 = -\frac{v^2}{4} \text{Tr}(D\Sigma)^2 - \frac{(\partial h)^2}{2} + \dots$$

(ex)

$$\therefore f = v$$

$$M_W = g_2 \frac{v}{2}, \quad \rho = 1$$

Fermion masses

$$\phi = (2, -1/2)$$

$$q\bar{d} : Y = 1/2 \Rightarrow$$

$$\lambda_i^d \epsilon_{\alpha\beta} \phi^\alpha q_i^\beta \bar{d}_i \quad \text{allowed} \dots \quad \underline{\text{Yukawa couplings}}$$

$$\langle \phi \rangle \rightsquigarrow m_i^d = \lambda_i^d v / \sqrt{2}$$

also



$$\text{likewise } e_i \text{'s} : m_i^e = \lambda_i^e v / \sqrt{2}$$

ups? $q\bar{u} : Y = -1/2$

can get as $\tilde{\phi} = \epsilon\phi^*$

alternately, $\tilde{\phi} \neq \epsilon\phi^*$ (SUSY, ...)

$$\lambda_i^u \tilde{\phi} q_i \bar{u}_i \rightsquigarrow m_i^u = \lambda_i^u v / \sqrt{2}$$

notes on symmetries:

a) w/out Higgs, also have chiral symmetry

$$q_i \rightarrow U_{ij} q_j$$

$$\bar{u}_i, \bar{d}_i \rightarrow U_{ij} \bar{u}_j, \bar{d}_j$$

l_i, \bar{e}_i also.

$$[U(3)]^5$$

Higgs explicitly breaks

2 Higgs restores $U(1)$ (classical) :

$$\phi \rightarrow e^{-2i\alpha} \phi$$

$$\tilde{\phi} \rightarrow e^{-2i\alpha} \tilde{\phi}$$

(\leftrightarrow SUSY ...)

b) custodial $\hat{\Sigma} \rightarrow \hat{\Sigma} U_R$
 \uparrow
 $(\phi \tilde{\phi})$

- not present if Higgs $\neq 2$ ("accidental")
- calculable violation $\propto \lambda_i^u - \lambda_i^d, g_2$

eg. if equal,

$$\lambda_i g_i \hat{\Sigma} \begin{pmatrix} \bar{u}_i \\ d_i \end{pmatrix} = \text{inv. t.}$$

↑ rotates by U_R - see p 2.19.

Couplings renormalizable

(Renormalizability w/SSB: 't Hooft-Veltman \rightarrow Nobel)

... minimal renormalizable theory

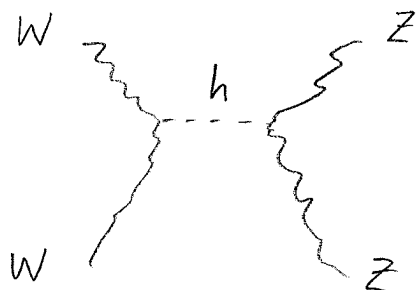
1 new degree of freedom.

mass:
$$V(\phi) = \frac{\lambda}{4} \left(\frac{v^2 + 2hv + h^2}{2} - \frac{v^2}{2} \right)^2$$

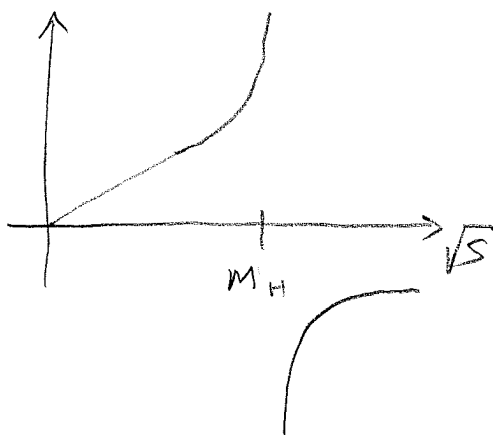
$$= \frac{\lambda}{4} (vh)^2 + \dots \Rightarrow$$

$$m_h = \frac{\sqrt{\lambda} v}{\sqrt{2}}$$

Unitarity $M(W_L W_L \rightarrow Z_L Z_L) \Rightarrow$



$$M_{\text{tot}} \approx \frac{g_Z^2 S}{4M_W^2} \frac{m_H^2}{m_H^2 - S} \quad (\text{HW})$$



(tree level only...)

\therefore m_H cuts off growth of Goldstone boson piece.

$$m_H \ll 1 \text{ TeV} \leftrightarrow \text{weak}$$

$$m_H \gtrsim 1 \text{ TeV} \leftrightarrow \text{strong}$$

$M_H \gtrsim 114 \text{ GeV}$ LEP 2 direct (hints?)

loop corrections (precise EW - later?):

$$M_H \approx 199 \text{ GeV}$$

(see Altarelli & other refs.)

The Standard Model

$$\mathcal{L}_{SM} = \mathcal{L}_{sym} + \mathcal{L}_{\phi}$$

gains: unitarity, renormalizability

problems: rest listed before, and ...

naturalness

- $V(\phi)$ renormalized
- expect $\delta M_h^2 \sim \delta(\Lambda^2) \sim \Lambda_{\text{New Phys}}^2$

unless fine tuning. (why not M_{Planck}^2 ??)

not as bad as ~~unitarity~~, ~~nonrenorm~~

but explanation needed. (environmental selection ??)

this "hierarchy problem" suggests:

new physics nearby

Also $m_h \approx 180 \text{ GeV} \Rightarrow \text{Landau pole } (\lambda \rightarrow \infty)$

$m_h \approx 130 \text{ GeV} \Rightarrow \text{instability : } \lambda \rightarrow < 0$

- 1/30/07