Quantum Transitions Between Vacua

Motivation: String Landscape
how are states populated?
(Which are populated with what probability TBD...)

Testing ground for semi-classical + quantum gravity.

Basic Model:

\[ V(\phi) \]

- Coleman + company
  - FV is generally metastable.
  - Tunneling corresponds to bubble nucleation.
- Lee + Weinberg
  - If \( V(\phi) > 0 \), then you can tunnel up as well.

No Gravity (Coleman, Callan + Coleman)

In WKB: \( \frac{\pi}{V} = A e^{-\frac{S_E}{2}} \)

\( O(4) \) - invariant instanton has lowest action:
\[ \phi = \phi(\rho) ; \rho \in \mathbb{R}^2 \]
EOM: \( \frac{\partial^2 \phi}{\partial \rho^2} + \frac{3}{\rho} \frac{\partial \phi}{\partial \rho} = \frac{\partial V}{\partial \phi} \)

BC: \( \phi(\rho \to 0) = \phi_F \) (Far away, Field is in FV)
\( \phi(\rho=0) \approx \phi_T \) (near origin, Field approaches the TV)
\( \frac{\partial \phi}{\partial \rho} \bigg|_{\rho=0} = 0 \) (EOM non-singular)

Solution:

\( \phi \)

\( \phi_T \)

\( \phi_F \)

\( \rho \)

This has a clear interpretation as a localized event
With gravity: Coleman + de Luccia

In WNB \[ \Pi = A e^{-S_{\text{eff}}} \]

\[ S_{\text{eff}} = S_I - S_{\text{BL}} \]

\[ S_I = - \int d^4 x e \sqrt{\gamma} \; V(\phi(x)) \]

\[ S_{\text{BL}} = - \int d^4 x e \sqrt{\gamma} \; V(\phi(x)) \approx - \frac{3}{8 \sqrt{2} \gamma \psi} \]

Assume \( O(4) \)-invariant:

\[ ds^2 = dt^2 + \rho^2(t) dS^2 \]

EOM:

\[ \frac{\partial^2 \phi}{\partial t^2} + \frac{3}{\rho} \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial \rho} = \frac{\partial V}{\partial \phi} \]

\[ \left( \frac{\partial \phi}{\partial t} \right)^2 = 1 + \frac{8\pi}{3} \rho^2 \left( \frac{1}{2} \left( \frac{\partial \phi}{\partial \rho} \right)^2 - V(\phi) \right) \]

BC: IF \( V_\psi > 0 \), instanton is compact (\( \rho \) has 2 zeros)

\[ \rho(z = 0) = 0 \; ; \; \rho(z_m) = 0 \]

\[ \left. \frac{\partial \phi}{\partial z} \right|_{z = 0} = 0 \; ; \; \left. \frac{\partial \phi}{\partial z} \right|_{z = z_m} = 0 \]

\[ \phi(z = 0) \approx \phi_I \; ; \; \phi(z_m) \approx \phi_F \]
Solution: ρ

$\phi$

$\phi_f$

$z_0$

$z_m$

Initial data slice

TV or FV

! Very Different w/ Gravity!

1) No part of instanton contains $\phi_F$ or $\phi_f$.

2) No nice interpretation as a localized fluctuation.

3) Solutions can have diverse properties:
   - $V_F=0$ vacua can be stabilized.
   - Multiple-pass instantons.
   - Hawking-Moss instanton.
The Great Divide (Aguirre, Banks, Johnson)

As $V_T \to 0$, $S_{BL} \to \infty$

Does $S_1$ cancel this?

For $M < M_c$ : Yes
For $M > M_c$ : No

Above the divide: $\Gamma \sim e^{-\frac{g}{3V_T}}$

Downward transitions entropically suppressed

This, together with detailed balance

For $V_T > 0$, suggests interpretation

of $TV$ as finite quantum system with

transitions as rare fluctuations.

Below the divide: Unclear interpretation.

Decays can be very fast:

$$S_E = \frac{8\pi^2}{M_{3/2}^2} \frac{(C-1)^4}{c^2(2c-1)^2}, \quad C = \frac{1}{IV_T^{S_{susy}}}$$

(crescote et al.)
Euclidean action of this trajectory:

$$ S_E = \frac{2\pi \pi^2 \sigma^4}{2(V_F - V_T)} \quad F(\sigma, V_F, V_T) $$

↑

Flat space result

gravitational "correction"

More on the interpretation:

$$ ds_{FF}^2 = (1 - H_{TF}^2 R^2) \, dt^2 + (1 - H_{TF}^2 R^2)^{-1} \, dR^2 + R^2 d\Omega^2 $$

Note: no initial data surface for FI bubbles (or on the real instanton ...)
Thin-walled Approximation

Match 2 3S spheres:

1) TV bubbles, when small, look similar to the flat-space case.

2) FV bubbles involve very large fluctuations.

3) The radius of the wall becomes a collective coordinate to describe tunneling.

Dynamics determined by: $\sigma$, $V_T$, $V_F$

Tunneling from $R=0$ to $R_c$ found from Euclidean GOM:

$$(\Delta R)^2 + \left(\frac{R}{R_c}\right)^2 = 1$$
Lorentzian thin-wall CDL - Matching 2 DS

$\Rightarrow$ wall + volume energies cancel

What if they don't?

$\Rightarrow$ Exterior is now SDS

new parameter $M$

Still have 1-D potential problem:

$\left( \frac{\partial R}{\partial L} \right)^2 - V(R, M) = -1$

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<tr>
<th>Bound</th>
<th>unbound</th>
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Can you tunnel through?

Semi-classically, seems like Yes!

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L geometry \[\text{(Faschi, butt, + Bruner)}\]

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R geometry

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Zero-Mass limit:

L geometry:

R geometry:

FV bubble universe created from nothing.

Standard LW FV bubble

There is also an $O(3)$ invariant instanton:

Thermal activation of $\Omega$ (Garriga & Negevand)
Puzzles: What is allowed?

Not surprisingly, puzzles arise when gravity is important: FV bubbles

1) Interpolating geometry - when turning points are separated by a horizon, not a manifold...
   problem w/ FV bubbles and L geometry TV bubbles

2) ADS/CFT (Freivogel et al) - L geometry is not a unitary transition.
   Perhaps not surprising:
   
   ![Diagram of region behind the wormhole](image)
   region behind the wormhole was not there before the tunnel...

3) Bound solutions are violently unstable:
   can we assume spherical symmetry?
   Effects on pre-factor?