Constructing bulk observables in AdS/CFT

work with HLLRS, also IX
related work by BDHMBGLBHMPSPR

Outline

1. Semiclassical results
2. Finite N and black holes
3. Open problems

Semiclassical results

Start in the semiclassical limit $N \to \infty$, $\ell_P \to 0$ in Poincare coordinates on AdS_{d+1}.

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 + |dX|^2 + dZ^2\right) \qquad 0 < Z < \infty$$

Take a scalar field of mass m in AdS, dual to an operator $\mathcal{O}$ of dimension Δ in the CFT.

$$\phi(T, X, Z) \sim Z^{\Delta}\mathcal{O}(T, X) \qquad \text{as} \quad Z \to 0$$

We can obtain CFT correlators by sending bulk points to the boundary.

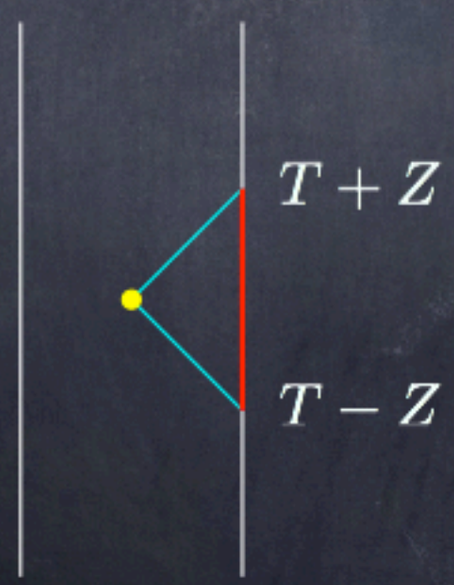
Can we go the other way, and recover the bulk field from its boundary behavior?

Yes. For example for a massless field in AdS_2

$$\Box\phi = 0 \qquad \phi(T,Z) \sim Z\mathcal{O}(T) \quad \text{as } Z \to 0$$

$$\Rightarrow \phi(T,Z) = \frac{1}{2}\int_{T-Z}^{T+Z} dT'\mathcal{O}(T')$$

Can represent this as



integrate over the spacelike separated region on the boundary

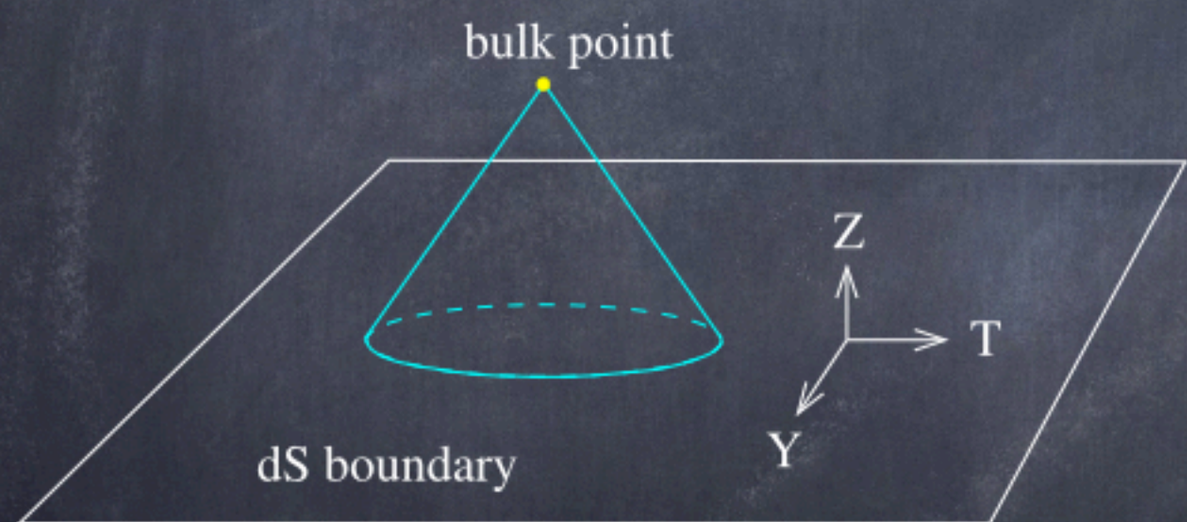
This generalizes to higher dimensions, provided you complexify the boundary.

HKLL, Leichenauer & Rosenhaus

$$X = iY$$

$$ds^2 = \frac{R^2}{Z^2}\left(dZ^2 - dT^2 - |dY|^2\right)$$

de Sitter space

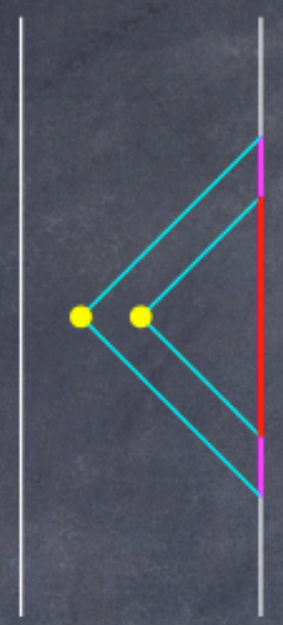


$$\phi = \int_{T^2+|Y|^2<Z^2} K\,\mathcal{O}$$

Also generalizes to fields with spin.

Heemskerk, KLRS, Sarkar & Xiao

Bulk locality isn't obvious.



overlapping operators on boundary

In the large N limit locality holds by construction.

In the 1/N expansion bulk locality can be satisfied by adding multi-trace, higher-dimension operators.

$$\phi(T,Z) = \int K\mathcal{O} + \sum_i a_i \int K_{\Delta_i}\mathcal{O}_i$$

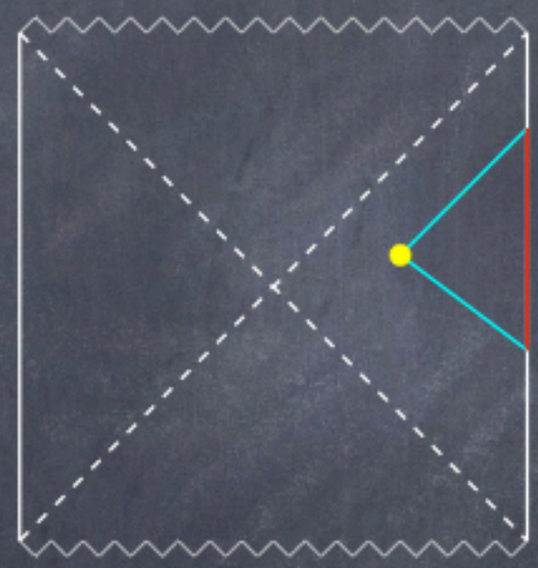
KLL

Requiring locality fixes the coefficients a_i.

=> a nice story in the semiclassical limit

Finite N and black holes

At finite N microcausality should break down: the CFT doesn't have enough d.o.f. to build local bulk fields. A simple context where we can see this happening in a 2-point function is an AdS-Schwarzschild black hole.



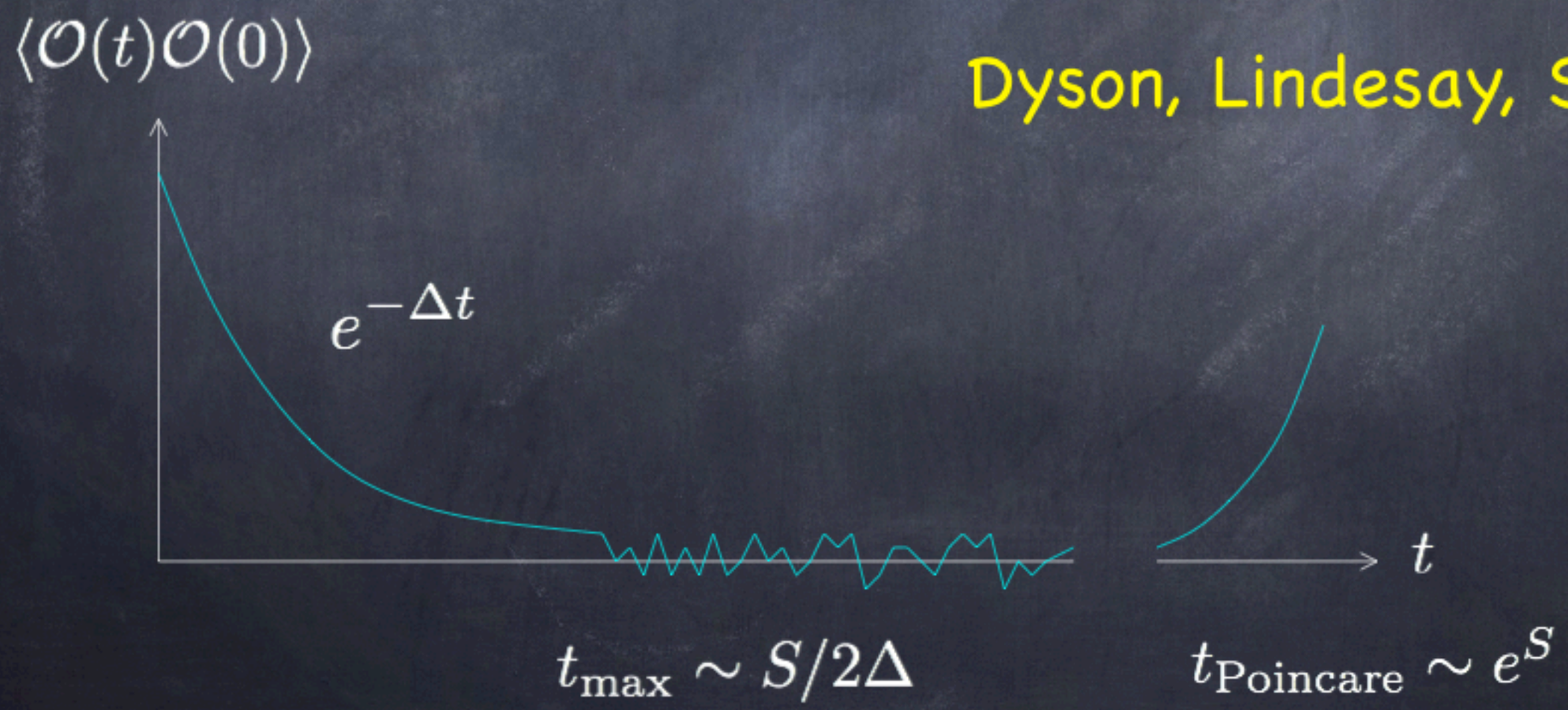
As the bulk point approaches the future horizon the smearing extends to $t = +\infty$ and becomes sensitive to the late-time behavior of CFT correlators.

This is problematic. At finite entropy a correlator can't go below the generic inner product of two normalized vectors.

$$\left|\langle\psi_1|\psi_2\rangle\right| \sim \frac{1}{\sqrt{\dim \mathcal{H}}} = e^{-S/2}$$

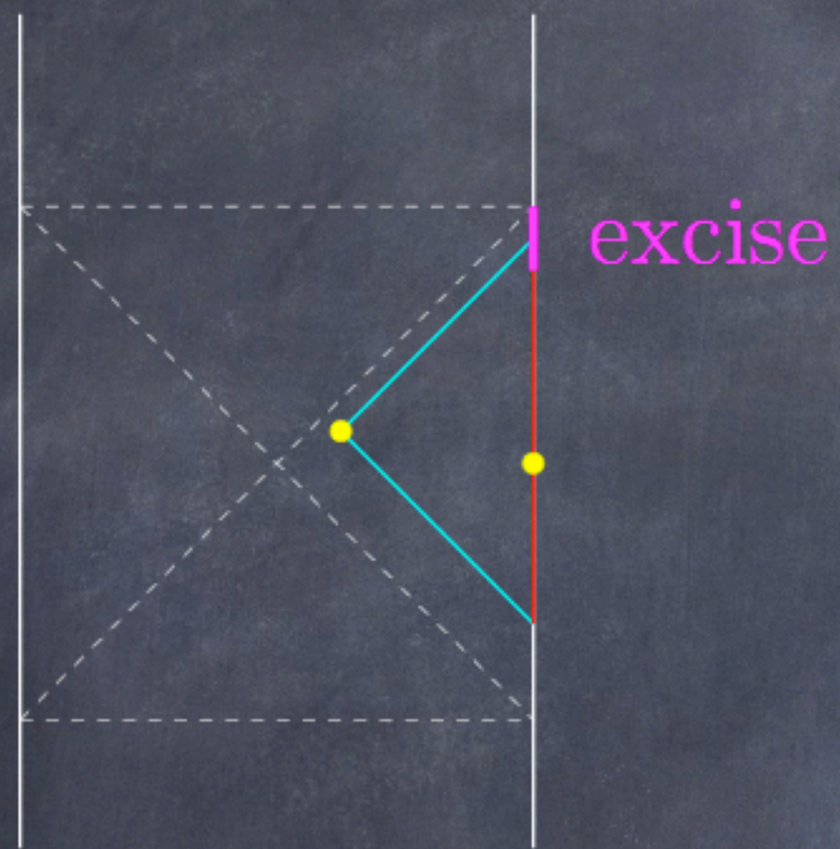
So a finite-N correlator should look like

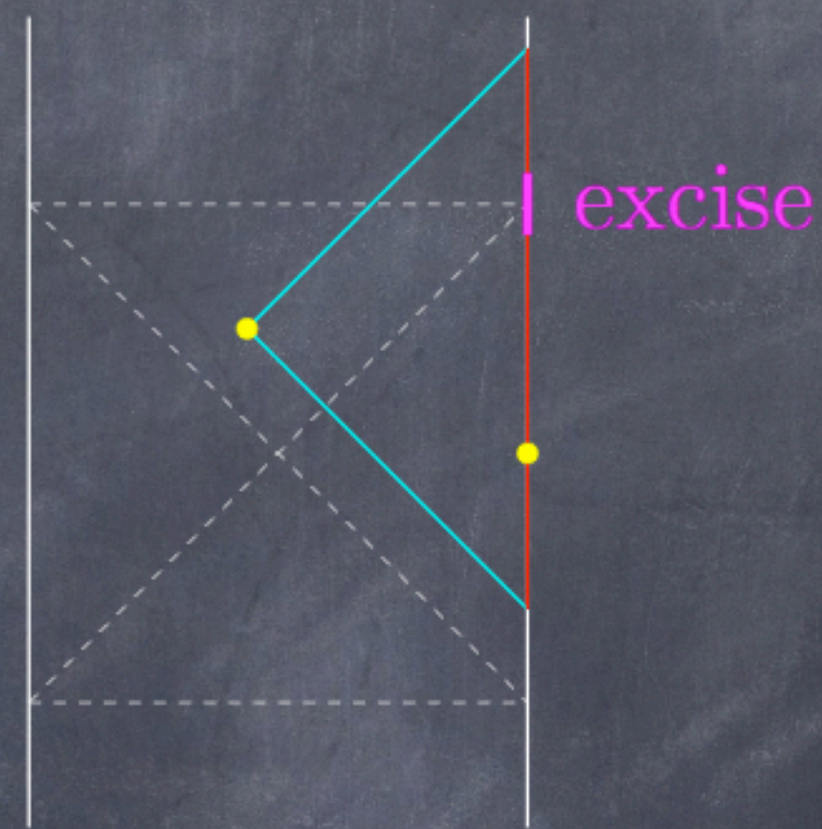
Dyson, Lindesay, Susskind



A simple cure is to cut off the smearing function at $t \sim t_{\max}$. In Rindler coordinates

HKLL





In Poincare coordinates we've cut out a ball of radius $e^{-S/2\Delta}$. In effect we've defined

$$\phi_{\text{modified}} = \phi_{\text{semiclass.}} - e^{-dS/2\Delta}\mathcal{O}$$

where $\mathcal{O}$ is a localized operator near the upper right corner of the Rindler patch.

KL

For SUGRA fields $\Delta = d$, so correlators are modified and microcausality is violated at $\mathcal{O}(e^{-S/2})$.

Consequences?

1. No horizons at finite N. Operators in the black hole interior don't commute with operators on $\mathcal{I}^+$.
2. These non-local commutators can account for correlations in outgoing Hawking particles.

Kabat & Iizuka

3. Measuring the state of the early HR is not innocuous! $\mathcal{O}(1)$ disturbance to the bh interior.

Open problems

1. For 2-point functions in a black hole background the cutoff procedure is a bit ad hoc. Is there a preferred cutoff? Or do we need to live with $\mathcal{O}\left(e^{-S/2}\right)$ ambiguities?
2. Locality should also fail in n-point functions. But when recovering bulk physics from an interacting finite-N CFT, what's the guiding principle?
3. Background dependence. Is there a procedure that can be used for any state in the CFT?