Constructing bulk observables in AdS/CFT

work with HLLRS, also IX
related work by BDHMBGLBHMPSPR

Outline

1. Semiclassical results
2. Finite N and black holes
3. Open problems
Semiclassical results

Start in the semiclassical limit $N \to \infty, \ell_P \to 0$ in Poincare coordinates on $\text{AdS}_{d+1}$.

$$ds^2 = \frac{R^2}{Z^2} \left( -dT^2 + |dX|^2 + dZ^2 \right) \quad 0 < Z < \infty$$

Take a scalar field of mass $m$ in AdS, dual to an operator $\mathcal{O}$ of dimension $\Delta$ in the CFT.

$$\phi(T, X, Z) \sim Z^\Delta \mathcal{O}(T, X) \quad \text{as} \quad Z \to 0$$

We can obtain CFT correlators by sending bulk points to the boundary.

Can we go the other way, and recover the bulk field from its boundary behavior?
Yes. For example for a massless field in $\text{AdS}_2$

\[ \Box \phi = 0 \quad \phi(T, Z) \sim Z \mathcal{O}(T) \quad \text{as} \quad Z \to 0 \]

\[ \Rightarrow \phi(T, Z) = \frac{1}{2} \int_{T-Z}^{T+Z} dT' \mathcal{O}(T') \]

Can represent this as

integrate over the spacelike separated region on the boundary
This generalizes to higher dimensions, provided you complexify the boundary.

\[ X = iY \]

\[ ds^2 = \frac{R^2}{Z^2} \left( dZ^2 - dT^2 - |dY|^2 \right) \]

HKLL, Leichenauer & Rosenhaus

de Sitter space

Also generalizes to fields with spin.

Heemskerk, KLRS, Sarkar & Xiao
Bulk locality isn't obvious.

In the large N limit locality holds by construction. In the 1/N expansion bulk locality can be satisfied by adding multi-trace, higher-dimension operators.

\[ \phi(T, Z) = \int KO + \sum_i a_i \int K_{\Delta_i} O_i \quad \text{KLL} \]

Requiring locality fixes the coefficients \( a_i \).

\( \Rightarrow \) a nice story in the semiclassical limit
Finite $N$ and black holes

At finite $N$ microcausality should break down: the CFT doesn't have enough d.o.f. to build local bulk fields. A simple context where we can see this happening in a 2-point function is an AdS-Schwarzschild black hole.

As the bulk point approaches the future horizon the smearing extends to $t = +\infty$ and becomes sensitive to the late-time behavior of CFT correlators.
This is problematic. At finite entropy a correlator can't go below the generic inner product of two normalized vectors.

\[ |\langle \psi_1 | \psi_2 \rangle| \sim \frac{1}{\sqrt{\dim \mathcal{H}}} = e^{-S/2} \]

So a finite-N correlator should look like

\[ \langle \mathcal{O}(t) \mathcal{O}(0) \rangle \]

\[ t_{\text{max}} \sim S/2\Delta \]

\[ t_{\text{Poincare}} \sim e^S \]

Dyson, Lindesay, Susskind
A simple cure is to cut off the smearing function at $t \sim t_{\text{max}}$. In Rindler coordinates

In Poincare coordinates we've cut out a ball of radius $e^{-S/2\Delta}$. In effect we've defined

$$\phi_{\text{modified}} = \phi_{\text{semiclass}} - e^{-dS/2\Delta} \mathcal{O}$$

where $\mathcal{O}$ is a localized operator near the upper right corner of the Rindler patch.
For SUGRA fields $\Delta = d$, so correlators are modified and microcausality is violated at $\mathcal{O}(e^{-S/2})$.

Consequences?

1. No horizons at finite N. Operators in the black hole interior don't commute with operators on $\mathcal{I}^+$.

2. These non-local commutators can account for correlations in outgoing Hawking particles.

3. Measuring the state of the early HR is not innocuous! $\mathcal{O}(1)$ disturbance to the bh interior.
Open problems

1. For 2-point functions in a black hole background the cutoff procedure is a bit ad hoc. Is there a preferred cutoff? Or do we need to live with $O(e^{-S/2})$ ambiguities?

2. Locality should also fail in n-point functions. But when recovering bulk physics from an interacting finite-N CFT, what’s the guiding principle?

3. Background dependence. Is there a procedure that can be used for any state in the CFT?