The black hole interior in AdS/CFT

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based on work with Suvrat Raju: 1211.6767, 1310.6334, 1310.6335 + in progress

Main questions

- Does CFT describe BH interior?
- What happens to the infalling observer?
- What can we learn about the information paradox

- Review of our proposal
- Issues
- Comments on non-equilibrium states

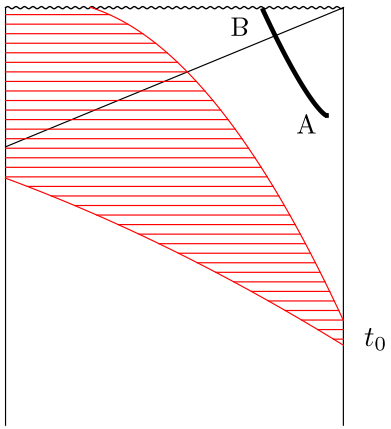
WORK IN PROGRESS

Many open questions...

Plan

Focus on big black hole in AdS

Black hole in AdS



Consider the $\mathcal{N} = 4$ SYM on $S^3 \times \text{time}$, at large N, large λ.
and typical pure state $|\Psi\rangle$ with energy of $\mathcal{O}(N^2)$.
Define

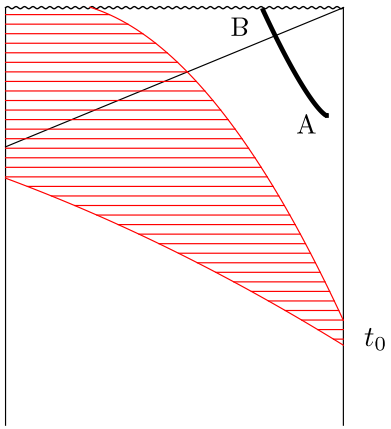
$$\mathcal{O}_{\omega,m} = \int_{-\infty}^{+\infty} dt\, e^{i\omega t} Y^*_{lm}(\Omega)\mathcal{O}(t,\Omega)$$

we have

$$[\mathcal{O}_{\omega,m}, \mathcal{O}^\dagger_{\omega',m'}] = \delta(\omega-\omega')\delta_{mm'}G(\omega,m)$$

where $G(\omega, m)$ satisfies the KMS condition.

Black hole in AdS



Define the CFT operator

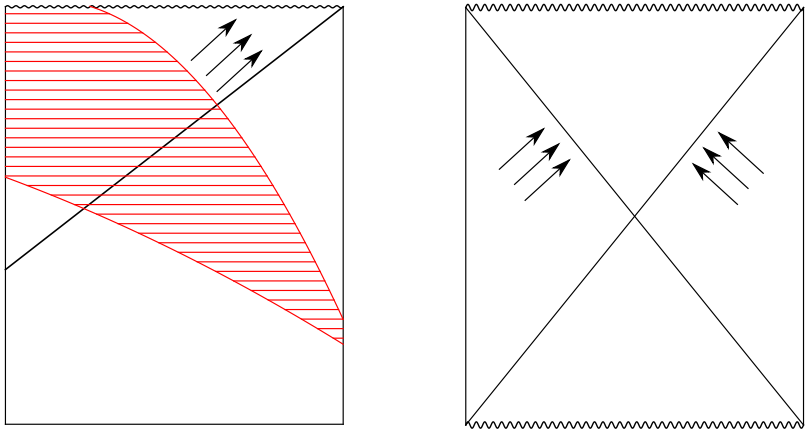
$$\phi_{\rm CFT}(t,\Omega,z)=\sum_m\int_0^\infty d\omega\,\mathcal{O}_{\omega,m}\,e^{-i\omega t}Y_m(\Omega)f_{\omega,m}(z)+\text{h.c.}$$

At large N (and late times) the correlators

$$\langle\Psi|\phi(t_1,\Omega_1,z_1)...\phi(t_n,\Omega_n,z_n)|\Psi\rangle$$

reproduce those of semiclassical QFT on the BH background (in Hartle-Hawking state)

Behind the horizon



Need new modes

If we have free infall we expect

$$\phi_{\rm CFT}(t,\Omega,z) = \sum_m \int_0^\infty d\omega \Big[\mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(z) + \text{h.c.}$$

$$+ \widetilde{\mathcal{O}}_{\omega,m}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(z) + \text{h.c.} \Big]$$

where the modes $\widetilde{\mathcal{O}}_{\omega,m}$ must satisfy certain conditions

Conditions for $\widetilde{\mathcal{O}}_{\omega,m}$

The $\widetilde{\mathcal{O}}_{\omega,m}$'s ("mirror" or "tilde" operators) must obey the following conditions:

- Correlators of all these operators on $|\Psi\rangle$ must reproduce (approximately) those of the thermofield double state

$$\langle\Psi|\mathcal{O}(t_1)...\widetilde{\mathcal{O}}(t_k)..\mathcal{O}(t_n)|\Psi\rangle \approx \frac{1}{Z}\mathrm{Tr}\big[\mathcal{O}(t_1)...\mathcal{O}(t_n)\mathcal{O}(t_k+i\frac{\beta}{2})...\mathcal{O}(t_m+i\frac{\beta}{2})\big]$$

or

- The algebra of $\widetilde{\mathcal{O}}$'s is isomorphic to that of the $\mathcal{O}$'s
- The $\widetilde{\mathcal{O}}$'s commute with the $\mathcal{O}$'s
- The $\mathcal{O}$'s and $\widetilde{\mathcal{O}}'s$ are correctly entangled

Does the CFT contain the operators $\widetilde{\mathcal{O}}$ with the desired properties?

If so, we will declare that the CFT describes the interior of the black hole and that we have free infall through the horizon.

Last year: argued for the existence of the $\widetilde{\mathcal{O}}$ by assuming an approximate decomposition of the Hilbert space as $\mathcal{H}_{coarse} \otimes \mathcal{H}_{\text{fine}}$ and using the entanglement.

Some progress: more precise understanding of tilde operators.

Construction of tildes

The "doubling" is generic whenever we have:

- A large quantum system in a typical state $|\Psi\rangle$
- We are probing it with a small algebra $\mathcal{A}$ of observables

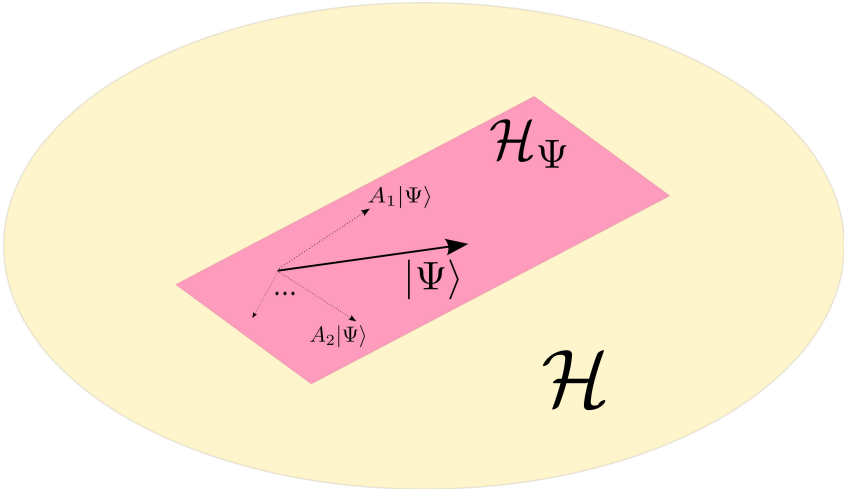
Under these conditions, the small algebra $\mathcal{A}$ is effectively "doubled".

For us:

- $|\Psi\rangle =$ BH microstate (typical QGP state of $E \sim \mathcal{O}(N^2)$)
- $\mathcal{A} =$ "algebra" of small number (i.e. $\mathcal{O}(1)$) of single trace operators

$$\mathcal{A} = \text{span of}\{\mathcal{O}_\omega,\ \mathcal{O}_{\omega_1}\mathcal{O}_{\omega_2}, ...\}$$

The Hilbert space $\mathcal{H}_\Psi$



For any given microstate $|\Psi\rangle$ consider the linear subspace $\mathcal{H}_\Psi$ of the full Hilbert space $\mathcal{H}$ of the CFT

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span} : A|\Psi\rangle,\ A \in \mathcal{A}\}$$

Reducibility of representation of $\mathcal{A}$

The "doubling" follows from the important property:

$$\boxed{A|\Psi\rangle \neq 0 \qquad \text{if} \qquad A \neq 0, \qquad \forall A \in \mathcal{A}}$$

(we cannot annihilate the QGP microstate by the action of a few single trace operators)

Physical interpretation:

"The state $|\Psi\rangle$ *appears to be entangled when probed by the algebra* $\mathcal{A}$".

Example 1: two spins

Consider the Hilbert space of two spins, and $\mathcal{A}$ = operators acting on the first. If the two spins are in the state

$$|\Psi\rangle = |\uparrow\uparrow\rangle$$

In this case there is no entanglement and indeed the previous condition is violated since

$$s_+^{(1)}|\Psi\rangle = 0 \qquad \text{while} \qquad s_+^{(1)} \neq 0$$

On the other hand consider the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Now there is entanglement and, relatedly, there is no non-vanishing operator acting on the first spin that annihilates the state.

Example 2: Relativistic QFT in vacuum state

Reeh-Schlieder theorem: consider any open set D in Minkowski space. The Minkowski vacuum $|0\rangle_M$ cannot be annihilated by acting with local operators in D.

$$\Rightarrow$$

In the Minkowski vacuum the local operator algebras at different regions in spacetime are entangled.

Special case: we cannot annihilate the Minkowski vacuum by acting with operators in the right Rindler wedge (we have entanglement between left and right wedges)

Why doubling?

Remember the important condition

$$A|\Psi\rangle \neq 0 \qquad \text{for} \qquad A \neq 0 \qquad (1)$$

Suppose that

$$\dim\mathcal{A} = n$$

Then from (1) follows that

$$\dim\mathcal{H}_\Psi = n$$

However the algebra $\mathcal{L}(\mathcal{H}_\Psi)$ of *all* operators that can act on $\mathcal{H}_\Psi$ has dimensionality

$$\dim\mathcal{L}(\mathcal{H}_\Psi) = n^2$$

while the original algebra $\mathcal{A}$ had only $\dim\mathcal{A} = n$!
This suggests that

$$\mathcal{L}(\mathcal{H}_\Psi) = A \otimes \widetilde{A}$$

where $\widetilde{A}$ is a "second copy" of A. We can choose basis so that $[A, \widetilde{A}] = 0$

Constructing the second copy

On the subspace $\mathcal{H}_\Psi$ we define the *antilinear* map S by

$$\boxed{SA|\Psi\rangle = A^\dagger|\Psi\rangle}$$

This is well defined *because* of the condition $A|\Psi\rangle \neq 0$ for $A \neq 0$.
We manifestly have

$$S|\Psi\rangle = |\Psi\rangle$$

and

$$S^2 = 1$$

For any operator $A \in \mathcal{A}$ acting on $\mathcal{H}_\Psi$ we define a new operator acting on the same space by

$$\hat{A} = SAS$$

Constructing the second copy

The hatted operators commute with those in $\mathcal{A}$:

$$\hat{B}A|\Psi\rangle = SBSA|\Psi\rangle = SBA^\dagger|\Psi\rangle = (BA^\dagger)^\dagger|\Psi\rangle = AB^\dagger|\Psi\rangle$$

and also

$$A\hat{B}|\Psi\rangle = ASBS|\Psi\rangle = AB^\dagger|\Psi\rangle$$

hence

$$[A, \hat{B}]|\Psi\rangle = 0$$

The "hatted" operators satisfy:

- Their algebra is isomorphic to $\mathcal{A}$
- They commute with $\mathcal{A}$

they are "almost" the tildes, but not quite (the mixed A-$\hat{A}$ correlators are not "canonically" normalized)

Constructing the second copy

The mapping S is not an isometry. We define the "magnitude" of the mapping

$$\Delta = S^\dagger S$$

and we "rescale" the operator S by its norm, so that it becomes (anti)-unitary. Equivalently we consider the "polar decomposition" and write it as

$$S = J\Delta^{1/2}$$

where J is anti unitary.
The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{“modular Hamiltonian”}$$

Equilibrium condition

If

$$K = \beta H + c$$

then we will call the state $|\Psi\rangle$ as an equilibrium state, as seen by the algebra $\mathcal{A}$.

In the large N gauge theory and using the KMS condition for correlators of single-trace operators we can argue that

$$K = \beta(H_{CFT} - E_0)$$

Definition of the tilde operators

To summarize, we have

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = e^{-\beta(H_{CFT}-E_0)}$$

We define the J by

$$S = J\Delta^{1/2}$$

Finally we define the tildes by

$$\boxed{\widetilde{O} = JOJ}$$

We can show that

- Algebra of $\widetilde{\mathcal{O}}$ is isomorphic to that of $\mathcal{O}$
- The $\widetilde{\mathcal{O}}$ operators commute with $\mathcal{O}$
- Correlators of $\mathcal{O}$ and $\widetilde{\mathcal{O}}$ reproduce those of the thermofield doubled system

Possible confusion: we defined

$$\mathcal{H}_\Psi = \mathrm{span}\{A|\Psi\rangle,\ A \in \mathcal{A}\}$$

While $[\widetilde{A}, A] = 0$ the tilde operators *act on the same Hilbert space* $\mathcal{H}_\Psi$

i.e. the set of states of the form $\widetilde{A}|\Psi\rangle$ is already contained in $\mathcal{H}_\Psi$!

Definition of the tilde operators

Equivalently we **define** the tilde operators by the following set of linear equations

$$\widetilde{\mathcal{O}}_\omega |\Psi\rangle = e^{-\frac{\beta\omega}{2}} \mathcal{O}^\dagger_\omega |\Psi\rangle$$

and

$$\widetilde{\mathcal{O}}_\omega \mathcal{O} .... \mathcal{O} |\Psi\rangle = \mathcal{O} ... \mathcal{O} \widetilde{\mathcal{O}}_\omega |\Psi\rangle$$

These conditions are consistent since the number of equations scales like P^k (where k does not scale with N) while the size of the Hilbert space scales like e^{N^2}.

State dependence of the operators

Notice that the operators $\widetilde{O}$ depend on the microstate $|\Psi\rangle$. For any given $|\Psi\rangle$ the $\widetilde{O}$'s are still linear operators. Will come to this later.

Definition of tilde operators

The construction is "mathematically canonical".

Consider a von-Neumann algebra $\mathcal{A}$ acting on a Hilbert space $\mathcal{H}$.

Question: what is the commutant $\mathcal{A}'$?

If $\exists$ a state $|\Psi\rangle$ in $\mathcal{H}$ for which

i) States $\mathcal{A}|\Psi\rangle$ span $\mathcal{H}$ (at least dense)

and

ii) $A|\Psi\rangle \neq 0$ for all $A \neq 0$

then

The commutant $\mathcal{A}'$ is isomorphic to $\mathcal{A}$ and there is a canonical (anti)-isomorphism J between the two. (Tomita-Takesaki theorem...)

Rindler space example

Consider QFT in Minkowski vaccum $|0\rangle_M$ and algebra

$$\mathcal{A} = \{\text{operators localized in right Rindler wedge}\}$$

The algebra $\mathcal{A}$ cannot annihilate the vacuum. Hence the conditions of the previous theorem apply.
The "commutant" of this algebra $\mathcal{A}'$ are the operators in the left Rindler wedge. We can construct them by defining the mapping $SA|\Psi\rangle = A^\dagger|\Psi\rangle$
Introducing the operator $\Delta = S^\dagger S = e^{-K}$ one can verify that K is proportional to the Lorentz boost generator. Then we decompose $S = J\Delta^{1/2}$ and we can prove that $\mathcal{A}' = J\mathcal{A}J$ i.e.

$$\phi_L(x) = J\phi_R(x')J$$

The "hidden" left wedge can be reconstructed by the modular structure of the theory.
(Bisognano-Wichmann theorem)

Large N gauge theory

In large N gauge theory:

$\mathcal{A} =$ "algebra of single trace operators"

Acting reducibly on Hilbert space $\mathcal{H}_\Psi$ (the algebra cannot annihilate the QGP state $|\Psi\rangle$)

The tilde operators are the commutant of the single trace algebra acting on this space.

We argued that the commutant is non-empty and isomorphic to the original algebra $\mathcal{A}$.

The commutant of the representation of the single trace operators on the microstate $|\Psi\rangle$ describes the region behind the horizon

Moreover, for equilibrium states the modular Hamiltonian coincides with the CFT Hamiltonian.

Reconstructing the interior

Using the $\mathcal{O}_\omega$'s and $\widetilde{\mathcal{O}}_\omega$'s we can reconstruct the black hole interior by operators of the form

$$\phi_{\rm CFT}(t,\Omega,z) = \sum_m \int_0^\infty d\omega \Big[\mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(z) + \text{h.c.}$$

$$+ \widetilde{\mathcal{O}}_{\omega,m}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(z) + \text{h.c.} \Big]$$

Low point functions of these operators reproduce those of effective field theory on the black hole background.

Realization of Complementarity

The operators $\widetilde{\mathcal{O}}$ seem to commute with the $\mathcal{O}$'s

This is only approximate: it works only inside low-point functions (by construction)

If we consider N^2-point functions, then we find that the construction cannot be performed since we will violate

$$A|\Psi\rangle \neq 0, \qquad \text{for} \qquad A \neq 0$$

or equivalentely we will find that

$$[\mathcal{O}, \widetilde{\mathcal{O}}] \neq 0$$

inside complicated correlators.

Example: spin chain

Consider a chain of N spins

$$\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow \ \ldots \ \uparrow$$

The Hilbert space is 2^N dimensional. We have local operators

$$\mathbf{s}^i_{x,y,z} \qquad , \qquad i = 1, ..., n$$

with standard commutation relations.

Also consider a typical pure state $|\Psi\rangle$. In such state each spin is maximally entangled (not with another simple spin, but with a "scrambled qubit")

The algebra of all operators acting on the Hilbert space is generated by products of the individual spins.

Example: spin chain

Question: can we find "scrambled operators" $\widetilde{s}^i_{x,y,z}$ which act on the same Hilbert space, satisfy isomorphic algebra and commute with ALL the $s^i_{x,y,z}$?

Sounds impossible but the answer is **YES**, if previous conditions are only demanded for low-point functions (relative to k).

The construction is exactly the same as before. We define the mapping

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

where $A = s^{i_1} s^{i_2} ... s^{i_k}$, with $k \ll N$. In this case we can verify that

$$\Delta = S^\dagger S \approx I$$

And hence $J = S$. So the "scrambled spin operators" are given by

$$\widetilde{s}^i_{x,y,z} = J s^i_{x,y,z} J$$

Example: spin chain

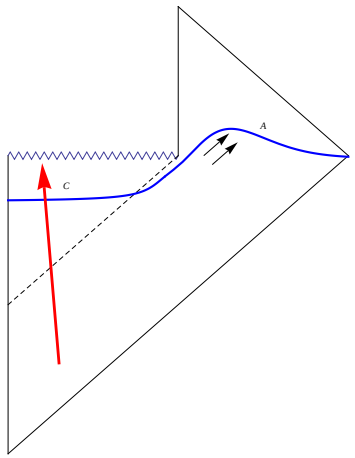
One can write down these operators *explicitly* and verify that

- For low point functions we have effectively $[\widetilde{s}^i, s^j] = 0$
- For $\approx N$-point functions the commutator is nonzero, which means that the $\widetilde{s}$ can be expressed in terms of highly complicated combinations of the s^i operators

This is a mathematically precise definition/realization of complementarity

Evaporating black hole

Black Hole interior is not independent Hilbert space, but scrambled copy of (part of) the exterior



In our construction:

- Exterior of black hole $\Rightarrow$ operators $\phi(x)$
- Interior of black hole $\Rightarrow$ operators $\widetilde{\phi}(y)$
- In low-point correlators ϕ, $\widetilde{\phi}$ seem to be independent
- If we act with too many (order S_{BH}) of ϕ's we can "reconstruct" the $\widetilde{\phi}$'s

Evaporating black hole

Complementarity can be realized consistently with locality in effective field theory.

commutator can (in principle) be acceptably small

On complementarity

What we learn about complementarity

- There is a large Hilbert space describing **both** the interior and the exterior of the black hole
- We can construct operators acting on this Hilbert space and describing observables outside the black hole
- We can construct operators acting on the same Hilbert space and describing observables inside the black hole
- For few, light observables, they approximately commute.
- But not for too accurate measurements, or measurements involving too many insertions
- "Super-observer" is acceptable (no spacetime interpretation)

Regarding the Information paradox

- Only exterior: small corrections can unitarize radiation

- Interior + exterior:
  -Quantum cloning is not an issue because the interior and exterior are not independent Hilbert spaces

  -Strong subadditivity argument of Mathur/AMPS does not apply because interior and exterior are not independent Hilbert spaces

Issues

Is state dependence really necessary?

- $\tilde{O}^\dagger_\omega$ has negative energy, number of states, left inverse
- $\mathrm{Tr}(N_a) \neq 0$ counting argument
- Small commutator
- ?

Issues

If interior operators *have* to be state dependent, then:

- Is state dependence **in principle** ok in QM? (if not, can we phrase a sharp paradox)
- State dependence, ambiguities in definition of equlibrium states and of $\widetilde{O}$
- Evaporation of BHs in flat space — interaction of Hawking radiation with environment
- More general systems: superpositions, ensembles, doubled CFT etc.

(non)-Equilibrium states

Observables obeying the Eigenstate Thermalization Hypothesis (ETH)

$$A_{ab} = f(E)\delta_{ab} + R_{ab}$$

where $f(E)$ is a smooth function. $R_{ab} \sim \mathcal{O}(e^{-S/2})$ and have erratic phases. Our modes

$$A_\omega = R_{ab}$$

connect states of energy E to $E + \omega$.

On equilibrium states we have

$$\langle \Psi | A_\omega | \Psi \rangle \approx 0$$

A typical state

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

with random phases.

We have

$$\langle\Psi|A_\omega|\Psi\rangle = \sum_{ij} c_i^* c_j R_{ij} \sim \mathcal{O}(e^{-S/2})$$

due to random cancellations of phases.

When we excite the state as

$$|\Psi'\rangle = U(O)|\Psi\rangle$$

we can have

$$\langle\Psi|U(O)^\dagger A_\omega U(O)|\Psi\rangle \neq 0$$

Even if $|\Psi\rangle$ is in the microcanonical, the state $|\Psi'\rangle = U(O)|\Psi\rangle$ has significant spread in energy.

(non)-Equilibrium states

Define projection operators P_E on energies between E and $E + \delta E$. We can decompose our original state as

$$|\Psi'\rangle = \sum_E P_E |\Psi'\rangle = \sum_E |\Psi'_E\rangle$$

then we have

$$\langle\Psi'|A_\omega|\Psi'\rangle = \sum_E \langle\Psi'_{E-\omega}|A_\omega|\Psi'_E\rangle = \sum_E f(E)$$

If $\langle\Psi'|A_\omega|\Psi'\rangle \sim \mathcal{O}(1)$ then we also have $f(E) \sim \mathcal{O}(1)$ (or at least, not exponentially suppressed)

(non)-Equilibrium states

In states of the form

$$|\Psi'\rangle = U(O)|\Psi\rangle$$

we get an $O(1)$ answer for

$$\langle\Psi'|A_\omega|\Psi'\rangle$$

because the phases of different energy bins are correlated — in relation to the matrix element of A.

What about states of the form

$$U(\widetilde{O})|\Psi\rangle$$

Since $[A_\omega, \widetilde{O}] = 0$ we also have $[A_\omega, U(\widetilde{O})] = 0$ and

$$\langle\Psi|U(\widetilde{O})^\dagger A_\omega U(\widetilde{O})|\Psi\rangle = \langle\Psi|U(\widetilde{O})^\dagger U(\widetilde{O})A_\omega|\Psi\rangle = \langle\Psi|A_\omega|\Psi\rangle = 0$$

(non)-Equilibrium states

States of the form

$$U(\widetilde{O})|\Psi\rangle$$

look like equilibrium states for operators in the algebra $\mathcal{A}$, but intuitively we think of them as excited, non-equilibrium states.

Excitation can be detected by correlation functions involving the Hamiltonian but let us also look at the phases. We have

$$U(\widetilde{O})|\Psi\rangle = ... = e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

States created by acting with unitaries of the tildes can be expressed in terms of ordinary unitaries, modulated by a function of the Hamiltonian.

Notice that the definition of these states is "state independent" — and independent of the existence/definition of the tildes.

(non)-Equilibrium states

So the claim is that if $|\Psi\rangle$ is an equilibrium state then on the state

$$|\Psi''\rangle = e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

correlators of $\mathcal{A}$ are the same (to leading order in $1/N$) with those on $|\Psi\rangle$. This follows from the KMS condition of thermal correlators (and large N factorization).

In particular we have

$$\langle\Psi''|A_\omega|\Psi''\rangle \approx 0$$

Remember that in the original state $|\Psi\rangle$ we had

$$\langle\Psi|A_\omega|\Psi\rangle \approx 0$$

due to the erratic phases. The reasons that $\langle\Psi''|A_\omega|\Psi''\rangle \approx 0$ is qualitatively different.

(non)-Equilibrium states

$$\langle \Psi'' | A_\omega | \Psi'' \rangle = \langle \Psi | U(O)^\dagger \, e^{-\beta(H-E_0)/2} \, A_\omega \, e^{-\beta(H-E_0)/2} \, U(O) | \Psi \rangle$$

Inserting again the projection operators in the coarse energy bins we find

$$\langle \Psi'' | A_\omega | \Psi'' \rangle = \sum_E e^{-\beta(E-E_0)} f(E)$$

Notice that the $f(E)$ are non-zero! The fact that this correlator is zero is because of cancellations between different energy bins (and the KMS condition). Comparing the states

$$U(O)|\Psi\rangle$$

and

$$e^{\frac{-\beta(H-E_0)}{2}} U(O)|\Psi\rangle$$

we see that they have the same microscopic phases, but in the second state we have some sort of "population inversion".

(non)-Equilibrium states

- "Proper" equilibrium states

$$|\Psi\rangle$$

phases between different energy bins (relative to O's) erratic.

- States with particles outside the horizon

$$U(O)|\Psi\rangle$$

phases betweeen different energy bins correlated. Under time evolution the phases decohere and the state equilibriates (particles get lost behind the horizon).

- States with particles behind the horizon

$$e^{-\beta\frac{H-E_0}{2}}U(O)|\Psi\rangle$$

phases betweeen different energy bins correlated. **cancellations between contributions from different bins**.

(non)-Equilibrium states

We are still confused about perturbations of the equilibrium state $|\Psi\rangle$ by operators which (almost) commute with the hamiltonian, like

$$e^{i\theta O_\omega^\dagger O_\omega}$$

or

$$e^{i\theta \widetilde{O}_\omega^\dagger \widetilde{O}_\omega}$$

(Daniel's talk...)

Summary on non-equilibrium states

We argued that there is a canonical class of "quasi-equilibrium" states, of the form

$$e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

which are parametrized in a similar way as perturbations in region I (i.e. by unitaries $U(O)$) — yet the perturbations are almost undetectable by single trace operators.

This indicates the existence of a seemingly causally disconnected region of spacetime in the bulk.

The tilde operators cause transitions between these states.

But the existence of these states is rather robust (no state-dependence, or need to define the tildes)

Conclusion

- Is it really impossible to have state-independent operators?
- Is there a sharp problem with state dependence — in principle?
- Refine notion of equilibrium states

THANK YOU