Errata for Instructor's Solutions Manual for Gravity, An Introduction to Einstein's General Relativity *1st printing*

Updated 7/7/2003

(Thanks to Scott Fraser who provided almost all of these.)

Statement of Problem 2.9:
(See the book Errata for page 29, Problem 9.)
In the last sentence, replace R with a, for consistency with equation (2.21).

Solutions to Chapter 3 Problems:
Problems 3.1–3.5 should be labeled as Problems 3.2–3.6. The solution to Problem 3.1 is missing but is given below:

Solution to Problem 3.1:
Deriving the equations of motion in a rotating frame is a standard topic in Newtonian mechanics which can be found in almost any text on the subject. If $\vec{x}'(t)$ is the vector with components $(x'(t), y'(t), z'(t))$ in the rotating frame and $\vec{V}'(t)$ is its time derivative, the equation of motion is:

$$\frac{d\vec{V}'}{dt} = 2\vec{V}' \times \vec{\omega} + \vec{\omega} \times (\vec{x}' \times \vec{\omega}) \ .$$

where $\vec{\omega}$ is the angular velocity of the rotating frame. The first term in this expression is the Coriolis force and the second the centrifugal force. The explicit equations for the components $V^{x\prime}$ and $V^{y\prime}$ for an angular velocity of magnitude ω pointing along the $z-$axis are:

$$\frac{dV^{x\prime}}{dt} = +2\omega V^{y\prime} + \omega^2 x'$$
$$\frac{dV^{y\prime}}{dt} = -2\omega V^{x\prime} + \omega^2 y' \ .$$

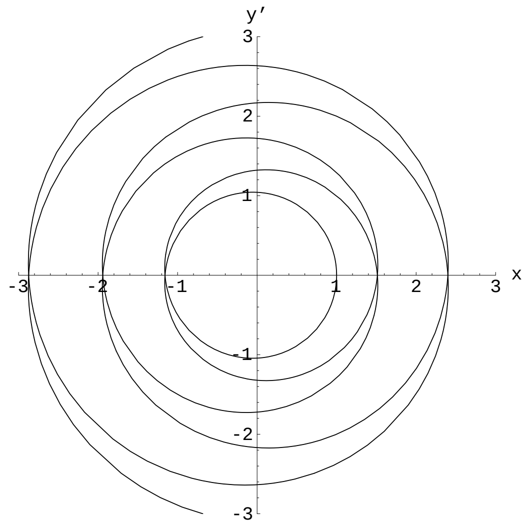
The given trajectory in the inertial frame

$$x(t) = d \ , \qquad y(t) = vt$$

becomes in the inertial frame

$$x'(t) = +x(t)\cos(\omega t) + y(t)\sin(\omega t) \ ,$$
$$y'(t) = -x(t)\sin(\omega t) + y(t)\cos(\omega t) \ .$$

A plot of the orbit for three periods of rotation with $V = 1$ and $d = 1$ is shown below:



Substituting the $x'(t)$ and $y'(t)$ given above into the equations of motion verifies that they are satisfied.

Solution to Problem 3.5:
(actually labeled as Problem 3.4, see above)
In the sentence which begins "For example, take the simple path ..." delete "take" (to fix the grammar). Although the problem states the second boundary condition as $x(T) = 1$, the solution actually uses the initial condition $x(T) = \sinh T$. Below is the revised solution, which results from using the initial condition $x(T) = 1$. The only changes are: the grammar editing mentioned above, the replacement of $x(T) = \sinh T$ with $x(T) = 1$ in the sentence which begins "The solution satisfying $x(0) = 0$ ...", and the modification of four of the displayed equations.

Revised Solution to Problem 3.5:
(actually labeled as Problem 3.4, see above)

Lagrange's equations

$$-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)+\frac{\partial L}{\partial x}=0 \tag{1}$$

are the necessary condition for an extremum to functionals of the form

$$S[x(t)]=\int dt\, L(\dot{x},x) \ . \tag{2}$$

In the present case, $L=\dot{x}^2+x^2$ and the Lagrange equation (1) is

$$\ddot{x}_{cl}=x_{cl}$$

whose general solution is a linear combination of $\sinh t$ and $\cosh t$. The solution satisfying $x(0)=0$, $x(T)=1$ is

$$x_{cl}(t)=\frac{\sinh t}{\sinh T} \ .$$

The value of the action at this extremum can be found by doing the integral directly, but is most easily computed by integrating (2) by parts to give

$$S[x(t)]=\dot{x}(t)\,x(t)\Big|_0^T+\int_0^T dt\, x(t)\ \left[-\ddot{x}(t)+x(t)\right] \ .$$

The second term vanishes because of Lagrange's equation, so

$$S[x_{cl}(t)]=\coth T \tag{3}$$

for the extremal path. The argument of the action is positive for any choice of $x(t)$ and can be made arbitrarily big by choosing a wiggly path with big $\dot{x}$. The extremum, therefore, cannot be a maximum but must be a minimum. For example, the simple path

$$x_*(t)=\frac{t}{T}$$

satisfies the boundary conditions, and

$$S[x_*(t)]=\frac{1}{T}\left(1+\frac{T^2}{3}\right)$$

which is greater than (3), as calculating a few values or a simple plot will show.

Solution to Problem 5.6:
In the equation line which immediately precedes equation (2), replace $\sinh^2(gt)$ with $\sinh^2(g\tau)$ inside the square root.

Solution to Problem 9.6:
There are three typographical errors (two signs and one integrand). The revised solution is below.

Revised Solution to Problem 9.6:
From (9.37) we have for the inward a radial orbit $(\ell = 0)$ starting from zero initial velocity $(e = 1)$

$$\left(\frac{dr}{d\tau}\right) = -\left(\frac{2M}{r}\right)^{\frac{1}{2}}$$

Therefore the proper time to pass between $6M$ and $2M$ is

$$\begin{aligned} \mathcal{T} &= \int d\tau = \int_{6M}^{2M} dr \, \left(\frac{d\tau}{dr}\right) = -\int_{6M}^{2M} dr \, \left(\frac{r}{2M}\right)^{\frac{1}{2}} \\ &= M \, \frac{2^{\frac{1}{2}}}{3} \, \left(6^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = 5.59M \, . \end{aligned}$$

Solution to Problem 10.5:
Since equation (9.57) gives the precession per orbit, in order to convert to the precession per year, the expression given in the solution for $\delta\phi$ needs to be multiplied by a ratio of orbital periods:

$$\delta\phi = (42.98''/\text{century}) \, \frac{a_{\text{merc}}(1-\varepsilon^2_{\text{merc}})}{a(1-\varepsilon^2)} \, \frac{P_{\text{merc}}}{P}$$

Solution to Problem 12.12:
In the sentence beginning with “Orthogonally to ...” replace “Orthogonally” with “Orthogonality” and replace $(n^v, n^r, 0, 0,)$ with $(n^v, n^r, 0, 0)$.

Solution to Problem 12.22 (a), (c):
In the figure for part (a), replace the label $r = R_1$ with $r = R$ (for consistency with

the problem's notation). In part (c), replace u and v (two occurrences each) with U and V (for consistency with the textbook's notation).

Solution to Problem 15.4:
Replace ρ^2 with ρ_+^2 in the displayed equation $\rho^2 = r_+^2 + a^2 \cos^2\theta$.

Solution to Problem 15.11:
In the second sentence, replace "the allowed values of b and M" with "the allowed values of b and r".

Solution to Problem 15.14:
In the displayed equation immediately following "As $r \to r_+$," the period after the expression for $g_{\phi\phi}$ should be deleted.

Solution to Problem 18.2 (b):
In the expression for T, the exponent in the first equality should be corrected so the equation reads

$$T = T_0 \left(\frac{a_0}{a}\right) = \left(\frac{t_0}{t}\right)^{\frac{1}{2}} .$$

Statement of Problem 18.6:
Replace "What is timescale" with "What is the timescale".

Solution to Problem 18.6:
In the first sentence, replace "relates that two" with "relates the two".

Solution to Problem 18.16:
In the final sentence beginning with "Evaluating this ..." replace $\tilde{t} = t_0$ with $\tilde{t} = \tilde{t}_0$ (or $t = t_0$) and "gives gives" with "gives".

Statement of Problem 18.18:
(See the book Errata for page 397, Problem 18.)
Delete the label *De Sitter Space.*

Solution to Problem 18.18:
Insert the following sentence before "This is an ellipse..." "The constant of inte-

gration fixing the zero of t has been chosen arbitrarily to give a simple formula."

Statement of Problem 18.19:
(See the book Errata for page 398, Problem 19.)
Add the label *de Sitter Space*.

Solution to Problem 18.24 (b):
In the second equation, replace ρ_{vac} with ρ_v (for consistency).

Solution to Problem 18.31:
In the first sentence, replace "become" with "becomes".

Solution to Problem 18.32:
Replace "See the solution to Problem 7" with "See the solution to Problem 22".

Solution to Problem 20.16:
Replace ∇_{ξ} with ∇_{ξ} in the last sentence.

Solution to Problem 20.19:
Replace ∇_{ℓ} with ∇_{ℓ} in the first sentence.

Solution to Problem 20.24:
Replace $\nabla_{\mathbf{u}}$ (three occurrences) with $\nabla_{\mathbf{u}}$ in the displayed equation.

Solution to Problem 20.25:
Replace $\nabla_{\mathbf{u}}$ with $\nabla_{\mathbf{u}}$ in the first displayed equation.

Solution to Problem 21.12 (c):
Replace $\nabla_{\mathbf{u}}\nabla_{\mathbf{u}}$ with $\nabla_{\mathbf{u}}\nabla_{\mathbf{u}}$ in the second displayed equation of part (c).

Solution to Problem 21.18 (a):
In the first sentence, the font following $f(r,t')$ should be corrected so that it reads: "Write $t = t' + f(r,t')$ and substitute into the given metric. The $drdt'$ cross term is ..."

Solution to Problem 21.21:

(Notation) Replace ∇^2 with $\Box$ at two places.

Statement of Problem 21.24:
(See the book Errata for page 469, Problem 24.)
Should be "Work out what happens to the test particles when the gravitational wave passes ..."

Solution to Problem 21.26 (a):
(Notation) Replace ∇^2 by $\Box$ in equation (1).

Solution to Problem 22.1:
Add this paragraph at the end:
"Eq. (1) shows that the left hand side of (2) vanishes when $\partial v^\alpha/\partial x^\alpha = 0$. On the right hand side the normals of the two three surfaces are pointing in opposite directions. If the normals are chosen consistently on both surfaces, then (2) becomes

$$\int_{t=\Delta t} d^3x\,\left(v^t n_t\right) = \int_{t=0} d^3x\,\left(v^t n_t\right) \ .$$

This says that the integral is the same at all times —- a conservation law!"

Solution to Problem 23.3:
In equation (4), replace $t = t\mathrm{ret}$ with $t = t_{\mathrm{ret}}$. See also the book Errata for page 496, equation (23.23).

Solution to Problem 23.5:
In the expression for $\bar{h}_{ij}$, replace the factor of 4 with 2. In the estimate for h, replace the factor (6.8×10^8 cm) in the denominator with (6.38×10^8 cm).

Solution to Problem 23.6 (a):
Replace $I^{t\alpha}$ by $T^{t\alpha}$, since only the latter appears in (23.30). Further, "the integrals in (23.21)" should be replaced by "the integrals in (23.30)" since (23.30) has the distance r outside the integral but (23.21) does not.

Solution to Problem 23.10:
Delete the middle equality and expression in the final equation.

Solution to Problem 23.11:
Delete the last sentence of the solution because of the modification of the problem given in the text errata.

Solution to Problem 23.12:
A negative sign is missing in equation (1), it should read

$$\frac{dP}{dt} = -\left(\frac{\tau}{P}\right)^{5/3} .$$