Errata for Instructor’s Solutions Manual for
Gravity, An Introduction to Einstein’s General
Relativity 1st printing

Updated 7/7/2003

(Thanks to Scott Fraser who provided almost all of these.)

Statement of Problem 2.9:
(See the book Errata for page 29, Problem 9.)
In the last sentence, replace $R$ with $a$, for consistency with equation (2.21).

Solutions to Chapter 3 Problems:
Problems 3.1–3.5 should be labeled as Problems 3.2–3.6. The solution to Problem 3.1 is missing but is given below:

Solution to Problem 3.1:
Deriving the equations of motion in a rotating frame is a standard topic in Newtonian mechanics which can be found in almost any text on the subject. If $\vec{x}'(t)$ is the vector with components $(x'(t), y'(t), z'(t))$ in the rotating frame and $\vec{V}'(t)$ is its time derivative, the equation of motion is:

$$\frac{d\vec{V}'}{dt} = 2\vec{V}' \times \vec{\omega} + \vec{\omega} \times (\vec{x}' \times \vec{\omega}) .$$

where $\vec{\omega}$ is the angular velocity of the rotating frame. The first term in this expression is the Coriolis force and the second the centrifugal force. The explicit equations for the components $V'x'$ and $V'y'$ for an angular velocity of magnitude $\omega$ pointing along the $z-$axis are:

$$\frac{dV'x'}{dt} = +2\omega V'y' + \omega^2 x'$$
$$\frac{dV'y'}{dt} = -2\omega V'x' + \omega^2 y' .$$

The given trajectory in the inertial frame

$$x(t) = d , \quad y(t) = vt$$

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becomes in the inertial frame

\[ \begin{align*}
    x'(t) &= +x(t) \cos(\omega t) + y(t) \sin(\omega t), \\
    y'(t) &= -x(t) \sin(\omega t) + y(t) \cos(\omega t).
\end{align*} \]

A plot of the orbit for three periods of rotation with \( V = 1 \) and \( d = 1 \) is shown below:

![Plot of orbit](image)

Substituting the \( x'(t) \) and \( y'(t) \) given above into the equations of motion verifies that they are satisfied.

**Solution to Problem 3.5:**
*(actually labeled as Problem 3.4, see above)*

In the sentence which begins “For example, take the simple path ...” delete “take” (to fix the grammar). Although the problem states the second boundary condition as \( x(T) = 1 \), the solution actually uses the initial condition \( x(T) = \sinh T \). Below is the revised solution, which results from using the initial condition \( x(T) = 1 \). The only changes are: the grammar editing mentioned above, the replacement of \( x(T) = \sinh T \) with \( x(T) = 1 \) in the sentence which begins “The solution satisfying \( x(0) = 0 \)” , and the modification of four of the displayed equations.

**Revised Solution to Problem 3.5:**
*(actually labeled as Problem 3.4, see above)*
Lagrange’s equations

\[-\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) + \frac{\partial L}{\partial x} = 0 \quad (1)\]

are the necessary condition for an extremum to functionals of the form

\[S[x(t)] = \int dt \, L(\dot{x}, x) \quad (2)\]

In the present case, \( L = x^2 + x^2 \) and the Lagrange equation (1) is

\[\ddot{x}_{cl} = x_{cl}\]

whose general solution is a linear combination of sinh \( t \) and cosh \( t \). The solution satisfying \( x(0) = 0, x(T) = 1 \) is

\[x_{cl}(t) = \frac{\sinh t}{\sinh T} .\]

The value of the action at this extremum can be found by doing the integral directly, but is most easily computed by integrating (2) by parts to give

\[S[x(t)] = \dot{x}(t) x(t) \bigg|^{T}_{0} + \int_{0}^{T} dt \, \dot{x}(t) \left[ -\ddot{x}(t) + x(t) \right] .\]

The second term vanishes because of Lagrange’s equation, so

\[S[x_{cl}(t)] = \coth T \quad (3)\]

for the extremal path. The argument of the action is positive for any choice of \( x(t) \) and can be made arbitrarily big by choosing a wiggly path with big \( \dot{x} \). The extremum, therefore, cannot be a maximum but must be a minimum. For example, the simple path

\[x_{s}(t) = \frac{t}{T}\]

satisfies the boundary conditions, and

\[S[x_{s}(t)] = \frac{1}{T} \left( 1 + \frac{T^2}{3} \right)\]

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which is greater than (3), as calculating a few values or a simple plot will show.

**Solution to Problem 5.6:**
In the equation line which immediately precedes equation (2), replace \( \sinh^2(gt) \) with \( \sinh^2(g\tau) \) inside the square root.

**Solution to Problem 9.6:**
There are three typographical errors (two signs and one integrand). The revised solution is below.

**Revised Solution to Problem 9.6:**
From (9.37) we have for the inward a radial orbit \((\ell = 0)\) starting from zero initial velocity \((e = 1)\)

\[
\left( \frac{dr}{d\tau} \right) = - \left( \frac{2M}{r} \right)^{\frac{1}{2}}
\]

Therefore the proper time to pass between \(6M\) and \(2M\) is

\[
\mathcal{T} = \int d\tau = \int_{6M}^{2M} dr \left( \frac{d\tau}{dr} \right) = - \int_{6M}^{2M} dr \left( \frac{r}{2M} \right)^{\frac{1}{2}}
\]

\[
= M \frac{2^{\frac{1}{2}}}{3} \left( 6^{\frac{3}{2}} - 2^{\frac{3}{2}} \right) = 5.59M .
\]

**Solution to Problem 10.5:**
Since equation (9.57) gives the precession per orbit, in order to convert to the precession per year, the expression given in the solution for \(\delta\phi\) needs to be multiplied by a ratio of orbital periods:

\[
\delta\phi = (42.98''/\text{century}) \frac{a_{\text{merc}}(1 - e_{\text{merc}}^2)}{a(1 - e^2)} \frac{P_{\text{merc}}}{P}
\]

**Solution to Problem 12.12:**
In the sentence beginning with “Orthogonally to ...” replace “Orthogonally” with “Orthogonality” and replace \((n^x, n^r, 0, 0, 0)\) with \((n^x, n^r, 0, 0, 0, 0)\).

**Solution to Problem 12.22 (a), (c):**
In the figure for part (a), replace the label \(r = R_1\) with \(r = R\) (for consistency with
the problem’s notation). In part (c), replace \( u \) and \( v \) (two occurrences each) with \( U \) and \( V \) (for consistency with the textbook’s notation).

**Solution to Problem 15.4:**
Replace \( \rho^2 \) with \( \rho^2_+ \) in the displayed equation \( \rho^2 = r^2_+ + a^2 \cos^2 \theta \).

**Solution to Problem 15.11:**
In the second sentence, replace “the allowed values of \( b \) and \( M \)” with “the allowed values of \( b \) and \( r \)”.

**Solution to Problem 15.14:**
In the displayed equation immediately following “As \( r \to r_+ \),” the period after the expression for \( g_{\phi\phi} \) should be deleted.

**Solution to Problem 18.2 (b):**
In the expression for \( T \), the exponent in the first equality should be corrected so the equation reads

\[
T = T_0 \left( \frac{a_0}{a} \right) = \left( \frac{t_0}{t} \right)^{\frac{1}{2}} .
\]

**Statement of Problem 18.6:**
Replace “What is timescale” with “What is the timescale”.

**Solution to Problem 18.6:**
In the first sentence, replace “relates that two” with “relates the two”.

**Solution to Problem 18.16:**
In the final sentence beginning with “Evaluating this ...” replace \( t = t_0 \) with \( \tilde{t} = \tilde{t}_0 \) (or \( t = t_0 \)) and “gives gives” with “gives”.

**Statement of Problem 18.18:**
(See the book Errata for page 397, Problem 18.)
Delete the label De Sitter Space.

**Solution to Problem 18.18:**
Insert the following sentence before “This is an ellipse...” “The constant of inte-
gration fixing the zero of \( t \) has been chosen arbitrarily to give a simple formula.”

**Statement of Problem 18.19:**
(See the book Errata for page 398, Problem 19.)
Add the label de Sitter Space.

**Solution to Problem 18.24 (b):**
In the second equation, replace \( \rho_{\text{vac}} \) with \( \rho_v \) (for consistency).

**Solution to Problem 18.31:**
In the first sentence, replace “become” with “becomes”.

**Solution to Problem 18.32:**
Replace “See the solution to Problem 7” with “See the solution to Problem 22”.

**Solution to Problem 20.16:**
Replace \( \nabla_\xi \) with \( \nabla_\xi \) in the last sentence.

**Solution to Problem 20.19:**
Replace \( \nabla_\ell \) with \( \nabla_\ell \) in the first sentence.

**Solution to Problem 20.24:**
Replace \( \nabla_u \) (three occurrences) with \( \nabla_u \) in the displayed equation.

**Solution to Problem 20.25:**
Replace \( \nabla_u \) with \( \nabla_u \) in the first displayed equation.

**Solution to Problem 21.12 (c):**
Replace \( \nabla_u \nabla_u \) with \( \nabla_u \nabla_u \) in the second displayed equation of part (c).

**Solution to Problem 21.18 (a):**
In the first sentence, the font following \( f(r,t') \) should be corrected so that it reads: “Write \( t = t' + f(r,t') \) and substitute into the given metric. The \( drdt' \) cross term is ...

**Solution to Problem 21.21:**
(Notation) Replace $\nabla^2$ with $\Box$ at two places.

Statement of Problem 21.24:
(See the book Errata for page 469, Problem 24.)
Should be “Work out what happens to the test particles when the gravitational wave passes ...”

Solution to Problem 21.26 (a):
(Notation) Replace $\nabla^2$ by $\Box$ in equation (1).

Solution to Problem 22.1:
Add this paragraph at the end:
“Eq. (1) shows that the left hand side of (2) vanishes when $\frac{\partial v}{\partial x} = 0$. On the right hand side the normals of the two three surfaces are pointing in opposite directions. If the normals are chosen consistently on both surfaces, then (2) becomes

$$\int_{t=\Delta} d^3x (v'n_t) = \int_{t=0} d^3x (v'n_t).$$

This says that the integral is the same at all times —- a conservation law!”

Solution to Problem 23.3:
In equation (4), replace $t = t_{ret}$ with $t = t_{ret}$. See also the book Errata for page 496, equation (23.23).

Solution to Problem 23.5:
In the expression for $h_{ij}$, replace the factor of 4 with 2. In the estimate for $h$, replace the factor $(6.8 \times 10^8 \text{ cm})$ in the denominator with $(6.38 \times 10^8 \text{ cm})$.

Solution to Problem 23.6 (a):
Replace $f^{\alpha}$ by $T^{\alpha t}$, since only the latter appears in (23.30). Further, “the integrals in (23.21)” should be replaced by “the integrals in (23.30)” since (23.30) has the distance $r$ outside the integral but (23.21) does not.

Solution to Problem 23.10:
Delete the middle equality and expression in the final equation.
Solution to Problem 23.11:
Delete the last sentence of the solution because of the modification of the problem given in the text errata.

Solution to Problem 23.12:
A negative sign is missing in equation (1), it should read

\[
\frac{dP}{dt} = -\left(\frac{\tau}{P}\right)^{5/3}.
\]