DISTANCES OF GLOBULAR CLUSTERS BASED ON HIPPARCOS PARALLAXES OF NEARBY SUBDWARFS

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ABSTRACT

In the present study we have determined the distance moduli for seven globular clusters M2, M3, M10, M12, NGC 2808, NGC 6229, and NGC 6752, whose metallicities are in the range $-1.54 < [\text{Fe/H}] < -1.10$. We have derived distances for them by the main-sequence fitting method using a sample of local subdwarfs, whose accurate parallaxes are taken from the Hipparcos Catalog. The derived distance moduli are 15.52 for M2, 14.98 for M3, 14.23 for M10, 14.03 for M12, 15.56 for NGC 2808, 17.34 for NGC 6229, and 13.29 for NGC 6752.

Key words: globular clusters: individual (M2, M3, M10, M12, NGC 2808, NGC 6229, & NGC 6752)

I. INTRODUCTION

There is no doubt that almost all types of astronomical objects, including stars, clusters, and galaxies play a part in the cosmological distance ladder, and knowledge of distances is essential to all branches of astronomy. As we climb the rungs of the cosmological distance ladder, we can follow the history of the universe.

Globular clusters (GCs) are considered to be one of the most active research subjects in stellar astrophysics. They are the oldest objects for which reliable ages can be derived. They set the lower limit to the age of the universe which is a fundamental cosmological constraint. Their spread in age as a function of metallicity and Galactocentric distance and their location in the halo provide vital clues about the formation of galaxies.

For a long time, GCs have been most effectively used in the determination of the distance scale and age of the universe. Much interest is now centered on revising the distances to these objects following the Hipparcos mission, which provides more accurate parallaxes of stars than hitherto obtained.

There are various methods which can be applied to estimate the distances to GCs. One of them is the main-sequence fitting method. This method requires two fundamental kinds of data, namely deep photometric data for globular clusters and accurate parallaxes for a sufficient number of local subdwarfs. The limits of photometry do not extend deep enough to give main sequence lines for globular clusters and this fact coupled with the scarcity of local subdwarfs with accurate parallaxes have been major obstacles to the adoption of this method for determining accurate distances of globular clusters. In recent years, deep photometry down to $M_V > 8$ has been obtained for several GCs using large ground-based telescopes as well as the HST. The astrometric satellite Hipparcos has measured precise parallaxes for a large number of subdwarf candidates. As a consequence, improved distance moduli could be obtained for some globular clusters by the main-sequence fitting method. By applying the main-sequence fitting method to GCs using local subdwarfs which have parallaxes with an accuracy ~1 mas provided by Hipparcos, we expect to achieve the most accurate distance determinations hitherto available.

Recently, this method has been widely applied to the estimation of the distance moduli of GCs by Gratton et al. (1997a; 1998), Pont et al. (1998), Chaboyer et al. (1998) and Reid (1997, 1998). They have all used local subdwarfs, however with different criteria and a variety of techniques to avoid various types of biases. The results of fitting globular cluster main sequences to local subdwarfs with Hipparcos parallaxes obtained by Gratton et al. (1997a) and by Reid (1997, 1998) disagree only by small amounts. Moreover in almost all cases both investigations find distance moduli that are larger than the values that had previously been accepted. The changes in distance moduli lead to the ages becoming smaller by 1-2 Gyr. There is still some spread in the results, but the Hipparcos distances are systematically larger than ground-based distances, and
the ages are lower, converging, for the oldest clusters, to 12-14 Gyr. Therefore it is of interest to estimate whether all distance moduli for old clusters are larger than the ones estimated previously. In this study, we intend to adopt the best available data and Lutz-Kelker corrections to this technique to obtain new distance moduli for seven old clusters. Since the accurate distance determination by main-sequence method is prevented for the case of metal-poor GCs because of few metal-poor subdwarfs with Hipparcos parallaxes, we intend to estimate the distance moduli for GCs with intermediate metallicities. We have selected GCs which have intermediate metallicities and rather deep and accurate available photometry. They comprise M2, M3, M10, M12, NGC 2808, NGC 6229 and NGC 6752. Deep photometric data for this study are taken from Lee & Carney (1999) for M2, from Ferraro et al. (1997) for M3, from Hurley et al. (1989) for M10, from Sato et al. (1989) for M12, from Walker (1999) for NGC 2808, from Borissova et al. (1999) for NGC 6229, and Thomson et al. (1999) for NGC 6752.

II. BASIC DATA

Data for local subdwarfs and globular clusters are the two basic ingredients for main sequence fitting. Considerable care is generally devoted to the subdwarf template stars as well as the program globular clusters since the essential parameters have to be obtained on the basis of the data from different sources.

(a) Data for Local Subdwarfs

To derive the best distance modulus for the GCs by calibrating their main sequences, all subdwarf stars in the lists of Carretta et al. (2000) and Reid (1998) are collected.

Since all our program GCs are in the metallicity range of $-1.54 < [\text{Fe/H}] < -1.10$, we have selected subdwarfs with relevant metallicity, in the range $-1.89 < [\text{Fe/H}] < -0.95$, as well as accurate parallax values, $\Delta \pi < 0.12$, from the lists.

The visual magnitudes, colors, metallicities, and reddening of subdwarfs are adopted as those in Carretta et al. (2000) and Reid (1998). Whenever the values in the two papers differ, we have adopted those in Carretta et al. (2000).

Photometric data for subdwarf candidates have been originally collected from the available data of Carney et al. (1994), Ryan & Norris (1991), and Schuster & Nissen (1989).

The metallicity values for the selected subdwarf stars have been obtained from Axer et al. (1994, hereafter AFG) and Gratton et al. (1997b hereafter GCC). AFG have obtained stellar parameters based on their model atmosphere identical to those of Kurucz (1979) and one thousand spectra for 115 stars of metal-poor with visual magnitudes $V < 12$ in the metallicity range $-3.04 < [\text{Fe/H}] < -0.1$. On the other hand, GCC obtained their parameters based on the model atmosphere of Kurucz (1993), and the published equivalent width measurements of FeI and FeII for 300 stars. The metallicity values of AFG and GCC are found to give systematically high abundances of $[\text{Fe/H}]_{\text{AFG}}$ for $[\text{Fe/H}]_{\text{GCC}} < -1.5$ although they are in good agreement for abundances $[\text{Fe/H}] > -1.30$ (Reid 1998). However, we have adopted GCC values if both GCC and AFG values are available. The absolute magnitudes, to which the Lutz-Kelker (1973) corrections have been applied by Carretta et al. (2000), are adopted. We extracted main-sequence template subdwarfs with absolute magnitudes $M_V > 5$ in order to avoid the evolved subdwarfs. An enlarged sample of 23 subdwarfs was collected and used to determine the distance moduli of GCs. Table 1 displays the basic data for the local subdwarfs.

(b) Globular Clusters (GCs)

Since the parts of interest in CMDs of GCs are the

<table>
<thead>
<tr>
<th>HIP</th>
<th>Other</th>
<th>$(B-V)_0$</th>
<th>$M_V$</th>
<th>$[\text{Fe/H}]$</th>
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</thead>
<tbody>
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<td>7459</td>
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</tr>
<tr>
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<td>25329</td>
<td>0.864</td>
<td>7.17</td>
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</tr>
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<td>5.66</td>
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<td>7.03</td>
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<td>0.767</td>
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<td>0.805</td>
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<td>99267</td>
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<td>0.580</td>
<td>5.96</td>
<td>-1.62</td>
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</table>
bona-fide main sequence, relatively accurate and deep CCD photometry covering the faint part of the main sequence has been collected. Photometry and reddening have been extracted from the most recent deep photometry for each GC. Since the metallicity value for each GC differs considerably from one source to another, in general we adopted a value which is either the most recently obtained one or based on high-resolution spectroscopic analysis.

i) M2
Lee & Carney (1999, hereafter LC99) presented CCD photometry in blue and visual magnitudes for Oosterhoff type II GC M2. Their observations were carried out using the 0.9 m telescopes at Kitt Peak National Observatory KPNO and Cerro Tololo Inter-American Observatory CTIO. They provide a fiducial main sequence reaching to magnitude $V$ = 22.50 mag. We adopted data in Table 2 of LC99 for the main sequence line of M2. A small reddening, $E(B-V) = 0.02$ (Zinn 1985) toward M2 is adopted, although a larger value of $E(B-V) = 0.06$ was estimated by Harris (1976). Zinn (1993) and Da Costa & Armandroff (1995) have categorized M2 as an “old halo” cluster, based on its metallicity and horizontal-branch morphology. LC99 confirmed that the Oosterhoff II cluster M2 is about 2Gyr older than the Oosterhoff I cluster M3, which has a similar metallicity. The metallicity for M2 has been estimated with values ranging from $[\text{Fe}/\text{H}] = -1.31$, measured by Rutledge et al. (1997), to $[\text{Fe}/\text{H}] = -1.61$ on the metallicity scale of Zinn & West (1984 hereafter ZW84), and to $[\text{Fe}/\text{H}] = -1.46$ on Carretta & Gratton’s (1997 hereafter CG97) metallicity scale. We adopted the later value from the GC97 metallicity scale for M2.

ii) M3
GC M3 has been the target of a large number of studies and analyses. Ferraro et al. (1997) obtained deep BVI CCD photometry for Oosterhoff I GC M3 using the 3.6 m telescope at CFHT and presented a combined CMD with the photometry of the recalibrated older photographic study of Buonanno et al. (1994). Although LC99 reanalyzed the data of Ferraro et al. (1997), we adopted the fiducial sequences of M3 in Table 3 of Ferraro et al. (1997). The metallicity of M3 is estimated to be $-1.66$ and $-1.46$ on the Zinn & West scale(ZW84) and CG97 scale, respectively, by LC99. We adopted the value of $[\text{Fe}/\text{H}] = -1.46$ from the CG97 scale.

Table 2. Basic data for globular clusters

<table>
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<tr>
<th>Cluster</th>
<th>$E(B-V)$</th>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>0.02</td>
<td>-1.46</td>
</tr>
<tr>
<td>M3</td>
<td>0.01</td>
<td>-1.46</td>
</tr>
<tr>
<td>M10</td>
<td>0.27</td>
<td>-1.54</td>
</tr>
<tr>
<td>M12</td>
<td>0.23</td>
<td>-1.40</td>
</tr>
<tr>
<td>NGC 2808</td>
<td>0.20</td>
<td>-1.15</td>
</tr>
<tr>
<td>NGC 6229</td>
<td>0.01</td>
<td>-1.10</td>
</tr>
<tr>
<td>NGC 6752</td>
<td>0.04</td>
<td>-1.43</td>
</tr>
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</table>

Table 3. Derived distance modulus

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$(m-M)_V$</th>
<th>$(m-M)_{V,cc}$</th>
<th>$(&lt;m-M)_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>15.61±0.04</td>
<td>15.47±0.03</td>
<td>15.52</td>
</tr>
<tr>
<td>M3</td>
<td>15.03±0.14</td>
<td>14.97±0.07</td>
<td>14.98</td>
</tr>
<tr>
<td>M10</td>
<td>14.34±0.10</td>
<td>14.23±0.01</td>
<td>14.23</td>
</tr>
<tr>
<td>M12</td>
<td>14.16±0.08</td>
<td>14.02±0.02</td>
<td>14.03</td>
</tr>
<tr>
<td>NGC 2808</td>
<td>15.45±0.11</td>
<td>15.57±0.03</td>
<td>15.56</td>
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<tr>
<td>NGC 6229</td>
<td>17.27±0.10</td>
<td>17.38±0.07</td>
<td>17.34</td>
</tr>
<tr>
<td>NGC 6752</td>
<td>13.37±0.04</td>
<td>13.26±0.03</td>
<td>13.29</td>
</tr>
</tbody>
</table>

M10 is a globular cluster with intermediate metallicity, very high reddening and extremely blue horizontal branch (Hurley et al. 1989). Main-sequence photometric data for this cluster was extracted from deep CCD photometry carried out by Hurley et al. (1989) using the Canada-France-Hawaii 3.6 m (CFHT) telescope. They provide a fiducial main sequence reaching to magnitude $V$ = 22.50 mag. We adopted the same values of $E(B-V) = 0.28$ has been assigned by both Harris (1996) and Zinn (1985). Using a sample of 50 subdwarf stars with well established ground-based trigonometric parallaxes and spectroscopic [Fe/H] determination, Lutz et al. (1987) derived an empirical relation, a Luminosity-Color-Metallicity formula for these subdwarfs:

$$M_V = 1.41 + 5.17(B - V) - 0.94[Fe/H]$$  \((1)\)

Hurley et al. (1989) used this relation to estimate a distance modulus of $(m-M)_V = 14.40±0.38$ for M10.

We adopt the same values of $E(B-V)$ and [Fe/H] as in Hurley et al. (1989) and take the fiducial main sequence of M10 from Table 4 of Hurley et al. (1989).

iv) M12
Although the investigation of GC M12 started from a very early date, in 1942, using photographic plates, the most comprehensive studies of M12 are probably those of Sato et al. (1989) and Brocato et al. (1996), which were based on CCD photometry. Brocato et al. (1996) concentrated their photometric studies on the luminous...
stars in nine globular clusters including M12, while Sato et al. (1989) presented deeper CCD photometry for M12. Sato et al.’s photometry, which was carried out using the 3.6 m CFHT telescope, reached well down the main sequence fainter than $V = 23.5$ mag and they estimated a reddening value of $(B-V) = 0.23 \pm 0.04$ for M12 using horizontal branch stars. However, Harris (1976) and Zinn (1985) measured two smaller reddening values of 0.19 and 0.17, respectively. Sato et al. (1989) have determined the metallicity of M12 to be $[\text{Fe/H}] = -1.4 \pm 0.1$ using values found in the literature in conjunction with their own estimate of UV excesses by taking the weighted mean of different sources. They also estimated the apparent distance modulus to be $(m-M)_V = 14.25 \pm 0.20$ for M12 by comparing their fiducial main sequence with that of equation (1) defined by the field subdwarfs of Lutz et al. (1987). A reddening of $(B-V) = 0.23$, a metallicity of $[\text{Fe/H}] = -1.40$, and the fiducial main sequence of Sato et al. (1989) are adopted in this study.

v) NGC 2808

Walker (1999) presented a deep color-magnitude diagram for the globular cluster NGC 2808 reaching 3 mag below the main sequence turn-off. The observations were obtained at the 4 m Blanco telescope and 0.9 m CTIO telescope. He adopted a reddening value near $(B-V) = 0.20$ and determined the metal abundance for this cluster based on calcium triplet measurements (Rutledge et al. 1997) to be $[\text{Fe/H}] = -1.36 \pm 0.05$ on the Zinn & West scale (ZW84). However, Ferraro et al. (1999) and Rutledge et al. (1997) estimated values of $-1.15$ and $-1.11$, respectively, on the CG97 metallicity scale. We adopted the recent value of $[\text{Fe/H}] = -1.15$ (Ferraro et al. 1999). Several reddening determinations towards NGC 2808 have been obtained since Burstein & McDonald (1975) estimated a large value of $(B-V) = 0.34$. We have applied the recent value of $(B-V) = 0.20$ and have derived a fiducial main sequence from Figure 2 CMD of Walker et al. (1999), ignoring the suspected binary sequence. Bedin et al. (2000) estimated an even smaller reddening of $(B-V) = 0.19$.

vi) NGC 6229

NGC 6229 is one of the most remote GCs associated with the Galaxy, lying about 30 kpc from the Galactic center (Harris 1996). There exist many previous photometric studies (Cohen 1986; Carney et al. 1991) for this cluster. None of these studies reached the magnitudes fainter than the main sequence turn-off (TO) point. Borissova et al. (1999), however, carried out observations at the Calar Alto Observatory (Spain) of the Max-Plank-Institute fur Astronomie (Heidelberg) using the 3.5m and 2m telescopes and presented deep
BV CCD photometry down to $V = 24$ mag. They determined a reddening value of $E(B-V) = 0.01$ similar to those of Harris (1999) and Zinn (1985) and estimated its metallicity to be equal to $[\text{Fe/H}] = -1.10 \pm 0.1$ on the CG97 scale however, on the ZW84 scale it was estimated to be $-1.54$. Although Ferraro et al. (1999) estimated an average value of $[\text{Fe/H}] = -1.30$, we have adopted $[\text{Fe/H}] = -1.10 \pm 0.10$ from the CG97 scale and the fiducial main sequence of Table 4 of Borissova et al. (1999), which extends only to three magnitudes fainter than the TO.

vii) NGC 6752

For NGC 6752 Thompson et al. (1999) presented deep CCD photometry to the faint main sequence reaching to $V = 20.6$ mag based on the CCD photometry time series with the 1m Swope telescope at Las Campanas Observatory. Reddening values of $E(B-V) = 0.04$ were estimated by Harris (1996) and Zinn (1985), while Gratton et al. (1997b) estimated a value of 0.035 by averaging Zinn’s value with other independent estimates. While the metallicity was determined to be $[\text{Fe/H}] = -1.43$ on the CG97 scale, it has been known to be $[\text{Fe/H}] = -1.54$ on the ZW84 scale. Penny & Dickens (1986) obtained an age of $16 \pm 1$ Gyr and a metallicity of $[\text{Fe/H}] = -1.5 \pm 0.2$ from matching to the theoretical isochrones of VandenBerg & Bell (1985). They estimated a distance modulus of $13.17 \pm 0.17$ based on the local subdwarfs with accurate trigonometric parallaxes from ground-based observations. We have adopted $E(B-V) = 0.04$, a CG97 scale value of $[\text{Fe/H}] = -1.43$, and derived main sequence line from Figure 4 of Thompson et al. (1999).

Table 2 summarizes the adopted reddening and metallicity values for seven program GCs. The reference photometric data for GCs is listed in the second column of Table 4.

## III. DETERMINATION OF THE DISTANCE MODULUS

Since our essential process is to match the absolute color-magnitude diagram of the subdwarfs with bona-fide main sequences of the well studied GCs, we have selected only unevolved subdwarfs with absolute magnitude fainter than $M_V > 5$ to avoid evolved stars for each GC.

For the calibration process of the GC main-sequence fitting, two relations between absolute magnitude and color for subdwarfs are used: one is obtained from all 23 subdwarfs and the other is from the color-corrected subdwarfs to the corresponding cluster metallicity.

The absolute color-magnitude relation for the 23 subdwarfs has been obtained by second-order least-squares fitting. Uncertainties in the absolute magnitude $\sigma_M$ due to uncertainties in parallax values $\sigma_\pi/\pi$ have been corrected.

$$ M_V = 2.44 + 6.05(B-V) - 0.72(B-V)^2 $$

Using the bona-fide main-sequence section of each cluster at more than eight color points in the color range $0.45 < (B-V) < 0.85$ (except for NGC 6229, whose available main sequence is so short that we use only three points), we have evaluated the absolute magnitudes on these main sequence points of each cluster. Therefore the corresponding visual magnitude for each color point leads to the visual distance modulus. The distance modulus for a given GC is obtained by averaging the values from the eight points and the uncertainty is the deviation from the average. However, the total uncertainty would be larger than 0.11 mag, which is the intrinsic uncertainty inherent in most of the photometric and reddening data used in this study. Figure 1 illustrates the subdwarf sequence.

Theoretical models predict that the location of the subdwarf main sequence is sensitive to metallicity. Therefore, the difference of $\sim 0.1$ dex between the abundances of the subdwarfs and the GCs may lead to an uncertainty of $\pm 0.08$ mag in the derived distance moduli (Carretta et al. 2000). However, the local subdwarfs are not numerous enough for a sufficient number to be selected with a metallicity equal to that of a given globular cluster. The only way is to correct the color of each subdwarf to that of a subdwarf with the same metallicity as that of a globular cluster by using theoretical isochrones. To obtain the absolute location of the subdwarf sequence as a function of metallicity, we fol-

![Fig. 1. The subdwarf sequence.](image-url)
lowed the procedure of Gratton et al. (1997a). We used the semi-empirical relation, \((B-V)M_{V,cc}^{cc}[\text{Fe/H}]\), of Gratton et al. (1997a), which is suitable for subdwarfs with absolute magnitude around \(M_V \approx 6\). Therefore we could only use the subdwarfs with \(5.5 < M_V < 6.5\). So the sample of subdwarfs is reduced to ten. We have corrected every subdwarf to a given metallicity scale on the color-magnitude diagram. We can reduce all subdwarfs to the same \([\text{Fe/H}]\) value of a globular cluster and applied a new calibrating subdwarf sequence for each globular cluster has been obtained.

Relations for the reduced subdwarf template for metallicity values of \([\text{Fe/H}] = -1.46\) for M2 and M3, \(-1.54\) for M10, \(-1.40\) for M12, \(-1.15\) for NGC2808, \(-1.10\) for NGC 6229, and \(-1.43\) for NGC 6752 have been derived as follows:

for M2 and M3 with \([\text{Fe/H}] = -1.46\)

\[
M_V = 2.818 + 5.243(B - V)_o
\]  
(3)

for M10 with \([\text{Fe/H}] = -1.54\)

\[
M_V = 2.863 + 5.245(B - V)_o
\]  
(4)

for M12 with \([\text{Fe/H}] = -1.40\)

\[
M_V = 2.771 + 5.243(B - V)_o
\]  
(5)

for NGC 2808 with \([\text{Fe/H}] = -1.15\)

\[
M_V = 2.606 + 5.245(B - V)_o
\]  
(6)

for NGC 6229 with \([\text{Fe/H}] = -1.10\)

\[
M_V = 2.564 + 5.245(B - V)_o
\]  
(7)

for NGC 6752 with \([\text{Fe/H}] = -1.43\)

\[
M_V = 2.807 + 5.236(B - V)_o
\]  
(8)

Using these equations, we have estimated the distance modulus \((m - M)_{V,cc}\) (suffix, cc means color correction) for each program GC, as previously.

Therefore, we have estimated new distances for seven GCs by fitting both the subdwarf sequence from the local subdwarfs with an abundance range of \(-1.89 < [\text{Fe/H}] < -0.95\) and the one from the color-corrected subdwarfs to the corresponding cluster metallicity.

IV. RESULTS AND DISCUSSION

We have obtained new distance moduli for seven GCs by fitting their main sequences with local sub-dwarf sequences in two ways.

First, we estimated and applied an empirical relation between color and absolute magnitude for the local subdwarfs in the metallicity range \(-1.89 < [\text{Fe/H}] < -0.95\) and determined the distance moduli for points in the color range of the subdwarf template for each cluster, averaging them to obtain the distance modulus for a given cluster.

Second, we obtained an empirical relation for the subdwarf sequence corrected to the corresponding cluster metallicity for each globular cluster, and applied main-sequence fitting.

Since we shifted the colors of subdwarfs using the semi-empirical relation, \((B-V)M_{V,cc}^{cc}[\text{Fe/H}]\), of Gratton et al. (1997a), we defined a mono-metallicity sequence around \(M_V \approx 6\). Applying each relation at several points in a narrower range of color than the first one we have derived the \((m - M)_{V,cc}\) distance modulus for each GC by averaging the distance moduli of these points. We have adopted a distance modulus for each cluster by using the average of the two results weighted according to their errors. Table 3 summarizes the derived distance moduli for each GC. The second and the third columns list results for each method and the last column lists the one we adopted. The errors in the second and the third columns are just standard deviations of fitting points. If we include the effects of the uncertainties in \(E(B-V)\), metallicity, and photometry, overall errors in the distance moduli are larger than 0.11 mag. Due to the steepness of the MS, small errors in the photometry may cause large errors in the derived distance moduli.

Photometric uncertainties for individual clusters may have large errors. For the best cases, a total photometric uncertainty of \(\approx 0.04\) mag (distance modulus) is attainable. If the reddenings adopted for GCs and template subdwarfs carry an uncertainty of \(\approx 0.01\), an uncertainty of \(\approx 0.07\) mag is translated in the derived distance moduli. Therefore, M2, M12, and NGC 2808 have larger uncertainties in their derived distances. Systemetic difference of 0.1 dex in the adopted metallicities of subdwarfs and GCs translates into a corresponding uncertainty of \(\approx 0.08\) mag in the derived distance moduli (Carretta et al. 2000). The uncertainties of the first method are expected to be larger than those of the second method at least by \(\approx 0.08\) mag.

Figures 2 to 8 display the fitting of the fiducial sequences of GCs with those of mono-metallicity subdwarfs. The adopted parameters, namely reddening, metallicity, distance modulus derived from the mono-metallicity color-magnitude relation of the subdwarf sequence, relation used, and adopted cluster distance moduli are shown in each panel of the figure.

Table 4 summarizes the distance moduli found in this
study along with the previously published results for the program GCs. The second column gives references for photometric data used in this study, and the fourth column gives references for published distance moduli.

a) Distance Modulus

i) M2
The apparent distance modulus of M2 is derived in the present work to be $(m-M)_V = 15.52$, slightly larger...
Cudworth & Rauscher (1987) determined the distance moduli to be $15.34 \pm 0.20$ and $15.38 \pm 0.30$, respectively, for M2. Using $V_{HB} = 16.05$, with a reddening of $E(B-V) = 0.06$ (both from Harris 1976) and an adopted absolute value for the horizontal branch $M_Y(HB) = +0.7$ (from RR-Lyrae statistical parallaxes), which is one of the results of Barnes & Howley (1986), they determined $(m-M)_o = 15.16$ with an estimated error of $\pm 0.20$. This corresponds to a distance of $10.76 \pm 1.0$ kpc. and an apparent distance modulus of $15.34$. By using the proper motion and radial velocities dispersions independent of any standard candle assumptions, they determined an astrometric distance modulus of $11.0 \pm 1.7$ kpc, which corresponds to $(m-M)_o = 15.20 \pm 0.30$ and an apparent distance modulus of $15.38$.

Harris (1976) adopted an absolute magnitude of the horizontal branch $M_Y(HB) = 0.6$ and used $E(B-V) = 0.06$ to determine $(m-M)_o = 15.45$. None of the previous distance moduli have been based on main-sequence fitting techniques. All the results of the previous works are from comparison of the level of the HB with the absolute magnitudes of RR-Lyrae stars. Although the absolute magnitude of the HB $M_Y(HB)$ is still a point of discord between different authors, all the published distance moduli of M2 are consistent within the errors. Our new distance modulus $(m-M)_o$ is in good agreement with that obtained by Harris (1976; 1996) and agrees with those of Cudworth & Rauscher (1987) within the errors. Even if we apply recent versions of the luminosity-metallicity relation for RR-Lyrae stars $M_Y(RR) = 0.20(\pm 0.04)[Fe/H] + 0.98(\pm 0.05)$ Fernley et al. (1998), $M_Y(RR) = 0.30[Fe/H] + 0.94$ Sandage (1993), and $E(B-V) = 0.06$ (both from Harris 1976) and an adopted absolute value for the horizontal branch $M_Y(HB) = +0.7$ (from RR-Lyrae statistical parallaxes), which is one of the results of Barnes & Howley (1986), they determined $(m-M)_o = 15.16$ with an estimated error of $\pm 0.20$. This corresponds to a distance of $10.76 \pm 1.0$ kpc. and an apparent distance modulus of $15.34$. By using the proper motion and radial velocities dispersions independent of any standard candle assumptions, they determined an astrometric distance modulus of $11.0 \pm 1.7$ kpc, which corresponds to $(m-M)_o = 15.20 \pm 0.30$ and an apparent distance modulus of $15.38$.

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$M_V(\text{RR}) = 0.17[\text{Fe/H}] + 0.79$ Lee (1990), we obtain $M_V(\text{RR}) = 0.69$, 0.50, and 0.54, respectively, at $[\text{Fe/H}] = -1.46$. Therefore, if we take $V_{\text{HB}} = 16.05$ for M2, we obtain apparent distance moduli $(m-M)_V = 15.36$, 15.55, and 15.51 respectively, which are in good agreement with the new result.

ii) M3

In the present study we have obtained the weighted average distance modulus for M3 as $(m-M)_V = 14.98$. Sandage (1970) calibrated the observed main sequence of M3 with the fiducial main sequence of the local subdwarfs in the $M_V-(B-V)_o$ plane and derived $(m-M)_V = 14.83$ with $E(B-V) = 0.0$, which is smaller than the new value. However, Sandage & Cacciari (1990), using three values for RR-Lyrae absolute luminosities, i.e., $M_V(\text{RR}) = 0.80$, $M_V(\text{RR}) = 0.19[\text{Fe/H}]+1.13$, and $M_V(\text{RR}) = 0.39[\text{Fe/H}]+1.27$, determined intrinsic distance moduli for M3 equal to 14.82, 14.81, and 15.00, respectively, assuming $E(B-V) = 0.01$, $[\text{Fe/H}] = -1.66$ and the level of the ZAHB $= 15.62$ $mag$. If we use the same relations with $[\text{Fe/H}] = -1.46$ and $V_{\text{HB}} = 15.65$ $mag$, we will have apparent distance moduli of 14.85, 14.80, and 14.95, respectively.

If we apply the different luminosity-metallicity relations of Fernley et al. (1998), Sandage (1993), and Lee (1990) we obtain distance moduli $(m-M)_V = 14.96$, 15.15, and 15.10, respectively. The new distance modulus agrees well with the average of all the previous values.

iii) M10

The new distance modulus for M10 was obtained in the present study to be $(m-M)_V = 14.23$. Harris, Racine, and Deroux (1976) studied M10 thoroughly based on photographic data and estimated a reddening of $E(B-V) = 0.26 \pm 0.02$ by comparing their CMD to that of M13. They estimated the distance to M10 by two methods, one of which assumed an absolute magnitude of $M_V = -1.46$, and the other involved fitting the M10 to the M13 fiducial sequence. Both methods yielded values of the distance modulus close to $(m-M)_V = 14.10$. Harris (1976; 1996) estimated distance moduli for M10 equal to 14.05 and 14.08, respectively. A larger distance modulus, $(m-M)_V = 14.40 \pm 0.38$ for M10, has been obtained by Hurley et al. (1989), using the main-sequence fitting technique to the subdwarf sample of Lutz, Hanson, and van Altena (1987). The present distance modulus agrees with the value of Hurley et al. (1989) within the errors. Determination of the distance modulus estimated by using the level of the HB seems to be not so fruitful for the case of GC M10 because the level of the HB is subject to uncertainty due to the scatter of the HB stars and the lack of RR-Lyrae stars in M10. However, the new distance modulus of 14.23 is in excellent agreement with the adopted distance modulus from the deep HST CMD for M10 by Pietto & Zoccali (1999).

iv) M12

Our distance modulus for M12 is $(m-M)_V = 14.03$, which is derived from the weighted average of $(m-M)_V = 14.16 \pm 0.08$ and $(m-M)_V = 14.02 \pm 0.02$. Sato et al. (1989) determined a distance modulus equal to $(m-M)_V = 14.25 \pm 0.20$ for M12 by the main-sequence fitting method, a value which is larger than ours. Most of other distance moduli have been obtained based on the level of the HB of M12. Racine (1971) derived a distance modulus equal to 14.30 $\pm 0.20$ which was based on his photographic CMD with an estimate of $V_{\text{HB}} = 14.9$ assuming $M_V(\text{HB}) = 0.6$. On the other hand, Harris (1996) estimated a distance modulus of $(m-M)_V = 14.02$ with $M_V(\text{RR}) = 0.58$ and $V_{\text{HB}} = 14.60$. Our result agrees well with that of Harris (1996).

v) NGC 2808

We derived an average weighted distance modulus for NGC 2808 equal to $(m-M)_V = 15.56$. NGC 2808 is one of the most interesting GCs since its very high concentration is likely to enhance any effect due to the dynamical evolution of binary or multiple star systems and is suspected to have a binary sequence along the main sequence. In addition, NGC 2808 displays an unusual horizontal branch morphology due to the presence of two well separated groups of stars with red HB and very blue HB. Harris (1976, 1996) estimated distance moduli of $(m-M)_V = 15.52$ and 15.56, respectively, which agree well with the present one. Buonanno et al. (1984) obtained a distance modulus equal to $(m-M)_V = 15.52 \pm 0.20$ by fitting the main sequence of NGC 2808 to the VandenBerg (1983) theoretical isochrones. Although NGC 2808 has an unusual CMD which could lead to diverse distance modulus estimates, all the results of studies including the present one are in good agreement. However, Bedin et al. (2000) estimated a slightly lower reddening value of $E(B-V) = 0.19 \pm 0.01$, and a larger distance modulus of $(m-M)_V = 15.74 \pm 0.10$ from RGB stars based on the method derived by Saviane et al. (2000).

vi) NGC 6229

Although the available part of the main-sequence for NGC 6229 is very short, Fig. 7 shows a good match to the main sequence of this cluster with the subdwarf fiducial line. A mean weighted distance modulus, $(m-M)_V = 17.34$, has been obtained for NGC 6229. Assuming $M_V(\text{HB}) = 0.6$ for the horizontal branch in all clusters, Harris & Racine (1979) obtained a distance modulus for this cluster equal to 17.50. Fusi et
al. (1993) who applied the apparent magnitude of HB stars \( V_{HB} \) and \( M_V^{HB} \) derived from the equation \( M_V(\text{RR}) = 0.17 \) \([\text{Fe/H}] + 0.79\) of Lee (1990), obtained an apparent distance modulus equal to 17.64 for NGC 6229 adopting \([\text{Fe/H}] = -1.40\) and \( V_{HB} = 18.20\). However if we apply to this equation a metallicity value \([\text{Fe/H}] = -1.1\), we will get a smaller distance modulus of 17.60. Although the published distance modulus for NGC 6229 varies from 17.20 to 17.64, the present distance modulus is very close to the one adopted by Borissova et al. (1999) and close to the average of all the published values.

**vii) NGC 6752**

There are many studies on NGC 6752. It has recent distance determinations based on Hipparcos parallaxes, carried out by Reid (1997, 1998), Gratton et al. (1998), Chaboyer et al. (1998) and Carretta et al. (2000). Gratton et al. (1997a) and Carretta et al. (2000) obtained the same distance modulus. They directly compared the apparent magnitudes of the cluster main sequence at a given color and the absolute magnitude of the template main-sequence at that color. Gratton et al. (1998) used nine subdwarfs which are fainter than \( M_V^o > 5.5 \). They fit the main sequence in the metallicity range \(-1.8 < [\text{Fe/H}] < -1.0\). They obtained a distance modulus of \((m-M)_V = 13.34 \pm 0.07\), corresponding to a distance \(4.65 \) kpc, while Carretta et al. (2000) obtained an identical value adopting the same technique based on 18 subdwarfs. They used all stars with \( 5 < M_V < 8 \) and considered only stars whose metal abundance was within 0.5 dex of that of NGC 6752. Using the main-sequence fitting technique, Reid (1997, 1998) determined distance moduli equal to \( 13.23 \pm 0.10 \) and \( 13.28 \pm 0.15 \), respectively, for NGC 6229. Penny & Dickence (1986) have estimated a distance modulus of \( 13.17 \pm 0.17 \) from local subdwarfs with accurate ground-based trigonometric parallaxes. Renzini et al. (1996) used the white dwarf cooling sequence and determined a value of \( (m-M)_V = 13.05 \pm 0.12 \) assuming a reddening value of \( E(B-V) = 0.04 \pm 0.02 \). Chaboyer et al. (1998) showed the existence of a 1 \( \sigma \) error in the distance modulus caused by possible metallicity errors for NGC 6752 using the main-sequence fitting technique with subdwarfs. They obtained a distance modulus of \( (m-M)_V = 13.32 \pm 0.11 \) with the metallicity, reddening, and fitting errors. In the present study the derived distance modulus is \( <(m-M)_V > = 13.29 \) obtained from the weighted average of two distance moduli. One of them is \( <(m-M)_V > = 13.37 \pm 0.04 \), obtained by calibrating eleven color points in the range 0.45 \((B-V)_o > 0.72\), while the other is \( (m-M)_{Vcc} = 13.26 \pm 0.03 \) derived using the color-corrected subdwarfs in the color range 0.52 \((B-V)_o < 0.69\) as in Gratton et al. (1997a). Our new distance modulus for this cluster is in good agreement with all the previous results which are based on post-Hipparcos parallax values. Especially it is in excellent agreement with those of Reid (1997; 1998) and Chaboyer et al. (1998), while it also agrees with those of Gratton et al. (1998) and Carretta et al. (2000) within the errors. These are all larger than the value obtained before Hipparcos data became available.

**V. SUMMARY**

A sample of 23 local subdwarfs with precise parallaxes provided by the Hipparcos satellite have been used to determine the distance moduli of seven GCs by the main-sequence fitting technique.

Subdwarfs in the abundance range \(-1.89 < [\text{Fe/H}] < -0.95\) were selected for seven GCs in the intermediate-metallicity range of \(-1.54 < [\text{Fe/H}] < -1.10\).

The weighted average distance modulus for each GC has been estimated from two distance moduli for each cluster based on the principal main-sequence fitting technique. They are as follows: \((m-M)_V\) is obtained from the fitting of the globular cluster main sequence to the weighted second-order empirical relation derived from 23 local subdwarfs, while \((m-M)_{Vcc}\) is estimated by fitting of the main sequences of globular clusters to the relation derived by the color-corrected subdwarfs to the corresponding metallicity of GC. For both the local subdwarfs and the GCs we have adopted the recent abundance scale, as well as abundances which are based on high-dispersion spectroscopic analysis.

We have estimated distance moduli of 15.52 for M2, 14.98 for M3, 14.23 for M10, 14.03 for M12, 15.56 for NGC 2808, 17.34 for NGC 6229, and 13.29 for NGC 6752. The new distance moduli show good agreement with those in the literature. All new results are within less than \( \pm 0.16 \) mag of the average values of previous results. Especially it is noted that suitable agreement was found with values obtained based on the Hipparcos parallaxes determined by Reid (1997; 1998), Chaboyer et al. (1998), Gratton et al. (1998), and Carretta et al. (2000) in the case of NGC 6752. However, not all new distances are larger than those of previous works.

Table 3 reveals that the estimated distance modulus based on color-corrected subdwarfs, \((m-M)_{Vcc}\) for each GC, differs from that obtained by the uncorrected subdwarfs template, \((m-M)_V\). The amount of the difference is at most \( \pm 0.14 \). This suggests that better distance moduli could be estimated if CMDs less affected by metallicity are used in the main-sequence fitting process. The study of such CMDs for program GCs is currently in progress.
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