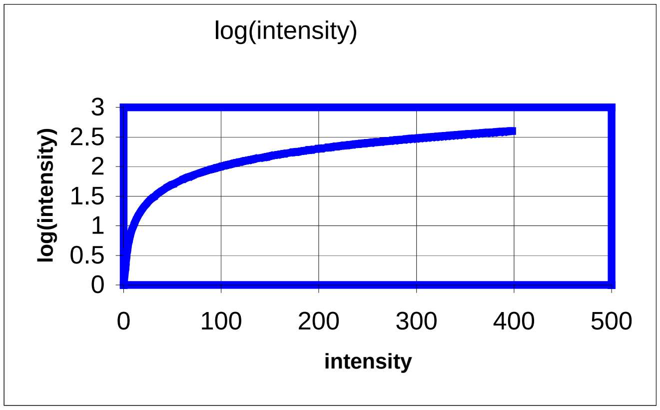
Magnitude Scale and Distance Measurements

1) Magnitude scale defined

Hipparchus, that ancient Greek astronomer, devised a scale by which to compare the brightness of stars he observed in the night sky. Ofcourse, he had no telescope back then, so he based it entirely on the way the eye distinguishes brightness levels of light.

Your eyes see LOGARITHMICALLY. That means, that if the intensity of a light source increases linearly, your eyes increase their response in proportion to the log of the light output from the source. This is a Good Thing evolutionarily, as it allows our eyes to detect very subtle changes at mid levels of light - such as hunting animals in the forrest, but not at very bright levels - like looking at the Sun, where we don't want to look anyway.



This plot shows a logarithmic response curve.

Remember, the smaller the magnitude the brighter the star. This is because Hypparchus called the brightest stars "first class", and dimmer stars were second, third, fourth, fifth class stars.

He set his magnitude scale as follows: if the apparent magnitude of star 1 is m_1 and the apparent magnitude of star 2 is m_2 , then if ($m_1 - m_2$) = 5, then I_2 (the relative intensity of star 2) is 100 times I_1 (the relative intensity of star 1).

$$m_1 - m_2 = 5 \Rightarrow \frac{I_2}{I_1} = \frac{100}{1} \qquad (1)$$

To develop a scale by which we can assign apparent magnitudes to any star based on its intensity relative to all other stars, we devise a scale that puts 5 equal divisions between 1 and 100, and is extendable without limit on either side. The appropriate scale for this is a logarithmic scale.

Recall: The logarithm of any number, in "base 10" is a power of 10 which, when you raise 10 to that power, gives you the number. For example:

10^2 = 100, so the log of 100 is 2.
10^1 = 10, so the log of 10 is 1.
10^0 = 1, so the log of 1 is 0.

You can also have fractional powers:
$10^{2.5}$ = 316.23

From the graph: If I_1 = 1 and I_2 = 100, the log of 1 is 0 and the log of 100 = 2. Relating this back to apparent magnitudes, that means that m_2 = 0 and m_1 = 5. So we need 5 divisions between 0 and 2; that is, 5 divisions between the *log of 1* and the *log of 100*.

Since 5 / 2 = 0.4, that means for each decrease in magnitude of 1, we have an increase of $10^{0.4}$ in relative intensity.

To visualize this, complete the following table. We use the notation Δm ("delta m") = $m_1 - m_2$.

Δm	$I_2/I_1 = 10^{(.4\Delta m)}$	Δm	$I_2/I_1 = 10^{(.4\Delta m)}$
0	$10^{(.4 * 0)} = 10^0 = 1$	6	
1		7	
2		8	
3		9	
4		10	
5	$10^{(.4 * 5)} = 10^2 = 100$	20	$10^{(.4 * 20)} = 10^8$

Δm is the difference in apparent magnitudes, and I_2/I_1 is the ratio of the intensities.

If $\Delta m = 0$, then both stars are equally bright.

If $\Delta m = 5$, then $star_2$ is 100 x brighter than $star_1$.

If $\Delta m = 20$, then how many times brigther is $star_2$ than $star_1$?

Now to develop the equation that allows you to calculate the relative intensities of any two stars, given their individual apparent magnitudes:

If $m_1 = 5$ and $m_2 = 0$ we know from equation (1) that $I_2 = 100$ and $I_1 = 1$. Recall that

$$log(A) - log(B) = log\left(\frac{A}{B}\right). \qquad (2)$$

So, $log(100) - log(1) = log\left(\frac{100}{1}\right) = \frac{2}{1} = 2$.

But we also know that $m_1 - m_2 = 5 - 0 = 5$.

We want to find the equation that relates these two relationships: $5 = x \times 2$. x = 2.5.

Generalizing this, we have: $m_1 - m_2 = 2.5 log\left(\frac{I_2}{I_1}\right) \qquad (3)$

Plug in our example and see if this equation works: $5 - 0 = 2.5 log\left(\frac{100}{1}\right)$

$$5 = 2.5 \times 2 = 5$$

Yes! It works!

Now we can generalize: $m_1 - m_2 = 2.5 log\left(\frac{I_2}{I_1}\right)$ (4)

and this give us a way to find the relative intensities of any two stars, based on their apparent magnitudes. Try a few examples:

1. The apparent magnitude of Spica is +0.98, and the apparent magnitude of Sirius A is -1.44. How many times brighter is Sirius A than Spica?

2. The apparent magnitude of Venus at its brightest is -4. How much brighter in the sky is Venus than Sirius A?

3. The apparent magnitude of the Sun is -26.7. How much brighter is the Sun than Venus at its brightest?

2) Finding the distance to a star from its absolute magnitude and apparent magnitude:

The visual magnitude you observe for a star depends both on its intrinsic luminosity and its distance. In order to bring all stars to the same "reference distance" so that we can really compare their magnitudes, we define the notion of ***absolute magnitude, M*** as the magnitude a star would have if it were at a standard distance of 10 parsecs from us. (1 parsec = 3.26 light years = 206,265 A.U.)

Recall that the flux of staright that you receive at a distance r from a star is proportional to $\frac{1}{r^2}$.

Suppose a star with apparent magnitude m has an absolute magnitude M = 0. Let this be a reference star, with intensity I that we observe at our distance r. Let I_0 be the intensity we would observe if we were at r = 10 pc. Then, from our equation (4) in part 1,

$$m - M = 5 - 0 = 5 = 2.5 log\left(\frac{I_0}{I}\right) .$$

The flux of starlight we receive from the star at distance r, f is proportional to $1/r^2$, and the flux of starlight we *would* receive from the star *if* we were at a distance of 10 pc is proportional to $1/10^2$. We substitute these values for f and f_0 into our equation for I and I_0:

$$m - M = 2.5 log\left(\frac{1/10^2}{1/r^2}\right) = 2.5 log\left[\frac{10^{-2}}{r^{-2}}\right] = 2.5\left(-2 - \left(-2 log\, r\right)\right) = -5 + 5 log(r)$$

$$m - M + 5 = 5 log(r)$$

$$\frac{m - M + 5}{5} = log(r) \Rightarrow\Rightarrow\Rightarrow 10^{\left[\frac{m-M+5}{5}\right]} = r$$

And this allows us to find the distance for any star whose apparent and absolute magnitudes are known, or the absolute magnitude for any star whose apparent magnitude and distance are known!

Distance formula: $\boxed{r = 10^{\left[\frac{m-M+5}{5}\right]}}$ You will need to apply this later on, when we determine the distance to a star cluster by curve-fitting on the H-R Diagram.

1) The absolute magnitude of the Sun is +4.83, of Sirius A is + 1.45, and of Spica is .119. Using the apparent magnitudes given in part 1 of this review, find the distances to these stars using the distance formula.

Other ways of determining cosmic distances are by

- stellar parallax,
- proper motions of stars,
- the period-luminosity relation for Cepheid variables,
- light curves of type Ia supernovae,
- and redshifts.

One of the key pieces of evidence for the assertion that the Universe is now beginning to accelerate in its expansion is that the distances to high-redshift supernovae of Type Ia that are obtained from their redshifts and from their distance moduli DON'T MATCH!