

## High $z$ Sne and the Accelerating Universe

### 1. Interpreting the Hubble Diagram:

In Hubble's original diagram, he graphed recession velocity that he measured for about twenty nearby galaxies from their redshifts as a function of their distance from us, which he measured from their magnitudes. He "managed" to fit a straight line through these data, and came up with the interpretation that the farther away a galaxy was from us, the faster it was receding.

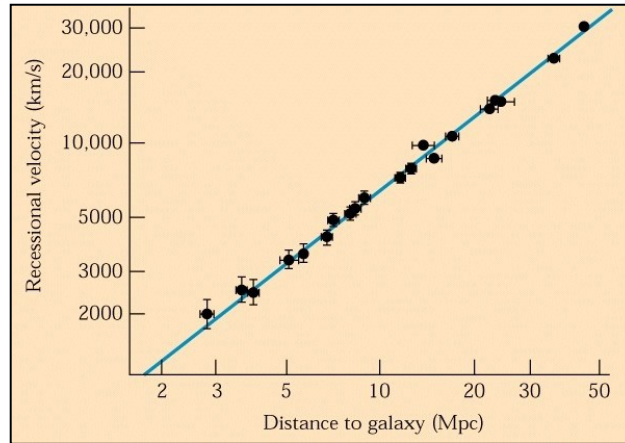


Figure 1. Hubble's Original Data Set, copied from somewhere on the Internet

**Graphing Hubble's Original Data:** Before graphing the supernova data, you get to mess around with Hubble's original data.

1) Open the file "**Hubble\_data.xls**" from link on your CD, which you will find in the directory "Hiz-Sne". This has Hubble's original data.

2) Open the file "**Hubble-data-graph.xls**" from the same directory.

3) Copy the data from the first spreadsheet into columns A and B of the second spreadsheet, to see the graph. Follow the instructions in the yellow box to add a trend line and display the equation.

\* *What can you deduce was Hubble's original estimate of his constant,  $H_0$  from this graph?*

**Interpreting what the diagram means:** The y – axis gives velocity, and using Hubble's initial assumption that the recession velocity was directly proportional to the galaxy's redshift,

$$\frac{\Delta\lambda}{\lambda} = z = \frac{v}{c} \quad \text{where } \Delta\lambda = \text{change in wavelength of the spectral line in the absorption spectrum}$$

you observe from the moving galaxy compared to the wavelength you would measure in its rest frame,  $\lambda$  = the wavelength of the same spectral line you would observe in the rest frame of the galaxy;  $z$  = red shift,  $v$  = velocity of recession, and  $c$  = speed of light.

Rearranging, we get  $v = zc$ .

Using Hubble's equation,  $v(r) = H_0 r$  and making the substitution that  $v = z$  we get

$$cz = H_0 r \text{ which leads to the linear approximation } r = \left( \frac{c}{H_0} \right) z, \text{ often written as } d = \left( \frac{c}{H_0} \right) z,$$

using  $d$  instead of  $r$  for distance.

Measuring distances in light years, however, gives us the age of a galaxy, so the farther away a galaxy is, the older it is, too.

So: graphing distance on the y axis and red shift on the x axis gives an idea of how much the universe has stretched over cosmic distances and time scales! This is nicely explained in the following figure from Saul Perlmutter, of the Supernova Cosmology Project at Lawrence Berkeley Lab:

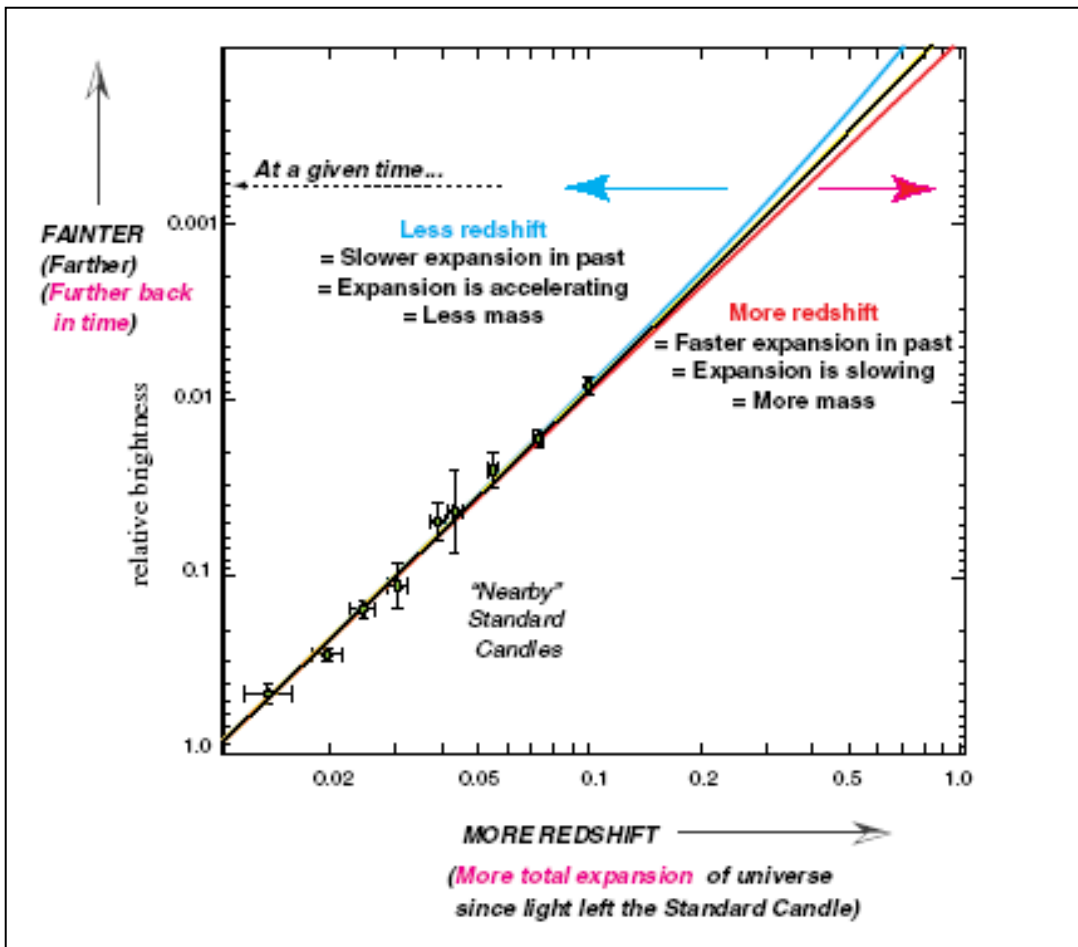


Figure 2. From Saul Perlmutter, Supernova Cosmology Project, Lawrence Berkeley Lab

On this diagram, instead of graphing distance on the y-axis, he has graphed **relative brightness**, or magnitude. For a "standard candle" – that is, a light source for which we know the intrinsic luminosity – its relative brightness gives us an idea of how far away it is.

Red shift is plotted on the x-axis, and should be thought of as a measure of the amount of stretching that the light waves have undergone since the time the light left its source. For a uniformly stretching universe, expanding with a constant velocity, the distance and the amount of stretch are directly proportional.

So: vertical axis = distance  $\Rightarrow$  age.  
horizontal axis = redshift  $\Rightarrow$  stretching of the universe.

*And that is the crux of understanding the Hubble diagram, because any deviations from this linear relationship back in time are caused by the cosmological parameters.*

Suppose you find a distant supernova; you are looking back in the history of the Universe. Suppose you find that there is MORE redshift than predicted from the Hubble diagram for that supernova; that would tell you that the expansion rate was FASTER back in time, and has been SLOWING DOWN. That would imply a higher mass density, which might lead you to wonder if the Universe will end in a "big crunch."

Suppose, on the other hand, that you found that your supernova was FARTHER away than you would expect from its redshift; that would tell you that the expansion rate was SLOWER back in time, and has been SPEEDING UP since then. That would imply a lower mass density, which might lead you to wonder if the Universe will expand forever.

That is, IF there is no cosmological constant! Taken together with measurements of the geometry of the Universe from the CMB data, which indicate a flat geometry, we have to conclude that there is a cosmological constant, although we don't yet know its nature!

## **2) Graphing the high redshift supernova data – revising the Hubble diagram!**

When we try to understand the cosmological constants from graphing supernovae at high red shifts, there are two parts to the task:

- 1) Getting accurate data! The data are accurate measurements of the distance moduli for high redshift supernovae and their red shifts. (Distance modulus compares the absolute brightness to the observed brightness, for a known source, and gives a measure of the distance to the source without making any assumptions about how to measure the distance.)
- 2) Fitting the model to the data for the "correct" interpretation.

### **1) Understanding the measurements:**

$\mu_0$  (called "mu-naught" or "mu-0" in the spreadsheet) is the *distance modulus*

$\mu = m - M$  = the difference between apparent and absolute magnitude of a source.

$\mu = 5 \log(d) + 15$  where d = distance to source

$\mu_0$  is measured directly from the data: magnitudes are accurately measured, and because people feel that they understand supernova theory quite well, they can calculate the absolute magnitude from the light curve, and thereby get a very good determination of  $\mu_0$ .

$z$  is the redshift, accurately measured from the spectrum of the supernovae.

**Graphing the High-z Sne data set:** (Sne = "supernovae")

- 1) Open the file "Reiss-Hiz-sne-data.xls" from your CD, from the Hiz-Sne directory. These data are 187 very accurately determined distance moduli and red shifts for type Ia supernovae. (Read the paper "Reiss-et-al-Hiz-SneIa.pdf" to see just how carefully they collected and reduced their data!) ("Reiss" is Adam Reiss, famous for proposing that the universe has undergone a "cosmic jerk." Note that in 2011 he was one of three recipients of the Nobel Prize in Physics for this discovery!)
- 2) Open the file "Hiz-Sne-data-graph.xls" from the same directory. There are four worksheets in this file. Make sure that sheet #1 is on top, the sheet labeled "1) mu vs z." You should see a blank graph.
- 3) Copy the data from the Reiss data spread sheet into this sheet, to see the plot.

**Understanding the models:**

It is not possible to measure the distance directly when we are looking that far out in space and back in time. So, we have to MODEL the expansion of the universe and calculate a distance based on our assumptions, and FIT the model to the data – and then we can make some predictions and intelligent statements about the expansion history of the Universe, and the cosmological parameters of density, expansion rate, etc.

For the simplest model, we assume a linear expansion rate.

$$d = \left( \frac{c}{H_0} \right) \mu \quad \text{and} \quad \mu = 5 \log(d) + 15 \quad \text{giving} \quad \mu = 5 \log \left( \frac{cz}{H_0} \right) + 15$$

as the relationship between the measurements of redshift and distance modulus.

**4) Next go to the second sheet, labeled "2) fitting a simple model."** You should see two columns that have "#DIV0!" warnings. That is just because there is nothing in the cell J2, under the label "H<sub>0</sub>" yet.

*Type different values for H<sub>0</sub> in units of km/sec/Mpc .*

**\*\* What value of H<sub>0</sub> fits the data best?**

**\*\* How does this compare with the "current best estimate" of 72 km/sec/Mpc?**

**5) Taking into account the expected change in the expansion rate over time.** In the last century, it was assumed that the expansion rate of the universe *should* be slowing down, due to the gravitational attraction of the matter within it. It was not known whether this slowing down would lead to an asymptotic approach to zero expansion velocity over an infinitely long time, or whether at some point the Universe would reach zero velocity, turn around, and start to accelerate the other way. The "deceleration parameter", **q<sub>0</sub>** was hypothesized as the parameter to search for that would tell us the answer.

It was *expected* that **q<sub>0</sub>** should be positive, indicating a slowing down of the expansion rate.

Now go to the third sheet, labeled "3) fitting a q-0 model". For a universe with a change in the expansion rate, we model this simply as

$$d = \left( \frac{cz}{H_0} \right) \left[ 1 + \frac{(1-q_0)z}{2} \right]$$
 and, again putting this into our equation for  $\mu_0$  we have

$$\mu = 5 \log d + 15 = 5 \log \left[ \frac{c}{H_0} z + \frac{c}{H_0} \frac{(1-q_0)}{2} z^2 \right] + 15$$

Try entering different values of  $q_0$  and  $H_0$  in this sheet in cells C2 and D2, respectively. Remember, a negative value for  $q_0$  means ACCELERATION, and positive means deceleration.

**\*\* What are the best combinations that fit the data for  $q_0$  and  $H_0$  fit the data best?**

Finally, we'll take a look at one of the models from Reiss, et al: the  $\lambda$  (Lambda) – "w" model.

$\lambda$ , or "Lambda," refers to Einstein's "cosmological constant", the so-called dark energy that we have NO idea what it is, but it seems to make up about 70% of the total mass-energy balance of the Universe! "w" is the "equation of state parameter" of the Universe. w is assumed to be equal to the ratio of pressure to density for any given component of the Universe: matter, dark energy, or radiation.

To really understand what "w" means, you need to get down and dirty with the equations of General Relativity, but in a nutshell, here are the basics:

\* We treat the Universe as homogeneous and isotropic on large scales, large enough to get away from local structures and into the Hubble flow. High redshift supernovae are certainly "way out there" in the Hubble flow.

\* Then we consider Einstein's model that the geometry of spacetime is determined by the total matter AND energy, including dark energy or  $\lambda$ , content of the universe. Pressure, p, refers to the net force at any point in spacetime, and is described by a "stress-energy tensor." Density,  $\rho$ , is the density of any component of the universe.

\* Next we treat the various components of the matter and energy as perfect fluids. We treat matter as "pressure-less dust" – a set of non-relativistic, non-interacting particles. . (This is somewhat like the "ideal gas" of molecules from kinetic gas theory, that don't collide or interact.) This includes all gravitating matter – baryonic and "dark matter."

We consider also radiation, and "vacuum energy" or  $\lambda$ . By definition, we have p and  $\rho$  for each component, and we can define w for each component. Let "a" be the "scale factor" – relative radius of the Universe at any time. We have:

$$\text{for matter: } p = 0, \quad \rho \sim a^{-3}, \quad \text{and } w = p/\rho = 0$$

for radiation:  $p = \rho/3$  and  $\rho \sim a^{-4}$  (because the wavelengths of light stretch as the universe expands)

for vacuum, we assume the model  $p = -\rho$  and  $\rho = \text{CONSTANT}$ .

Whoa... wait a minute! For normal matter, the density decreases as the volume increases, if the mass is constant. But for the dark energy, we assume that the DENSITY IS CONSTANT, therefore the amount of dark energy INCREASES as the volume increases! (And that is totally counter-intuitive, but that is what the data are telling us!)

**Now, go to the final sheet, labeled "4) fitting a lambda-w model".**

Taking the vacuum energy to be a cosmological constant (and not messing around with a "quintessence field"), we have the following equation for the distance to a high redshift source:

$$d = \left( \frac{c}{H_0} \right) \left( 1 + z \int_0^z dz \left[ \Omega_m (1+z)^3 + (1 - \Omega_m)(1+z)^{3(1+w)} \right]^{-1/2} \right)$$

where  $\Omega_m$  is the density parameter for all the matter content of the Universe,  $\Omega_m = \rho_m/\rho_c$  where  $\rho_c$  is the critical density of the Universe necessary for a "flat geometry."

The value of  $1 - \Omega_m$  then is  $\Omega_\Lambda$ , the density parameter for the vacuum, or dark, energy.

$w$  is the equation of state parameter, and  $z$  is again the red shift.

The integral is handled in Excel with an "add in" called "Xnumbers" which you have on your CD, and is installed on the computers in the lab for this workshop. (You will need to install it on your computers back home in order to run this model. OR – you can enter the formula into Mathematica, or any other software you like.)

***Try various combinations of  $\Omega_m$ ,  $w$ , and  $H_0$  that fit the data. Note that typing in the values for "Omega-m" and "w" in the spread sheet gives you the parameters for the equation. Xnumbers cannot handle a cell reference, so you must type in the numeric values into the equation in cell H2, where the "red numbers" are.***

***\*\* What values of  $\Omega_m$ ,  $w$ , and  $H_0$  best fit the data??***

***\*\* What do you notice about all the models at low redshifts??***

The conclusions from these data that have been made are that the Universe is accelerating, and that this acceleration began around a redshift of 0.5. But, as you can see from your modeling experience, there are OTHER estimates of density,  $w$ , and  $H_0$  that can fit these data, and in fact ALL the models seem to fit the data at low redshifts. Only the high redshift data

are sensitive to the cosmological parameters. Fortunately, there are other independent ways to measure the age of the Universe and the geometry.

The age of the Universe can be measured from the turnoff points on the H-R diagram for globular clusters – the oldest stars we can measure. This puts a lower limit on  $H_0$ , which is itself an upper limit on the age of the Universe.

The geometry of the Universe can be gauged by accurate measurements of the Cosmic Microwave Background (CMB). By mapping the small-scale fluctuations in temperature of the Universe, and calculating the two-dimensional power spectrum of the distribution of these tiny temperature variations across the whole sky, we can get an independent check on the density and  $H_0$ .

In the next two Lambda Labs you will see how these measurements are made, and how they all converge and support the current model:

about 30% matter; ~ 4% baryonic (“normal” ) matter and ~ 26% dark matter  
about 70% dark energy  
 $H_0 = 70$  km/sec/Mpc (or so)  
and the age of the Universe at approximately 13.7 billion years.  
and a distribution of temperature anisotropies that describes a spatially flat geometry for the Universe!